

# Imperfect price discrimination, market structure, and efficiency

Qihong Liu *Department of Economics, University of Oklahoma*  
Konstantinos Serfes *Department of Economics and International Business, Drexel University*

*Abstract.* We introduce a flexible third-degree price discrimination framework by modeling the information firms possess about consumers' locations (preferences) on the Salop circle as a partition. Higher information quality is translated into a partition refinement. In the limit, we obtain the perfect price discrimination paradigm. We show that the free-entry equilibrium number of firms exhibits a U-shape as a function of the quality of information. This implies that *imperfect* price discrimination generates the most efficient free-entry outcome. JEL classification: D43, L11, L43

*Discrimination imparfaite par les prix, structure de marché et efficacité.* Les auteurs produisent un cadre d'analyse flexible de la discrimination par les prix au troisième degré en modélisant l'information que les firmes possèdent à propos de la localisation (préférences) comme base de découpage dans un cercle de Salop. Une plus grande qualité d'information se traduit par un découpage plus fin. A la limite, on obtient le paradigme parfait de discrimination par les prix. On montre que le nombre de firmes en équilibre quand l'entrée est libre est une fonction en forme de U dans son rapport avec la qualité de l'information. Cela implique que la discrimination imparfaite par les prix engendre le résultat le plus efficace quand l'entrée est libre.

## 1. Introduction

Social efficiency in monopolistically competitive markets has been traditionally one of the central issues in industrial organization. It is well known that in

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models of spatial competition, free entry usually leads to an excessive number of firms (brands) (e.g., Salop 1979). The reason is that an entrant's profits are excessive relative to its marginal contribution to social welfare. On the other hand, *perfect* price discrimination *mitigates* this inefficiency (e.g., Bhaskar and To 2004). When firms can target individual consumers perfectly, competition intensifies and profits become less excessive, resulting in lower entry (but still excessive from the social perspective).

Our aim in this paper is to investigate the effect of *imperfect* price discrimination (of a *varying* degree of imperfection) on the number of entrants (product variety).<sup>1</sup> We formulate a two-stage game, based on Salop's circular model. In stage 1, firms make their entry decisions. In stage 2, firms are endowed with customer-specific information of a certain quality and set their prices. This information, modelled as a partition of the characteristics space, helps firms to segment the consumers into different groups and charge each group a different price.<sup>2</sup> Information of a higher quality is translated into a refinement of the partition. In the limit, we obtain the case of perfect discrimination.<sup>3</sup>

The motivation for our modelling framework is twofold. First, most of the literature on product variety based on location models builds upon one of the following two *extreme* assumptions. Firms either charge uniform prices (e.g., Salop), or are able to discriminate perfectly (Bhaskar and To 2004; Lederer and Hurter 1986; MacLeod, Norman, and Thisse 1988). Our framework is more flexible and can generate the perfect discrimination paradigm as its limit point. More important, we also obtain a comprehensive picture of firms' entry decisions when information about consumers is *less* than perfect. Second, our formulation of consumer-specific information comes closer to the real world practice, where firms use their databases to segment consumers imperfectly into different groups. The recent rapid growth of the Internet, and information technology (IT) in general, has allowed retailers and marketing firms to collect a vast amount of consumer data. These data are analysed with the aid of sophisticated software tools and techniques with the purpose of assembling a detailed picture of consumers' preferences and segmenting consumers into groups. As the IT advances, the quantity and quality of customer data increase, and the resulting segmentation becomes increasingly refined.

The most interesting result we obtain is that the equilibrium number of firms exhibits a U-shape pattern as a function of the information quality. This

1 For a comprehensive survey on the price discrimination literature see Stole (2003).

2 There are many examples of firms that segment consumers into groups based on observable characteristics and offer a different price to each group. Dell, for example, follows this practice. According to the 8 June 2001 *Wall Street Journal*: 'One day recently, the Dell Latitude L400 ultralight laptop was listed at \$2,307 on the company's Web page catering to small businesses. On the Web page for sales to health-care companies, the same machine was listed at \$2,228, or 3% less. For state and local governments, it was priced at \$2,072.04, or 10% less than the price for small businesses.'

3 In Liu and Serfes (2004) we use the same kind of partition in a Hotelling duopoly model to study the firms' incentives to acquire information, as well as the evolution of equilibrium profits and welfare as a function of the information quality.

implies that although perfect price discrimination mitigates the social inefficiency due to excessive entry relative to the non-discriminatory outcome (e.g., Bhaskar and To 2004), it *does not* minimize this difference. Imperfect price discrimination based on ‘moderate’ quality of consumer information yields the most efficient non-cooperative outcome. Since consumer and social welfare in this zero-profit framework are aligned, policy implications based on this model are clear-cut. Regulatory authorities should create an environment that allows and fosters *only a limited* (but strictly positive) collection and application of consumer information.

The rest of the paper is organized as follows. In section 2 we formulate the model, and in section 3 we present the analysis along with the main results. We conclude in section 4. The proof of proposition 1 can be found in the appendix.

## 2. The model

There is a continuum of consumers of measure one uniformly distributed on the unit circle (Salop). Each consumer is identified by his location on the circle, which corresponds to his most preferred brand and buys one unit of a product, or does not buy at all. We assume that each consumer derives a benefit equal to  $V$  if he buys from any of the firms. Suppose there are already  $n$  firms, equidistantly located from each other in the market, which implies that the distance between any two adjacent firms is  $1/n$ . We assume that information about the location (preferences) of all consumers is available to all that enter the market. The information allows firms to segment the consumers into different groups and is modelled as a partition of the intervals between any two neighbouring firms into  $2^k$  segments, with  $k = 0, 1, 2, \dots$ . Hence,  $k$  will parameterize the information quality, with higher  $k$ 's being associated with higher information precision (information refinement). In practice, firms can obtain such information from a number of different sources, such as (i) directly through repeated past transactions with the customers, (ii) via a telemarketing or direct-mail survey, (iii) from credit card reports, or (iv) from a marketing firm (see Shaffer and Zhang 2000, 2002, for a more extensive discussion and more references on this issue).<sup>4</sup>

Owing to symmetry, all intervals exhibit the same partition and it suffices to focus only on one interval, that is, the  $[0, 1/n]$  interval, with firm 1 located at 0 and firm 2 located at  $1/n$ . The  $m$ th segment is denoted by  $[(m-1)/2^k n, m/2^k n]$ , where  $m = 1, \dots, 2^k$  (see figure 1, where we depict the partition of the characteristics space between the above mentioned adjacent firms). Let  $p_{im}$

4 One example is the *Abacus Catalog Alliance*, a database that contains transactional data with detailed information on consumer and business-to-business purchasing and spending behaviour. It is a blind alliance of 1,800 merchants offering shared data representing over 90 million households and is the largest proprietary database of consumer transactions used for target marketing purposes (see <http://www.doubleclick.com/us/>).

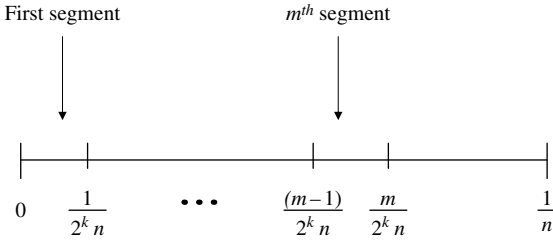


FIGURE 1 Partition of the characteristics space

denote firm  $i$ 's ( $i = 1, 2$ ) price in the  $m$ th segment. Firms can charge different prices to consumers in different segments, which amounts to third-degree price discrimination. Each firm's marginal cost is normalized to zero. In addition, each consumer incurs a constant per-unit of distance transportation cost, denoted by  $t > 0$ . Therefore, a consumer who is located at point  $x$  (in segment  $m$ ) derives an indirect utility equal to  $V - tx - p_{1m}$  if he buys from firm 1 and an indirect utility equal to  $V - t((1/n) - x) - p_{2m}$  if he buys from firm 2. Each consumer buys the product that gives him the highest utility. We assume that  $V$  is sufficiently high, ensuring that the market is covered. There are sufficiently many identical firms ready to enter the market. Firms incur an entry fee  $F > 0$  upon entering the market. Arbitrage between consumers is not feasible.

*A remark on the information structure.* The kind of information we have in mind is about consumer characteristics (e.g., sex, age, income group, purchase history). This information, after it has been processed and analysed, helps firms to segment the consumers into different groups. Firms can now price according to each group's willingness to pay for the different brands. More data about consumers (and/or more sophisticated techniques employed to analyse these data) lead to a finer segmentation. This is consistent with the way most practitioners and empirical researchers view market segments (e.g., Besanko, Dubé, and Gupta 2003; Rossi, McCulloch, and Allenby 1996). We have made two simplifying assumptions, which are nevertheless necessary in order to reduce the complexity of the model: (i) the size of all segments are equal and (ii) the distribution is uniform. Moreover, in practice, a firm's strategy regarding customer information consists of at least two main elements: (i) whether to collect detailed information and if so (ii) how much to invest in such a process. More firm resources directed towards this goal should result in consumer databases of higher quality. More important, the state of the existing technology imposes an exogenous bound on the quality. For tractability, we assume that the existing technology – which is beyond a firm's control – is

entirely responsible for the quality of a customer database, and a firm that enters the market is automatically endowed with it. Finally, our model is static and the effect of information improvements on the equilibrium of the game is in the form of a comparative statics analysis.

We analyse the following two-stage game:

- Stage 1: *Entry decisions*. Firms make their entry decisions simultaneously and independently. After entry, firms locate equidistantly from each other.<sup>5</sup>
- Stage 2: *Pricing decisions*. The entrants are endowed with consumer information of quality  $k$  and simultaneously and independently set their prices.<sup>6</sup>

It is worth emphasizing that the number of firms does not interact with the quality of information. Although as  $n$  increases the number of segments on the entire circle increase, this does not affect each firm's ability to segment its neighbouring consumers. In the Salop model, where competition is localized, this is what matters.

We are interested in obtaining the zero-profit equilibrium number of firms as a function of the information quality  $k$  and comparing it with the social optimum. We search for a symmetric subgame perfect Nash equilibrium.

### 3. Analysis

We solve the game backwards.

#### 3.1. Stage 2: Pricing decisions

We solve for the symmetric equilibrium vector of prices. Suppose  $n$  firms have entered the market and all have acquired customer-specific information. Since firms have consumer information, they know in which of the  $2^k$  segments each consumer is located and therefore are able to charge different prices for different segments, although the price within each segment must be the same. In segment  $m$ , firms 1 and 2 charge prices  $p_{1m}$  and  $p_{2m}$ . The marginal consumer in segment  $m$  is located at

5 As in Salop (1979) and Grossman and Shapiro (1984), we do not study product choice competition. Whatever the number of firms happens to be, they are equally spaced around the circle.

6 Alternatively, we could have endogenized information acquisition. If the decision to acquire information is made simultaneously with the pricing decisions, then firms will have strict incentives to acquire information, provided that the cost of acquiring information is sufficiently low. Therefore, a market configuration where all firms acquire information will be an equilibrium.

$$V - tx - p_{1m} = V - t\left(\frac{1}{n} - x\right) - p_{2m} \implies x_m = \frac{p_{2m}n - p_{1m}n + t}{2nt}.$$

The demands<sup>7</sup> of the firms' products (in segment  $m$ ) are<sup>8</sup>

$$d_{1m} = x_m - \frac{(m-1)}{2^k n} \text{ and } d_{2m} = \frac{m}{2^k n} - x_m,$$

and their profits are

$$\pi_{1m}(p_{1m}, p_{2m}) = p_{1m}d_{1m} \text{ and } \pi_{2m}(p_{1m}, p_{2m}) = p_{2m}d_{2m}.$$

Firm  $i$ 's, problem (which is separable segment by segment) can be expressed as follows:

$$\max_{p_{im} \geq 0} \pi_{im}(p_{1m}, p_{2m}) \text{ for each } m, \text{ where } m = 1, \dots, 2^k, \text{ and } i = 1, 2.$$

The ability of both firms to treat each segment independently of the other ones allows us to solve separately for the equilibrium in each subinterval and then aggregate over all subintervals to find the equilibrium profits, denoted by  $\pi_i^*(k, n)$ , as a function of the information quality and the number of firms. Proposition 1 summarizes the solution to the above problem.

**PROPOSITION 1** (equilibrium prices for a fixed number of firms). *For each  $k$ , ( $k \geq 1$ ) there exist two thresholds (integers)  $m_1$  and  $m_2$  (with  $0 \leq m_1 < m_2 \leq 2^k + 1$ ), where*

$$m_1 = 2^{(k-1)} - 1 \text{ and } m_2 = 2^{(k-1)} + 2$$

such that

- i) [This case is valid only when  $m_1 \geq 1$ ]. Firm 1's equilibrium demand is equal to  $1/2^k n$  in all segments from 1 to  $m_1$ , i.e., firm 1 is a constrained monopolist in these segments. Firm 2's equilibrium demand in these segments is zero. Moreover, firm 1's prices are:  $p_{1m}^* = t(2^k - 2m)/2^k n$ , while firm 2 sets  $p_{2m}^* = 0$ ,  $m = 1, \dots, m_1$ .
- ii) Both firms sell positive quantities in the segments from  $m_1 + 1$  to  $m_2 - 1$ . Moreover, firm 1's prices are:  $p_{1m}^* = t(2^k - 2m + 4)/(3 \times 2^k n)$ , and firm 2's prices are:  $p_{2m}^* = t(2m - 2^k + 2)/(3 \times 2^k n)$ ,  $m = m_1 + 1, \dots, m_2 - 1$ .
- iii) [This case is valid only when  $m_2 \leq 2^k$ ]. Firm 2's equilibrium demand is equal to  $1/2^k n$  in all segments from  $m_2$  to  $2^k$ ; that is, firm 2 is a constrained monopolist in these segments. Firm 1's equilibrium demand in these segments

7 Throughout the paper, demand in each segment must be within the interval  $[0, 1/2^k n]$ .

8 It can be easily observed that each firm's market share is zero in non-adjacent intervals, that is, in intervals that are between any two firms other than firm  $i$  and its two immediate neighbouring firms.

is zero. Moreover, firm 2's prices are:  $p_{2m}^* = t(2m - 2^k - 2)/2^k n$ , while firm 1 sets  $p_{1m}^* = 0$ ,  $m = m_2, \dots, 2^k$ .

Finally, the equilibrium (gross) profits of each firm, over all segments, as a function of  $k$  are

$$\pi_i^*(k, n) = \frac{t(9 \times 4^k - 18 \times 2^k + 40)}{36 \times 4^k n^2}, \quad i = 1, 2.$$

*Proof.* See appendix.<sup>9</sup>

Since each firm makes sales to its two adjacent (symmetric) intervals of length  $1/n$ , firm  $i$ 's total (net) profits are

$$\Pi_i(k, n) = 2\pi_i^*(k, n) - F = \frac{t(9 \times 4^k - 18 \times 2^k + 40)}{18 \times 4^k n^2} - F. \tag{1}$$

Proposition 1 says that both firms have strictly positive market shares only in the middle two segments, that is, in segments  $2^{(k-1)}$  and  $2^{(k-1)} + 1$ . For example, if  $k = 3$  firms can segment the consumers into  $2^3 = 8$  groups. The middle two segments are  $2^{(3-1)} = 4$  and  $2^{(3-1)} + 1 = 5$ . In all segments located to the left of  $2^{(k-1)}$ , firm 2's market share is zero. Likewise, in all segments to the right of  $2^{(k-1)} + 1$ , firm 1's market share is zero. The equilibrium outcome is inefficient, since some consumers do not purchase their favourite brand. In particular, all consumers in the  $2^{(k-1)}$  segment prefer firm 1's brand, all else equal. However, a fraction of them buys firm 2's product. Similarly, some consumers in the  $2^{(k-1)} + 1$  segment buy from firm 1, although their favourite firm is firm 2. This inefficiency is caused by the firms' ability to target consumers, that is, customer poaching (e.g., Fudenberg and Tirole 2000; Liu and Serfes).

### 3.2. Stage 1: Entry decisions

*Free-entry equilibrium.* By setting  $\Pi_i(k, n)$  [see (1)] equal to zero, we can derive the free-entry equilibrium number of firms in the market as a function of the information quality  $k$

$$n^*(k) = \frac{\sqrt{2t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}}{6\sqrt{F}}. \tag{2}$$

We can easily see that the equilibrium number of firms follows a U-shape pattern as a function  $k$  (see figure 2). Note that

<sup>9</sup> The proof is almost the same as the proof of proposition 1 in Liu and Serfes, where the number of firms is two (rather than  $n$ , which is the case in the present paper). For the sake of completeness, we provide the proof in the appendix.

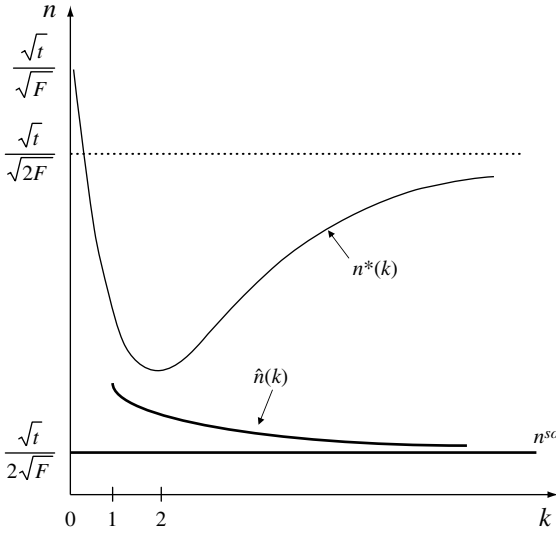


FIGURE 2 Number of entrants

$$\lim_{k \rightarrow \infty} n^*(k) = \frac{\sqrt{t}}{\sqrt{2F}}. \tag{3}$$

Equation (3) represents the equilibrium number of firms under perfect price discrimination. Note (from proposition 1) that (1) holds for  $k \geq 1$ . When  $k = 0$  (i.e., no price discrimination) the equilibrium number of firms is (e.g., Tirole 1988, 283)

$$n^*(0) = \frac{\sqrt{t}}{\sqrt{F}}. \tag{4}$$

*A regulator controls entry and can prevent price discrimination (first-best).* As we have already mentioned above, the existence of customer-specific information creates inefficiencies, owing to the fact that some consumers do not purchase their favourite brand (customer poaching). Therefore, in the *first-best* social outcome firms should not engage in price discrimination. The social planner then chooses the number of firms to minimize the sum of transportation and fixed costs (see Tirole 1988, 284)

$$C = \underbrace{\frac{t}{4n}}_{\text{Transportation cost}} + \underbrace{nF}_{\text{Fixed cost}}.$$

The cost is minimized at

$$n^{so} = \frac{\sqrt{t}}{2\sqrt{F}}. \tag{5}$$

*A regulator controls entry but cannot prevent price discrimination.* To induce the first-best outcome, in our context, a social planner must be able to control entry and to prevent price discrimination. Alternatively, the social planner can choose the number of firms to minimize the sum of transportation and fixed costs, knowing that firms will price discriminate. Under this assumption, it can be shown that the total costs in the market are given by

$$C' = \underbrace{\frac{t}{4n}}_{\text{Transportation cost}} + \underbrace{\frac{8t}{36 \times 4^k n}}_{\text{Customer poaching cost}} + \underbrace{nF}_{\text{Fixed cost}}, \text{ for } k \geq 1.$$

Imperfect price discrimination generates additional social costs. Note that  $C' > C$ , owing to customer poaching that takes place when firms have the ability to segment the market imperfectly. The impact of customer poaching on transportation cost is captured by the second term in  $C'$ . (Note that  $C' = C$  when  $k = 0$  or  $k = \infty$ .) The total cost  $C'$  is minimized at

$$\hat{n}(k) = \frac{\sqrt{t(9 + 8 \times 4^{-k})}}{6\sqrt{F}}, \text{ for } k \geq 1. \tag{6}$$

Observe that  $\hat{n}(k)$  is a decreasing function of the quality of information  $k$ . As  $k$  increases, the customer poaching cost decreases, and the social planner prefers fewer firms in the market in order to save on the fixed costs. In addition,  $\hat{n}(k) > n^{so}$  for all finite  $k \geq 1$ . The social planner prefers excessive entry, relative to the uniform price case, in order to partially offset the increase in customer poaching cost as a result of the pricing flexibility. Under perfect or no price discrimination,  $\hat{n} = n^{so}$  (see figure 2). When firms target consumers perfectly (or when firms charge one price), each consumer buys from his most preferred firm (i.e., the second term in  $C'$  disappears), and therefore there is no need for excessive entry.

Proposition 2 summarizes the comparisons: (i) between the free-entry outcome and the first best and (ii) between the outcome that arises when a regulator controls only entry (but not price discrimination) and the first best.

**PROPOSITION 2 (main results)**

- *The free-entry equilibrium number of firms is always excessive from the first-best point of view (i.e.,  $n^*(k) > n^{so}$  for any  $k \geq 0$ ) and U-shaped as a function of the information quality. Social inefficiency, under free-entry, is minimized at ‘moderate’ levels of information quality, that is, at  $k = 3$ . In other words, imperfect-price discrimination yields a more efficient free-entry equilibrium than either no or perfect price discrimination.*

- *Imperfect price discrimination (i.e., when  $1 \leq k < \infty$ ) creates additional inefficiencies, owing to customer poaching. Therefore, a social planner, who controls entry but cannot prevent price discrimination, prefers more firms than in the first best in order to mitigate the customer poaching costs (i.e.,  $\hat{n}(k) > n^{so}$ ).*

*Proof.* We plug (2) into  $C'$ . The function we obtain is convex in  $k$  and is minimized at  $k = 3$ .<sup>10</sup> ■

The intuition behind the U-shape of the free-entry number of firms as a function of the quality of consumer information is as follows. The number of entrants is directly linked to the level of profits. Firm profits,  $\Pi_i$ , exhibit a U-shape as a function of  $k$ , and that is why the equilibrium number of firms in the market is also U-shaped. There are two opposing effects that govern market interaction and are responsible for the non-monotonicity of profits. The ability to classify consumers into different segments allows firms to target the consumers of rival firms, while at the same time they do not lose rents from their own loyal customers. This leads to an all-out competition (*intensified competition effect*). On the other hand, better information allows firms to extract more surplus (*surplus extraction effect*). At low levels of information quality the first effect is more dominant than the second, and therefore profits fall. After a certain threshold of  $k$ , the surplus extraction effect becomes relatively more important and profits rebound. Since entry is always excessive, market inefficiency is minimized when the number of firms is as low as possible. This is achieved when the competition between firms is the fiercest (and hence profits are at their lowest levels), which happens at moderate levels of customer-specific information quality.

#### 4. Conclusion

The main result of this paper is that some (*but not too much*) price discrimination yields the most efficient outcome in a monopolistically competitive location model. Firms are located on the Salop circle, and entry occurs as long as profits are positive. Firms that enter the market are endowed with information about consumer preferences. The main innovation that we introduce in this paper is the way consumer information is modelled. In particular, we assume that the available information partitions the characteristics space, which allows firms to segment the market. This formulation is consistent with the way most practitioners and empirical researchers view market segments (e.g., Besanko, Dubé, and Gupta 2003; Rossi, McCulloch, and Allenby 1996). In addition, we allow the quality of information to vary (exogenously). The limit of this process is the perfect price discrimination paradigm. We show that the most efficient free-entry outcome occurs when the quality of consumer information

10 The remaining details of the proof follow from the analysis that precedes proposition 2.

is moderate. In other words, no or too much information leads to a less efficient outcome.

**Appendix**

*Proof of proposition 1.* We conjecture the following structure. There exist two integers,  $m_1$  and  $m_2$ , with  $0 \leq m_1 < m_2 \leq 2^k + 1$  such that: (i) [*left segments*] firm 1 is a constrained monopolist in all segments from 1 to  $m_1$  (if  $m_1 = 0$ , then firm 1 is never a constrained monopolist), (ii) [*middle segments*] in all segments from  $m_1 + 1$  to  $m_2 - 1$  the two firms sell positive quantities and (iii) [*right segments*] in all segments from  $m_2$  to  $2^k$  firm 2 is a constrained monopolist (again, if  $m_2 = 2^k + 1$  firm 2 is never a constrained monopolist). Next, we set out to prove that this structure indeed holds.

◆ Both firms charge strictly positive prices (middle segments).

Ignoring the non-negativity constraints and setting  $\partial\pi_{im}/\partial p_{im} = 0$ ,  $i = 1, 2$ , we obtain following solutions for the prices:

$$p_{1m} = \frac{t(2^k - 2m + 4)}{3 \times 2^k n} \text{ and } p_{2m} = \frac{t(2m - 2^k + 2)}{3 \times 2^k n}.$$

Using these prices we obtain the demands,

$$d_{1m} = \frac{-2m + 2^k + 4}{6 \times 2^k n} \text{ and } d_{2m} = \frac{2m - 2^k + 2}{6 \times 2^k n}.$$

We can see that  $d_{1m}$  is decreasing in  $m$ , and  $d_{2m}$  is increasing in  $m$ . This means that firm 1 may decide to charge a zero price and give the entire segment demand to firm 2 for segments that are in firm 2's territory. Analogously, for firm 2 in segments that are in firm 1's territory. For segments in the middle of the interval both firms charge positive prices. Observe that  $d_{2m} = (2m - 2^k + 2)/(6 \times 2^k n) \leq 0$  for any  $m \leq 2^{k-1} - 1$  and  $d_{1m} = (2^k - 2m + 4)/(6 \times 2^k n) \leq 0$  for any  $m \geq 2^{k-1} + 2$ . Now define  $m_1(k)$  to be the largest integer that is less than or equal to  $2^{k-1} - 1$ , and  $m_2(k)$  to be the smallest integer that is greater than or equal to  $2^{k-1} + 2$ . Obviously,

$$m_1 = 2^{(k-1)} - 1 \text{ and } m_2 = 2^{(k-1)} + 2.$$

This will be used later in the proof. Hence, for any  $m = m_1 + 1, \dots, m_2 - 1$ , both firms charge strictly positive prices and have strictly positive segment demands.

◆ Firm 1 charges strictly positive prices, while firm 2 charges a zero price (left segments).

Following the analysis above, this case is valid for  $m \leq m_1$ . Then  $d_{2m} \leq 0$ . This implies that  $d_{2m} = 0$  and  $d_{1m} = 1/2^k n$ . This further implies that  $p_{2m} = 0$ , and  $p_{1m}$  is the solution to  $d_{1m}(p_{2m} = 0) = 1/2^k n$ , which yields  $p_{1m} = t(2^k - 2m)/2^k n$ .

- ◆ Firm 2 charges strictly positive prices, while firm 1 charges a zero price (right segments).

This case is valid for  $m \geq m_2$ . This case is symmetric to case 2. Firm 2's prices in these segments are:  $p_{2m} = t(2m - 2^k - 2)/2^k n$ . Below, we summarize the results:

The equilibrium prices and profits are as follows:

- i) if  $m_1 + 1 \leq m \leq m_2 - 1$  (middle segments),

$$p_{1m} = \frac{t(2^k - 2m + 4)}{3 \times 2^k n} \text{ and } p_{2m} = \frac{t(2m - 2^k + 2)}{3 \times 2^k n}$$

$$d_{1m} = \frac{-2m + 2^k + 4}{6 \times 2^k n} \text{ and } d_{2m} = \frac{2m - 2^k + 2}{6 \times 2^k n}$$

$$\pi_{1m}(k, n) = \frac{t(2m - 2^k - 4)^2}{18 \times 4^k n^2} \text{ and } \pi_{2m}(k, n) = \frac{t(2m - 2^k + 2)^2}{18 \times 4^k n^2},$$

- ii) if  $m \leq m_1$  (left segments),

$$p_{1m} = \frac{t(2^k - 2m)}{2^k n} \text{ and } p_{2m} = 0$$

$$d_{1m} = \frac{1}{2^k n} \text{ and } d_{2m} = 0$$

$$\pi_{1m}(k, n) = \frac{t(2^k - 2m)^2}{4^k n^2} \text{ and } \pi_{2m}(k, n) = 0,$$

- iii) and, if  $m \geq m_2$  (right segments),

$$p_{1m} = 0 \text{ and } p_{2m} = \frac{t(2m - 2^k - 2)}{2^k n}$$

$$d_{1m} = 0 \text{ and } d_{2m} = \frac{1}{2^k n}$$

$$\pi_{1m}(k, n) = 0 \text{ and } \pi_{2m}(k, n) = \frac{t(2m - 2^k - 2)^2}{4^k n^2}.$$

Therefore, firms' profits for each  $k$  are

$$\pi_1(k, n) = \sum_{m=1}^{m_1} \frac{t(2^k - 2m)^2}{4^k n^2} + \sum_{m=m_1+1}^{m_2-1} \frac{t(2m - 2^k - 4)^2}{18 \times 4^k n^2},$$

$$\pi_2(k, n) = \sum_{m=m_1+1}^{m_2-1} \frac{t(2m - 2^k + 2)^2}{18 \times 4^k n^2} + \sum_{m=m_2}^{2^k} \frac{t(2m - 2^k - 2)^2}{4^k n^2}.$$

By performing the summation and using  $m_1 = 2^{(k-1)} - 1$  and  $m_2 = 2^{(k-1)} + 2$ , we obtain

$$\pi_i^*(k, n) = \frac{t(9 - 18 \times 2^{-k} + 40 \times 4^{-k})}{36n^2}, \quad i = 1, 2.$$

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