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When vagueness induces indirect competition: strategic incompleteness of contracts*

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Summary. A class of employment contracts entailing production targets and consequent rewards is studied. In a nondiscriminatory environment, a principal hiring many agents faces the problem of writing a common contract which induces the highest possible effort from each one of his agents. While a very high target may get the best out of highly skilled agents, low skilled ones tend to shirk. On the other hand, although low targets make every agent put positive effort, there are efficiency losses from the high skilled agents. Also, in such environments the principal often has better information regarding the skills of all his agents than what each agent has regarding the rest of the agents at work. We show that if skills of agents are sufficiently close, the informed principal earns strictly higher profits by offering incomplete contracts as against being specific, as incomplete contracts reduce flow of information and induce indirect competition amongst agents.

Keywords and Phrases: Informed principal, Strategic vagueness, Information flow, Incomplete contracts.

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1 Introduction

A workplace comprises of an employer and many employees who are hired to carry out some productive activity. In such environments, employment is regarded

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as an agreement (or contract) between the employer and his employees which states that the employee is appointed for a stipulated period of time in order to produce output (tangible or otherwise) and earn wages and salaries in return. It is also universally known to all parties that if an employee is found to be highly productive, such contracts are renewed with the worker receiving some kind of a reward. Alternatively, everyone knows and understands that agents with poor performance will be laid off once the stipulated time period is over. The employer has the option of setting his standards explicitly in the contract or to remain vague. Real world contracts are hardly explicit (or complete) in this sense. For example, a contract written between an academic department and a newly hired assistant professor rarely specifies explicitly the number and the quality of research papers that the assistant professor has to publish and the scores in the teaching evaluations he needs to satisfy. Research and teaching are important factors for continuation of employment and salary increases, yet the exact thresholds are kept *excessively* unspecified.

Observed incompleteness (or vagueness) of contracts is in general attributed to several factors like unforeseen contingencies, transaction costs, asymmetric information, strategic ambiguity and ambiguity aversion.[1]

The contribution of this paper is to show that *observed incompleteness of contracts may arise even in an environment where all aspects of activities are verifiable and contracting costs are negligible.* As we will argue in greater detail, *flow of information* and expectations from all parties play a central role, and the nature of employment contracts may depend crucially upon them. In particular, the employer can control this flow of information and affect beliefs held by the employees. By offering incomplete contracts, the employer is able to hide information and induce *indirect competition*, resulting in higher efforts.

We consider an environment where an institution has employed two agents/workers and has appointed a principal/manager to meet some production target through effort exerted by the hired agents. Each agent is endowed with individual specific ability or productivity which can be either high or low. Before production takes place, such abilities are observed by the institution and consequently made

[1] (i) *Unforeseen contingencies* (Simon, 1981): future uncertainties have endless sources and it is impossible for even sophisticated minds to foresee all contingencies; (ii) *Transaction costs* (see Williamson, 1975): even if agents can perfectly foresee every possible future contingency, if the number of states is very large, complete contracting may become prohibitively expensive. Maskin and Tirole (1999) show that even when there are many contingencies, a clever and costless contract may do the job. On the other hand, Hart and Moore (1999) show that even if full contingency contracts are costless, null contracts may turn out to be optimal; (iii) *Asymmetric information:* when the contracting parties are better informed than any third party, incompleteness of contracts may arise out of the inability of the third party to verify certain events (e.g. Maskin and Moore, 1999; Segal, 1999). Using a principal-agent model, Spier (1992) shows that asymmetric information may lead to contractual incompleteness as in the presence of transaction costs, incomplete contracts may act as a signal of the principal's type; (iv) *Strategic ambiguity*: Bernheim and Whinston (1998) show that it is often optimal to leave some verifiable aspects of performance unspecified as specification of too many dimensions may worsen performance in the non-contractible ones; (v) *Ambiguity aversion*: Mukerji (1998) shows that when uncertainties cannot be described by unique probability distributions and agents are averse to such ambiguities, incomplete contracts may increase the surplus.

known to the principal,[2] while each agent knows only his or her own ability.[3] Each worker produces an individual output by exerting effort and the principal is asked to motivate work by promising a reward for good performance.[4] We assume that the reward is set a priori by the institution and is therefore fixed and costless to the principal.[5] For every given profile of possible worker skills, the institution decides upon the threshold individual output requirement which is made available to the principal, and an agent wins the prize if and only if he fulfills it while the principal is only required by the institution to fulfill the prescribed output requirement. This schedule of performance can be computed by all players involved and is therefore common knowledge (as we will show in our analysis) excepting that the agents are not sure of the exact profile of skills since at the time they decide how much effort to exert, they do not observe the skill of their fellow workers. As a consequence, before production takes place, the principal has the option of explicitly announcing the exact output requirement. Given such an announcement, agents decide upon how much effort to exert. Whenever the principal explicitly announces the threshold, the contractual arrangement is called complete. However, the principal always has the option of not announcing this threshold and thereby not revealing the true profile of skills to his workers. Whenever this is the case, the contractual arrangement is called incomplete.[6]

In a standard setting, the first best contracts will typically be complete and discriminatory. In particular, in our model the principal will then set different target performances depending upon the skills of the agents and induce them to put their highest efforts. Thus, low skilled agents will be asked to produce less than their high skilled counterparts. However such discriminatory environments may either be unlawful or may lead to unhealthy relationships amongst the agents. We therefore concentrate on nondiscriminatory environments only and address briefly the issue of discriminatory contracts in Section 3.2.

In the absence of discriminatory contracts, the institution along with the principal faces the problem of writing a common contract which induces the

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[2] Information found in a CV, recommendation letters, sample works, experience, etc.

[3] One may argue that if the employees communicate right after they have been hired, then each one will find out the true skills of his co-workers. We rule this possibility out by assuming that this type of communication is either not possible or is prohibitively costly and takes time. For instance, employees may not meet very often (e.g. salesmen), or be too busy to engage in any activity other than production, or find it ethically unacceptable to intrude into others' privacy and personal information.

[4] We assume for simplicity that agents work for zero wages as their reservation payoffs are also zero. Such contracts are typically called *spot-contracts* containing minimal current financial incentives and motivation to work is driven mostly by expected future market returns. In our simple environment this becomes the best class of contracts and shares the spirit with the literature concerning the economics of career concerns (see Holmstrom, 1982; Dewatripont et al., 1999). This literature is concerned with the implications of current performance on future contracts and possibilities of re-employment.

[5] The principal is judged by his superiors not on how many workers get the bonus, but on how much work his workers produce.

[6] Observe that this problem is equivalent to the following information revelation problem: the principal announces the performance schedule which specifies the output level that, if met by a worker, entitles him to the reward. The principal also chooses whether or not to reveal to the two workers their true productivity parameters before production occurs.

highest possible effort from each one of his agents. While a very high target may get the best out of highly skilled agents, low skilled ones tend to shirk. On the other hand, although low targets make every agent put positive effort, there are efficiency losses from the high skilled agents. Which specification should the institution and principal opt for? There are two possible cases here. The first is when the full efficiency output of the low skilled agent is sufficiently close to that of a high skilled one. In such a scenario, the principal would find it best to specify the output that extracts all the surplus from the low skilled agent. Both agents will put positive efforts, with the high skilled agent earning a positive surplus. Alternatively, if the full efficiency output of the low skilled agent falls significantly short of that of a high skilled one, the principal is better off specifying the output that extracts all the surplus from the high skilled agent. Consequently, the low skilled agent shirks while the high skilled agent earns a zero surplus.

We move on to ask the following question: can the principal do better, and in particular is there a way the principal can achieve the first best outcome without being discriminatory? The answer is surprisingly affirmative and this is where the role of incomplete contracts comes into play. The idea stems from the fact that in extensive form games with incomplete information, players may choose some moves in order to *strategically reveal information* to players who follow. Notice that the difference between observing a complete contract as against an incomplete one is solely in the timing of information flow amongst the agents. While a complete contract immediately reveals information, an incomplete one may or may not. In any case, with a complete contract the game played between the principal and the agents becomes trivial as once the target is specified, the agents simply decide whether to meet the target and win the reward or to shirk. However with an incomplete contract, the agents must form expectations regarding the true skill of their fellow agent in order to form a belief about the target which is implicitly included in the contract according to the optimal performance schedule designed by the institution. Each agent knows that this target increases (at least weakly) with the skills of the two agents. This lack of information created by such an incomplete contract therefore induces an indirect competition amongst the two agents. In particular, the high skilled agent may now put his highest effort in order to insure himself against the event that his fellow agent is high skilled too. On the other hand, incompleteness of contracts may also make the low skilled agents attach a positive probability to their fellow agent being of a low skill, thereby not shirking completely. Thus in an event where one agent is high skilled while the other is low skilled, an incomplete contract may lead to each agent producing efficiently, an outcome which no nondiscriminatory complete contract can achieve if the two agents have different skills. We show that there exist a continuum of perfect Bayesian equilibria in the game of incomplete information induced by the above environment where the principal is strictly better off by choosing to remain vague.[7]

[7] One may argue that the institution along with the principal may not abide by the actual performance schedule once production takes place if it is not explicitly announced. In this sense while

The rest of the paper is organized as follows. In Section 2 we describe the model. Section 3 studies the optimal complete contracts. Section 4 first proves an impossibility result by which we argue that if the skills of the agents are sufficiently far apart, incomplete contracts provide no extra benefit to the principal. We then show that when such skills are sufficiently close, there exist some states of the world where the principal is strictly better off by offering incomplete contracts. The section concludes with a numerical example. Finally, we draw our conclusions in Section 5.

2 The model

Consider an environment with one informed principal and two agents ($i = 1, 2$).[8] The principal offers a non-discriminatory contract[9] to hire the two agents. We assume that the principal has all the bargaining power and that the reservation utility of each agent is zero. Each agent has his own production function $y_i = f(e_i)$, with e_i being the level of effort exerted by him to produce individual output y_i. We assume that $f(\cdot)$ is strictly increasing with $f(0) \geq 0$.[10] Total output is denoted by $y = y_1 + y_2$. Effort is costly with $\theta_i e_i$ being the cost incurred by agent i when he puts an effort e_i. The true productive efficiency (or skill) of an agent is reflected by θ_i which is a random variable. There are two possible levels of efficiencies (types), h (high) and ℓ (low).[11] Consequently, $\theta_i \in \Theta = \{h, \ell\}$ with $\ell > h > 0$. Let Ω be the state space with

$$\Omega = \{(\ell, \ell), (\ell, h), (h, \ell), (h, h)\}, \quad (\theta_1, \theta_2) \in \Omega,$$

where the first element of each component in the state space denotes the type of agent 1 and the second the type of agent 2. Play unfolds in two stages. At stage 1,

explicit contracts can be easily enforceable, many implicit and vague contracts (like the one we have described) may suffer from the issue of enforceability in a court of law. However, as Lawler (1971) and Hamner (1975) argue, organizational practices involving implicit agreements are most of the time backed by trust between workers and supervisors. Moreover, Holmstrom (1981) and Bull (1987) show that many implicit contracts based on subjective performance assessments can be enforced by virtue of the firm's concern for its reputation in the labor market. In our model the threshold schedule as a function of the skills profile is common knowledge and we implicitly assume that the skills and the output ex-post are verified, or become common knowledge. Then a third party (court) or the firm itself fearing loss of reputation can enforce the contracts.

[8] Our results can be generalized to many agents. However such a generalization will not provide any additional insight to the problem at hand.

[9] We define a contract (or a contracting mechanism) to be nondiscriminatory if all agents work under the same terms. Any contract violating this criterion is called discriminatory.

[10] Zero effort does not necessarily imply zero output. For example, any job has certain minimum tasks that have to be performed. Precisely why even if an agent exerts zero effort the institution may still enjoy some surplus from hiring him. A positive effort is any improvement over such minimum requirements.

[11] The production function specification implies that workers have the same productivity if effort is considered to be the input. It is only when disutility of effort is considered to be the input in the production function that one worker type becomes less or more "efficient" than the other worker type (because effort is relatively more or less costly to him). This is equivalent to a framework in which workers have the same disutility of effort but differ directly in the productivity of their effort on final output.

nature moves by choosing a pair of types for the two agents with $\mu(h) \in (0,1)$ and $\mu(\ell) \in (0,1)$ being the respective probabilities of an agent being of type h and of type ℓ.[12] Nature's choice is perfectly observable by the principal but not by the agents who know only their own types.[13] Therefore the information partitions of the principal and the two agents are respectively,

$$\begin{aligned} \Pi(\Omega) &= \{\{(\ell,\ell)\}, \{(\ell,h)\}, \{(h,\ell)\}, \{(h,h)\}\} \\ \pi_1(\Omega) &= \{\{(\ell,\ell), (\ell,h)\}, \{(h,\ell), (h,h)\}\} \\ \pi_2(\Omega) &= \{\{(\ell,\ell), (h,\ell)\}, \{(\ell,h), (h,h)\}\}. \end{aligned}$$

At this stage the principal offers to the two agents a formal contract which comprises of a *wage* equal to zero (the agent accepts employment since his opportunity cost is also zero). In addition, he also promises to offer a *reward* $V \in \mathbb{R}_{++}$ to each one of the agents if the agent concerned produces an output of at least $\overline{y}(\theta_1,\theta_2)$. While V is exogenously fixed and costless to the principal in our model, $\overline{y}(\theta_1,\theta_2)$ depends on the true realization of the state which may or may not be stated explicitly. If the contract is not explicit and the principal chooses to remain *vague*, it is common knowledge that the contract has an underwritten clause which states "*good performance is rewarded,*" the reward being V and good performance implying the true value of $\overline{y}(\theta_1,\theta_2)$ which remains unknown to the agents. We call a contract *complete* if it is explicit and *incomplete* if it is vague.

At stage 2, the two agents observe the contract on offer and choose their levels of effort. If the principal specifies the threshold level of output that the individual production of each agent must meet, then each agent decides whether to work and meet the standard or shirk completely in which case he incurs no cost but he also receives no reward. On the other hand, upon observing a vague contract, the agents try to update their beliefs regarding the true state and based upon such updated beliefs take similar decisions. Finally production takes place and contract terms are met.

The above environment induces a game of incomplete information. At stage 1 after nature has drawn the true state $(\theta_1,\theta_2) \in \Omega$, the principal faces the information partition $\Pi(\Omega)$ and makes a move. Denote by $S = \mathbb{R}_+ \cup \{\emptyset\}$ the pure strategy set of the principal which is available at each information set of $\Pi(\Omega)$, with $s \in S$ being a particular choice of contract on offer. Thus, if he chooses to announce $\overline{y}(\theta_1,\theta_2)$, then $s = \overline{y}(\theta_1,\theta_2) \in \mathbb{R}_+$, while if he chooses not to announce this threshold, then $s = \emptyset$. At stage 2, the agents simultaneously make their moves after observing the offered contract. Their prior information partitions are given

[12] We assume that the probability that worker i is of a given type is independent of the type of worker j. Also, $\mu(h) = 1 - \mu(\ell)$.

[13] In the academic job market for assistant professors, stage 1 is in other words the stage of the hiring process when all fly-back job talks are over. Thus, the hiring committee has "sufficiently good" (or at least the best possible) information about the job candidates who they are very close to hiring. However, the quality of a fellow applicant remains completely unknown to the pool of applicants (they were neither together during the first round interviews, nor were they present in each others seminars!).

by π_1 and π_2. Each $s \in S$ induces a posterior probability distribution subjectively held by each agent i of type θ_i regarding the choice of nature (θ_1, θ_2). This is a Bayesian update of $\mu(\cdot)$ given s and is denoted by $\mu_{\theta_i}\left(\theta_j|s\right)$ which is agent i's subjectively held posterior belief of his fellow agent to be of type θ_j when agent i is of type θ_i and the principal has offered the contract s. The agents update the probability distributions over the elements in π_1 and π_2 by using their subjectively held posteriors. Denote by $E = \mathbb{R}_+$ the pure strategy set of each agent which is available at each information set of π_1 and π_1, with $e_i \in E$. This brings us to the end of stage 2.

The payoffs to the principal and the agents are given respectively by

$$u(s, e_1, e_2) = f(e_1) + f(e_2), \tag{1}$$

and

$$v_i\left(s, e_i, e_j\right) = \begin{cases} V - \theta_i e_i & \text{if} \quad f(e_i) \geq \overline{y}(\theta_1, \theta_2) \\ -\theta_i e_i & \text{if} \quad f(e_i) < \overline{y}(\theta_1, \theta_2). \end{cases} \tag{2}$$

The reservation payoff of each agent is set equal to zero. We denote this game of incomplete information by $G\left(S, E^2, u, (v_i)_{i=1,2}, \Theta, \mu\right)$.

In what follows we study issues relating to the existence as well as non-existence of *perfect Bayesian equilibria* of G such that the principal finds it strictly beneficial, at least for some realizations of the states in Ω, to offer an incomplete contract in stage 1 as opposed to offering a complete one.

3 Complete contracts

We first study the nature of an optimal complete contract. We begin by finding an optimal nondiscriminatory contract, that is a contract which specifies explicitly a uniform threshold for both agents. This in turn will define the optimal threshold schedule. Then we discuss a class of contracts where the principal offers different terms to each agent. Clearly such contracts are discriminatory by our definition.

3.1 Nondiscriminatory contracts

We begin by finding the equilibrium of the game G with the restriction that the principal cannot offer incomplete contracts. Thus the restricted strategy set of the principal is $\hat{S} = S \setminus \{\emptyset\}$ and denote the restricted game by $\hat{G} = G\left(\hat{S}, E^2, u, (v_i)_{i=1,2}, \Theta, \mu\right)$. The following lemma will be useful for our main results. It shows that optimal complete nondiscriminatory contracts can only be of two types. The first where all types of agents exert positive effort; the low skilled agent earns a zero surplus while the high skilled one earns a positive surplus. On the other hand, in the second type, only the high skilled agents exert a positive effort and earn zero surplus.

Lemma 1 *Let* $(\theta_1, \theta_2) \in \Omega$ *be any arbitrary state and let* $s^* = \overline{y}^*(\theta_1, \theta_2)$ *be the corresponding equilibrium strategy of the principal in the restricted game* $\hat{G}$*. Then* $\overline{y}^*(\theta_1, \theta_2) \in \{y_\ell, y_h\}$ *where*

$$y_\ell = f\left(V/\ell\right) \text{ and } y_h = f\left(V/h\right).$$

Consequently, the unique equilibrium point of $\hat{G}$ *is the strategy profile*

$$\left(s^*, e_1^*, e_2^*\right)$$

satisfying the following:

(i) *if* $s^* = y_\ell$ *then* $e_i^* = \frac{V}{\ell}$ *for* $i = 1, 2,$
(ii) *if* $s^* = y_h$ *then* $e_i^* = \frac{V}{h}$ *for* $i = 1, 2$ *if* $\theta_i = h$*; otherwise,* $e_i^* = 0.$

Proof. Take any arbitrary strategy $s \in \mathbb{R}_+$. Agent i's best response function is given by

$$b_i\left(\theta_i | s\right) = \begin{cases} 0 & \text{if} \quad V - \theta_i f^{-1}(s) < 0 \\ f^{-1}(s) & \text{otherwise}. \end{cases}$$

If $s = f\left(V/k\right)$ then, $V - \theta_i f^{-1}(s) = V\left(1 - \theta_i/k\right)$ for some $\theta_i \in \Theta$ and some $k \in [0, \infty)$. Clearly therefore, $V\left(1 - \theta_i/k\right) \geq 0$ whenever $\theta_i \leq k$, and $V\left(1 - \theta_i/k\right) < 0$ whenever $\theta_i > k$. Suppose $k > \ell$. Then, all types of agents receive a positive surplus and the principal can increase his payoff by setting $k = \ell$. Similarly, if a $k \in (h, \ell)$ yields a higher payoff to the principal than $k = \ell$ then the principal is strictly better off setting $k = h$. Finally, $k = h$ is always better for the principal than any $k > h$. The rest of the lemma can be easily seen and needs no proof. □

The principal's problem becomes trivial whenever the two agents are of the same type in which case he either specifies y_ℓ when both agents are of low type or y_h when both are of high type. In the event the types of the two agents differ, the principal has two choices. Clearly, if he chooses any specification greater than y_h, neither agent will work and the principal is strictly better off by reducing his specification to at least y_h so that the high type agent works. On the other hand, although both agents work whenever he chooses a specification less than y_ℓ, the principal becomes strictly better off by asking both agents to produce y_ℓ. Finally any specification strictly between y_ℓ and y_h implies that only the high type works and receives a positive surplus. Therefore the principal can do strictly better and yet keep the high type agent working by specifying a higher threshold up to a point where it reaches y_h. Whether the principal finds it optimal to specify y_ℓ or y_h depends upon whether $2y_\ell$ is greater or less than $y_h + f(0)$. We summarize this in the following proposition.

Proposition 2 *The optimal threshold schedule which generates the optimal complete contracts satisfies the following:*

(i) $\overline{y}^*(h, h) = y_h$ *and* $\overline{y}^*(\ell, \ell) = y_\ell$*;*
(ii) $\overline{y}^*(h, \ell) = \overline{y}^*(\ell, h) = y_h$ *whenever* $f(V/h) + f(0) > 2f(V/\ell)$*;*
(iii) $\overline{y}^*(h, \ell) = \overline{y}^*(\ell, h) = y_\ell$ *whenever* $f(V/h) + f(0) < 2f(V/\ell)$*.*

Even if agents are unable to tell the true profile of skills, it is straightforward for them to compute out the optimal threshold schedule as in Proposition 2. Thus Proposition 2 is common knowledge amongst all players.

3.2 Discriminatory contracts

In this paper we assume that the principal is prohibited from offering discriminatory contracts which would typically specify skill-dependent thresholds of outputs extracting the surplus from both types of agents. In this subsection, we digress from our main objective and discuss a class of contracts which entails different thresholds for different types of agents. Since this becomes interesting only when the agents have different skills, let us assume so. The principal offers contracts with different thresholds to each agent (y_h and y_l for the high and the low type respectively) and extracts the entire surplus from each one of them. However, these contracts are likely to create unhealthy relationships in the work environment as the high type agent would strictly prefer the low type's contract to his own. To mitigate this problem while still remaining discriminatory, the principal may have to make both contracts available to both agents who in turn will select one according to their types. This is the standard class of *separating contracts,* where in order to induce the high type to choose the contract designed for his type the principal has to incur a cost by giving him a positive surplus. Subsequently, in Theorem 4 we show that when the principal offers non-separating (pooling) contracts which also do not specify the thresholds explicitly (incomplete) the payoff to the principal (under some conditions) is equal to the discriminatory outcome where the principal specifies different thresholds of production extracting all the surplus from both agents. Therefore in this case separating contracts will not be offered by the principal as they entail the additional cost of separation in the form of information rents given to the high type through different V's, since with the same V offered to both agents the principal cannot separate the types. However, under the complementary set of conditions in Theorem 4 separating contracts may perform better than the contracts we have proposed. This issue is not further examined as in this paper our aim is just to demonstrate the benefits of offering an incomplete contract in a non-discriminatory environment.

4 Choosing to remain vague

Suppose now that the strategy set of the principal is extended to include the option of an incomplete contract, i.e. the strategy set is now $\hat{S} \cup \{\emptyset\}$. We show that if the skills of the agents are reasonably far apart, a complete contract always remains optimal with the qualification that under some states in Ω the principal becomes indifferent between offering incomplete and complete contracts. Then we show that when skills are reasonably close, there exist equilibria in G such that under certain states the principal is strictly better off by offering incomplete contracts.

4.1 An impossibility result

Suppose a high skilled agent is significantly more productive than a low skilled one. In the following theorem we show that in this case the principal is at most indifferent between announcing the threshold explicitly (that is offering a complete contract) in stage 1 and remaining vague (that is offering an incomplete contract). In order to do so, we first introduce some notations. Define the set $S^* := \{y_\ell, y_h\} \cup \{\emptyset\}$ and let Σ be the set of all probability distributions over S^*. A typical element of Σ is denoted by σ where $\sigma = \left(\sigma(y_\ell), \sigma(y_h), \sigma(\emptyset)\right)$ with $\sigma(s)$ being the probability that σ attaches to the pure strategy $s \in S^*$ with $\sigma(s) \geq 0$ for every $s \in S^*$ and $\sum_{s \in S^*} \sigma(s) = 1$. Thus, σ is a mixed strategy available to the principal. Given Lemma 1 and Proposition 2, it is easy to see that in no circumstance will the choice of a pure strategy by the principal lie outside S^*, and therefore no mixed strategy will attach a positive probability to any $s \in S \backslash S^*$. Thus restricting the mixed strategy set to Σ comes without any loss of generality.

Theorem 3 *Suppose a high skilled agent is significantly more productive than a low skilled one, that is $y_h + f(0) > 2y_\ell$. Then there does not exist an equilibrium point in G such that the principal is strictly better off by offering an incomplete contract.*

Proof. With $y_h + f(0) > 2y_\ell$, the optimal complete contract is $\bar{y}^*(\theta_1, \theta_2) = y_h$ if and only if $(\theta_1, \theta_2) \neq (\ell, \ell)$, with only the agent(s) with type h choosing a positive effort. However, if $(\theta_1, \theta_2) = (\ell, \ell)$, then $\bar{y}^*(\theta_1, \theta_2) = y_\ell$. This follows directly from Lemma 1 and Proposition 2.

Suppose the principal chooses the strategy $\sigma = (0, 0, 1)$ for any (θ_1, θ_2). Denote by $\bar{y}\left(\theta_i | s = \emptyset\right)$ the induced lottery faced by agent i of type θ when $s = \emptyset$. Then,

$$\begin{aligned} \bar{y}\left(\ell | s = \emptyset\right) &= \begin{cases} y_\ell & \text{with probability} \quad \mu(\ell) \\ y_h & \text{with probability} \quad 1 - \mu(\ell), \text{and} \end{cases} \\ \bar{y}\left(h | s = \emptyset\right) &= y_h \quad \text{with probability 1.} \end{aligned}$$

If agent i is of type ℓ and chooses $e_i = V/\ell$, his expected payoff is $-(1 - \mu(\ell)) V < 0$, while if he chooses $e_i = V/h$, his expected payoff is $V\left(1 - \ell/h\right) < 0$. Therefore, facing the lottery $\bar{y}\left(\ell | s = \emptyset\right)$, agent i of type ℓ chooses $e_i = 0$. On the other hand, if agent i is of type h, $e_i = V/h$ is his strictly dominant strategy (with appropriate tie-breaking assumptions) associated with a payoff equal to 0. Thus the payoff to the principal with $\sigma = (0, 0, 1)$ is $2f(V/h)$ if $\theta_1 = \theta_2 = h$, $2f(0)$ if $\theta_1 = \theta_2 = \ell$, and $f(V/h) + f(0)$ if $\theta_1 \neq \theta_2$. Now consider the following state-contingent subgame perfect pure strategy $\hat{\sigma}$ defined by

$$\hat{\sigma} = \begin{cases} (1, 0, 0) & \text{if } (\theta_1, \theta_2) = (\ell, \ell) \\ (0, 1, 0) & \text{if } (\theta_1, \theta_2) \in \{(\ell, h), (h, \ell)\} \\ (0, 1, 0) & \text{if } (\theta_1, \theta_2) = (h, h), \end{cases}$$

where $s = \emptyset$ occurs with zero probability for every possible realization of the state. Notice that $\hat{\sigma}$ can achieve whatever $\sigma = (0, 0, 1)$ can and can do strictly better whenever $(\theta_1, \theta_2) = (\ell, \ell)$ in which case the output is $2f(V/\ell)$. In what follows we show that there does not exist any state-contingent mixed strategy for the principal yielding a higher payoff than under $\hat{\sigma}$, establishing the fact that complete contracts are weakly dominant with respect to the state space. We proceed as follows.

Consider the following set of state-contingent subgame perfect (and possibly mixed) strategies σ^* for the principal, defined by

$$\sigma^* = \begin{cases} (1, 0, 0) & \text{if } (\theta_1, \theta_2) = (\ell, \ell) \\ \left(0, \sigma_1^*(y_h), \sigma_1^*\left(\emptyset\right)\right) & \text{if } (\theta_1, \theta_2) \in \{(\ell, h), (h, \ell)\} \\ \left(0, \sigma_2^*(y_h), \sigma_2^*\left(\emptyset\right)\right) & \text{if } (\theta_1, \theta_2) = (h, h), \end{cases}$$

for any mutually consistent $\sigma_1^*(y_h), \sigma_1^*\left(\emptyset\right), \sigma_2^*(y_h), \sigma_2^*\left(\emptyset\right) \in [0, 1]$. Notice that the principal plays $(1, 0, 0)$ if $(\theta_1, \theta_2) = (\ell, \ell)$ because any other strategy (e.g. mixing with $\emptyset$) would not be subgame perfect as the low types would not exert any positive effort, thereby lowering the principal's payoff.

If agent i of type ℓ observes $s = \emptyset$, his posterior belief is

$$\mu_\ell\left(h|\emptyset\right) = 1,$$

implying that $e_i = 0$. On the other hand, if agent i of type h observes $s = \emptyset$, his posterior belief is

$$\begin{aligned} \mu_h\left(h|\emptyset\right) &= \frac{\sigma_2^*\left(\emptyset\right)\mu(h)}{\sigma_2^*\left(\emptyset\right)\mu(h) + \sigma_1^*\left(\emptyset\right)\mu(\ell)} \text{ and} \\ \mu_h\left(\ell|\emptyset\right) &= \frac{\sigma_1^*\left(\emptyset\right)\mu(\ell)}{\sigma_1^*\left(\emptyset\right)\mu(\ell) + \sigma_2^*\left(\emptyset\right)\mu(h)}. \end{aligned}$$

No matter what the values of $\mu_h\left(h|\emptyset\right)$ and $\mu_h\left(\ell|\emptyset\right)$ are, an agent i of type h always chooses $e_i = V/h$. Thus in the proposed set of strategies σ^*, the payoff to the principal is $2f(V/h)$ if $\theta_1 = \theta_2 = h$, $2f(V/\ell)$ if $\theta_1 = \theta_2 = \ell$ and $f(V/h) + f(0)$ if $\theta_1 \neq \theta_2$. By comparing the payoffs from σ^* where the principal offers incomplete contracts in 2 out of the 3 states with $\hat{\sigma}$ where no incomplete contracts are offered we see that by being vague (incomplete) the principal does not derive any additional benefit. □

First observe that since a high skilled agent is significantly more productive than a low skilled agent, whenever the skills are different, the principal's optimal choice (explicit or implicit) is to set a high threshold (y_h) and exclude the low agent from getting V. When both agents are of the same type, the principal derives no strict benefit from offering an incomplete contract. If both agents are high skilled, a complete contract yields a payoff equal to $2y_h$ to the principal. If the principal offers an incomplete contract in this event, both agents, not knowing the type of their fellow agents, find it optimal to still produce y_h since each knows that even in the event of their fellow agent being a low type, the implicit threshold is y_h. On the other hand, if both agents are of a low type, a complete contract yields a payoff of $2y_\ell$ to the principal. Now if the principal offers an incomplete contract, both agents shirk completely since with some positive updated probability they believe that the implicit threshold is y_h (in the event their fellow agent is of a high type) and therefore foresee negative expected surplus from any positive level of effort. Hence with a perfectly homogenous workforce, a complete contract becomes a weakly dominant strategy for the principal. Finally suppose the agents are of different types. An incomplete contract does not affect the decision of the high type(s) as a high type always produces y_h. However, it only hurts the principal as the low type agent shirks completely. Therefore the payoff to the principal with an incomplete contract is $y_h + f(0)$ which he can always guarantee by offering his optimal complete contract with the specification of y_h.

4.2 Existence of equilibria with incomplete contracts

In this section we show that when skills are close, the principal may find it strictly beneficial to offer incomplete contracts. First we introduce some more notation. Given any choice of nature $(\theta_1, \theta_2) \in \Omega$, $G_{(\theta_1,\theta_2)}$ is the subgame defined by,

$$G_{(\theta_1,\theta_2)} \equiv G\left(S, E^2, u, (v_i)_{i=1,2}, \Theta \backslash \left\{\left(\theta_1', \theta_2'\right)\right\}, \mu\right)$$

with $\left(\theta_1', \theta_2'\right) \in \Omega$, $\theta_1' \neq \theta_1$ and $\theta_2' \neq \theta_2$.

For example, consider Figure 1 where it is assumed that nature has chosen (h, h). If the contract offered is incomplete, then both agents clearly realize that the state (ℓ, ℓ) has not occurred. However agent 1 cannot exclude the possibility of the state being (h, ℓ) while agent 2 cannot exclude the possibility of the state being (ℓ, h). Furthermore, neither agent can exclude the possibility of the state being (h, h). Therefore, if state (h, h) occurs, G is reduced to $G_{(h,h)}$ as defined above and constitutes the first three edges (from the left) of the graph.

Theorem 4 *Suppose the skills of the agents are sufficiently close and the probability of a low skilled agent is sufficiently small, that is $y_h + f(0) < 2y_\ell$ and $\mu(\ell) < h/\ell$ respectively. Then there exist equilibrium points in G such that the principal is strictly better off by offering an incomplete contract with positive probability independent of the true profile of skills. In particular, the strategy where the principal offers an incomplete contract with probability 1 independent of the skill profile of the two agents is an equilibrium. Moreover, incomplete contracts yield first best discriminatory outcomes if $\mu(\ell) < h/\ell$.*

Proof. With $y_h + f(0) < 2y_\ell$, the optimal complete contract is $\bar{y}^*(\theta_1, \theta_2) = y_\ell$ if and only if $(\theta_1, \theta_2) \neq (h, h)$. However, if $(\theta_1, \theta_2) = (h, h)$, then $\bar{y}^*(\theta_1, \theta_2) = y_h$. This follows from Lemma 1 and Proposition 2.

Suppose the principal chooses the strategy $\sigma = (0, 0, 1)$ for all (θ_1, θ_2). Then,

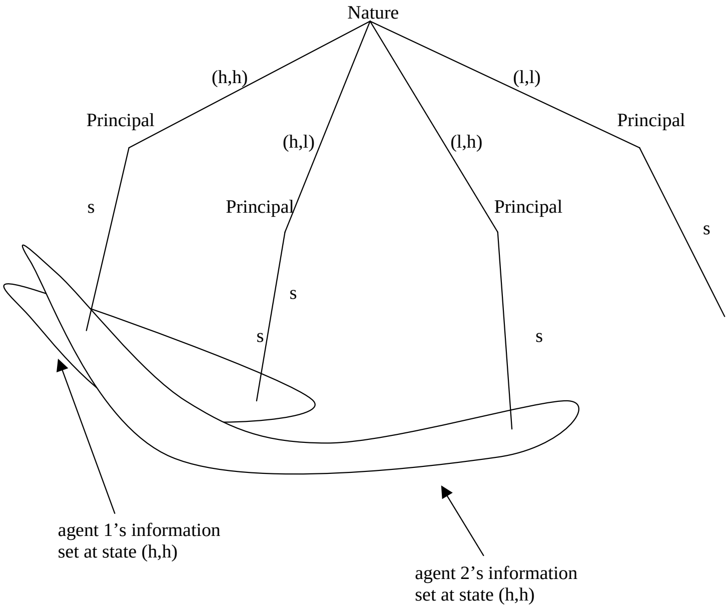

Figure 1. The subgame G(h,h)

$$\overline{y}\left(h|s=\emptyset\right) = \begin{cases} y_\ell & \text{with probability } \mu(\ell) \\ y_h & \text{with probability } 1-\mu(\ell), \textit{and} \end{cases}$$
$$\overline{y}\left(\ell|s=\emptyset\right) = y_\ell \quad \text{with probability } 1,$$

with $\overline{y}\left(\theta_i|s=\emptyset\right)$ being the induced lottery as defined in the proof of Theorem 3.

If agent i is of type ℓ, $e_i = V/\ell$ is his strictly dominant strategy associated with a payoff equal to zero. On the other hand if agent i is of type h and chooses $e_i = V/\ell$, his expected payoff is $V\left(\mu(\ell) - h/\ell\right)$, while if he chooses $e_i = V/h$, his expected payoff is 0. Thus, his best-response function is

$$b_i\left(h|\emptyset\right) = \begin{cases} V/\ell & \text{if } \mu(\ell) \geq \frac{h}{\ell} \\ V/h & \text{otherwise.} \end{cases}$$

The payoff to the principal with the strategy $\sigma = (0,0,1)$ is $2f(V/\ell)$ if $\theta_1 = \theta_2 = \ell$, $f(V/\ell) + f(V/h)$ if $\theta_1 \neq \theta_2$ and $\mu(\ell) < h/\ell$, $2f(V/\ell)$ if $\theta_1 \neq \theta_2$ and $\mu(\ell) \geq h/\ell$, $2f(V/h)$ if $\theta_1 = \theta_2 = h$ and $\mu(\ell) < h/\ell$, and lastly $2f(V/\ell)$ if $\theta_1 = \theta_2 = h$ and $\mu(\ell) \geq h/\ell$. Consider now the following state-contingent subgame perfect pure strategy $\hat{\sigma}$ defined as

$$\hat{\sigma} = \begin{cases} (1,0,0) & \text{if } (\theta_1,\theta_2) = (\ell,\ell), \text{payoff } 2f(V/\ell) \\ (1,0,0) & \text{if } (\theta_1,\theta_2) \in \{(\ell,h),(h,\ell)\}, \text{payoff } 2f(V/\ell) \\ (0,1,0) & \text{if } (\theta_1,\theta_2) = (h,h), \text{payoff } 2f(V/h) \end{cases}$$

where the contract with $s = \emptyset$ occurs with zero probability for any given state. Let us begin with the following observation. Notice that whenever $\mu(\ell) < \frac{h}{\ell}$

and $\theta_1 \neq \theta_2$, the principal strictly prefers $\sigma = (0,0,1)$ to $\hat{\sigma}$, while if $\mu(\ell) \geq \frac{h}{\ell}$ and $\theta_1 = \theta_2 = h$, he strictly prefers $\hat{\sigma}$ to $\sigma = (0,0,1)$. But this implies that if agents observe $s = \emptyset$ (which is possible under $\sigma = (0,0,1)$ and impossible under $\hat{\sigma}$) and $\mu(\ell) < \frac{h}{\ell}$, it may reveal to the agents that in fact $\theta_1 \neq \theta_2$ and the very benefit of $\sigma = (0,0,1)$ may cease to exist as now the high skilled agent will update his priors and may only exert an effort equal to V/ℓ. Thus the strategy $\sigma = (0,0,1)$ may not be a perfect Bayesian equilibrium with consistent beliefs unless the agents believe that occurrence of $s = \emptyset$ is out of the use of some possible state-contingent mixed strategy of the principal such that given the mixture, the updated priors (or posteriors in our case) of the high skilled agents still fall short of h/ℓ. The question remains as to whether such mixed strategies exist along with the required set of consistent beliefs. Clearly, such existence will maximize the principal's payoffs and even achieve the first best discriminatory outcomes whenever $\theta_1 \neq \theta_2$. The rest of the proposition is concerned with this existence issue. We proceed as follows.

Consider the following sets of state-contingent subgame perfect and possibly mixed strategies of the principal:

(a) if $(\theta_1, \theta_2) = (\ell, \ell)$, the principal chooses from the set

$$\sigma' = \left\{ \left(\sigma'(y_\ell), 0, \sigma'\left(\emptyset\right) \right) \right\},$$

for any $\sigma'(y_\ell) \in [0,1]$.

(b) if $(\theta_1, \theta_2) = (h, h)$, the principal chooses from the set

$$\sigma'' = \begin{cases} (0,1,0) & \text{if } \mu(\ell) \geq \frac{h}{\ell} \\ \left(0, \sigma''(y_h), \sigma''\left(\emptyset\right)\right) & \text{if } \mu(\ell) < \frac{h}{\ell}, \end{cases}$$

for any $\sigma''(y_h) \in [0,1]$.

(c) if $(\theta_1, \theta_2) \in \{(h,\ell), (\ell, h)\}$, the principal chooses from the set

$$\sigma''' = \begin{cases} \left(\sigma'''(y_\ell), 0, \sigma'''\left(\emptyset\right)\right) & \text{if } \mu(\ell) \geq \frac{h}{\ell} \\ (0,0,1) & \textit{if } \mu(\ell) < \frac{h}{\ell}, \end{cases}$$

for any $\sigma'''(y_\ell) \in [0,1]$.

Given these sets of strategies of the principal, *first suppose* $\mu(\ell) < h/\ell$. If agent i of type ℓ observes $s = \emptyset$, his posterior belief is

$$\mu_\ell\left(h|\emptyset\right) = \frac{\mu(h)}{\mu(h) + \sigma'\left(\emptyset\right)\mu(\ell)},$$

while if agent i of type h observes $s = \emptyset$, his posterior belief is

$$\mu_h\left(h|\emptyset\right) = \frac{\sigma''\left(\emptyset\right)\mu(h)}{\sigma''\left(\emptyset\right)\mu(h) + \mu(\ell)}. \tag{*}$$

Let $(\theta_1, \theta_2) = (\ell, \ell)$. Then given ($a$), any $\sigma'(y_\ell) \in [0,1]$ will constitute an equilibrium point in the subgame $G_{(\ell,\ell)}$. This is because independent of the nature of the observed contract under (a), each agent being of low skilled will exert

$e_i = V/\ell$ and the principal will receive $2f(V/\ell)$. On the other hand with any positive probability attached to y_h, the principal risks the payoff of $2f(0)$ with positive probability.

Now suppose $(\theta_1, \theta_2) = (h, h)$. The agents update their priors to obtain Eq.($*$). So, if $1 - \mu_h\left(h|\emptyset\right) < h/\ell$, that is

$$\frac{\mu(\ell)}{\sigma''\left(\emptyset\right)\mu(h) + \mu(\ell)} < \frac{h}{\ell}, \tag{3}$$

then $\sigma'' = \left(0, \sigma''(y_h), \sigma''\left(\emptyset\right)\right)$ is an equilibrium point in the subgame $G_{(h,h)}$. This is because the principal then receives $2f(V/h)$ from which he cannot improve. What remains to be shown in this case is that such a σ'' exists. Rewriting Eq.(3), we have

$$\sigma''\left(\emptyset\right) > \frac{\ell - h}{h}\frac{\mu(\ell)}{\mu(h)} \in (0, 1).$$

Therefore for any $\mu(\ell) \in \left[0, h/\ell\right)$, there exists such a $\sigma''\left(\emptyset\right) \in (0, 1]$.

Finally, let $(\theta_1, \theta_2) \in \{(h, \ell), (\ell, h)\}$. Then, $\sigma''' = (0, 0, 1)$ is an equilibrium point in the subgames $G_{(h,\ell)}$ and $G_{(\ell,h)}$ if Eq.(3) holds as now the payoff to the principal is $f(V/h) + f(V/\ell)$ which is the full efficiency outcome and thus the principal cannot improve. Clearly from the above, existence of σ'' guarantees existence of σ'''.

Thus, the principal by offering incomplete contracts with positive probability in all states, induces the high type to exert more effort, which is not possible under complete contracts.

Second suppose that $\mu(\ell) \geq h/\ell$. If agent i of type ℓ observes $s = \emptyset$, his posterior belief is

$$\mu_\ell\left(h|\emptyset\right) = \frac{\sigma'''\left(\emptyset\right)\mu(h)}{\sigma'''\left(\emptyset\right)\mu(h) + \sigma'\left(\emptyset\right)\mu(\ell)},$$

while if the agent is of type h and observes $s = \emptyset$, his posterior is $\mu_h\left(h|\emptyset\right) = 0$. Clearly, if $(\theta_1, \theta_2) = (\ell, \ell)$ then any $\sigma'\left(\emptyset\right) \in [0, 1]$ is an equilibrium point in the subgame $G_{(\ell,\ell)}$ as the payoff to the principal attains its maximum at $2f(V/\ell)$. Now suppose $(\theta_1, \theta_2) = (h, h)$. Then $\sigma'' = (0, 1, 0)$ is an equilibrium point in the subgame $G_{(h,h)}$ yielding a maximum attainable payoff of $2f(V/h)$ to the principle. Finally, suppose $(\theta_1, \theta_2) \in \{(h, \ell), (\ell, h)\}$. Then, $\sigma''' = \left(\sigma'''(y_\ell), 0, \sigma'''\left(\emptyset\right)\right)$ is an equilibrium point in the subgames $G_{(h,\ell)}$ and $G_{(\ell,h)}$ if $1 - \mu_h\left(h|\emptyset\right) \geq h/\ell$, which is clearly true since $\mu_h(h|\emptyset) = 0$. The high type will exert effort V/ℓ and the payoff to the principal is $2f(V/\ell)$ which is what the principal would have obtained by not being vague. Hence, in this case offering an incomplete contract does not make the principal strictly better off. □

Since types are close, whenever at least one agent is of a low type, the optimal contract comes with the specification (explicit or implicit) of y_ℓ. Only in the event both agents are of high type, the optimal complete contract is y_h. Therefore an agent with low type produces y_ℓ independent of whether the contract offered is complete or incomplete since an incomplete contract with at least one low type agent implicitly comes with the threshold y_ℓ. Given this, whenever both agents are of a low type, the principal is indifferent between a complete and an incomplete contract. On the other hand, if a high type agent observes an incomplete contract, he knows that with some probability his fellow agent is of a low type and has an incentive to reduce effort to produce y_ℓ in order to earn a positive surplus. However in doing so, he faces an indirect threat of his fellow agent being of a high type in which case he knows the implicit threshold becomes y_h. Producing y_ℓ in such an event leaves him with a negative surplus. Clearly if the probability of a low type fellow agent is sufficiently small, this threat outweighs the benefit of reducing his effort. Thus when the probability of a low type agent is sufficiently small and the agents are of different types, the principal is strictly better off by offering an incomplete contract since it induces the high type agent to indirectly compete with his fellow agent of a type unknown to him. This induces more effort from the high type agent without lowering the effort of the low type. In particular the high type produces y_h while the low type still produces y_ℓ. Notice that a high type agent cannot infer the true type of his fellow agent when an incomplete contract is offered. This is due to the fact that under the same conditions (that the probability of a low type agent is sufficiently small), a high type agent expects to observe (with some positive probability) an incomplete contract even when his fellow agent is of a high type since in this case the principal is indifferent between the two types of contracts.

4.3 A numerical example

We conclude this section with a simple numerical example to illustrate the working of the two theorems. Suppose $f(e_i) = e_i$ and let the cost of effort for agent i of skill θ_i be $\theta_i e_i$. Suppose $V = 10$. First let $\theta_i \in \{1/3, 1\}$. So in our notation we have $h = 1/3$ and $\ell = 1$. Consequently the zero surplus effort level of a low skilled agent is 10 while that of a high skilled agent is 30. Notice also that the optimal performance schedule as in Proposition 2 is given by

$$\begin{aligned} \bar{y}^*(h,h) &= 30,\ \bar{y}^*(\ell,\ell) = 10 \text{ and} \\ \bar{y}^*(\ell,h) &= \bar{y}^*(h,\ell) = 30 \text{ since } f(30) + f(0) = 30 > 2f(10) = 20. \end{aligned}$$

Suppose the principal offers an incomplete contract. Suppose agent i has skill 1. Clearly, with $e_i = 10$, he receives $\mu(\ell)[0] + (1 - \mu(\ell))[-10] < 0$ for any $\mu(\ell) \in (0, 1)$. Also, it is easy to see that for any effort greater than 10 the low skilled agent for sure receives a negative expected payoff and therefore it is optimal for agent i of skill 1 to exert zero effort. Similarly if agent i is of skill 1/3 and exerts an effort equal to 10 he knows from the optimal performance schedule that independent of the skill of his fellow agent, he will not be rewarded and thus earns a negative payoff as well. He can clearly do best by setting $e_i = 30$ which is the maximum effort he is willing to exert under any circumstance as well as the minimum effort required to earn the reward. In this case he earns the reward

10 but derives a disutility out of effort equal to $\frac{30}{3} = 10$ and thus earns a zero surplus. Any other choice of effort reduces his surplus below zero (excepting when effort is zero in which case he is indifferent between zero effort and effort equal to 30 and we assume in such situations he always chooses $e_i = 30$). So with $s = \emptyset$ the payoff to the principal is 0 if $\theta_1 = \theta_2 = \ell$, 60 if $\theta_1 = \theta_2 = h$ and 30 if $\theta_1 \neq \theta_2$. The principal can do better by the following state-contingent strategy:

$$s = \begin{cases} 10 \text{ if } \theta_1 = \theta_2 = \ell \\ 30 \text{ if } \theta_1 = \theta_2 = h \\ 30 \text{ if } \theta_1 \neq \theta_2, \end{cases}$$

yielding a payoff equal to 20 if $\theta_1 = \theta_2 = \ell$, 60 if $\theta_1 = \theta_2 = h$ and 30 if $\theta_1 \neq \theta_2$. Thus the principal is indifferent between a complete and an incomplete contract whenever $\theta_i = h$ for at least some $i = 1, 2$ and strictly prefers being complete if $\theta_1 = \theta_2 = \ell$. This establishes the result as in Theorem 3.

Now let us bring the two skills closer by assuming that $\theta_i \in \{2/3, 1\}$. Then the zero surplus effort level of a low skilled agent remains at 10 while that of a high skilled agent drops to 15. The optimal schedule is now

$$\begin{aligned} \overline{y}^*(h,h) &= 15, \overline{y}^*(\ell,\ell) = 10 \text{ and} \\ \overline{y}^*(\ell,h) &= \overline{y}^*(h,\ell) = 10 \text{ since } f(15) + f(0) = 15 < 2f(10) = 20. \end{aligned}$$

Suppose the contract observed is $s = \emptyset$. It is easy to see that if agent i is of skill 1, his strictly dominant strategy is to set $e_i = 10$ as by the very presence of himself in the workforce he knows that the hidden threshold requirement is 10. On the other hand as shown in Theorem 4, the best response of the high skilled agent with $h = 2/3$ is

$$b_i\left(\frac{2}{3}|\emptyset\right) = \begin{cases} 10 & \text{if } \mu(\ell) \geq \frac{2}{3} \\ 15 & \text{if } \mu(\ell) < \frac{2}{3}. \end{cases}$$

The payoff to the principal under $s = \emptyset$ is 20 if $\theta_1 = \theta_2 = \ell$, $\left.\begin{matrix} 20 \text{ if } \mu(\ell) \geq 2/3 \\ 30 \text{ if } \mu(\ell) < 2/3 \end{matrix}\right\}$ if $\theta_1 = \theta_2 = h$ and $\left.\begin{matrix} 20 \text{ if } \mu(\ell) \geq 2/3 \\ 25 \text{ if } \mu(\ell) < 2/3 \end{matrix}\right\}$ if $\theta_1 \neq \theta_2$. Suppose $\mu(\ell) \geq 2/3$. Then an incomplete contract has no additional benefit over the optimal state-contingent complete contracting strategy given by

$$s = \begin{cases} 10 \text{ if } \theta_1 = \theta_2 = \ell \\ 15 \text{ if } \theta_1 = \theta_2 = h \\ 10 \text{ if } \theta_1 \neq \theta_2, \end{cases}$$

yielding a payoff of 20 if $\theta_1 = \theta_2 = \ell$, 30 if $\theta_1 = \theta_2 = h$ and 20 if $\theta_1 \neq \theta_2$ to the principal where in fact the principal prefers to be complete when $\theta_1 = \theta_2 = h$. On the other hand when $\mu(\ell) < 2/3$, the principal strictly prefers to remain incomplete whenever $\theta_1 \neq \theta_2$. Suppose the true state is in fact $\theta_1 \neq \theta_2$ and suppose the principal uses the above pure strategy by which he offers an incomplete contract. Each agent will immediately realize that they are in a heterogeneous work force. Consequently the high skilled agent will choose an effort equal to 10 (instead of 15) and payoff to the principal will be 20 (instead of 25) and therefore incomplete contracts will lose its supposed advantage. Thus observing incompleteness of the contract as an outcome of such a pure strategy will not be enough to induce indirect competition. This leads us to look for mixed strategies such that given the probabilities attached to each possible contract type, the posteriors of the high skill agents still fall short of 2/3. Suppose $\mu(\ell) = \mu(h) = 1/2$. Notice that by construction we have $\mu(\ell) = 1/2 < h/\ell = 2/3$. So invoking theorem 4, consider the following state-contingent mixed strategy of the principal defined by

(a) if $(\theta_1, \theta_2) = (\ell, \ell)$, the principal chooses

$$\sigma' = (0, 0, 1),$$

(b) if $(\theta_1, \theta_2) = (h, h)$, the principal chooses

$$\sigma'' = \left(0, 1/3, 2/3\right) \text{ , and}$$

(c) if $(\theta_1, \theta_2) \in \{(h, \ell), (\ell, h)\}$, the principal chooses

$$\sigma''' = (0, 0, 1).$$

Then

$$\mu_\ell(h \mid \emptyset) = \frac{1}{2} \text{ and } \mu_h(h \mid \emptyset) = \frac{2}{5}.$$

Notice first that under this strategy an incomplete contract occurs with positive probability under all states. Suppose $(\theta_1, \theta_2) = (\ell, \ell)$. This is not a problem because independent of the nature of the contract, since $f(15) + f(0) < 2f(10)$, each agent exerts effort equal to 10 and the principal receives a payoff of 20. Now suppose $(\theta_1, \theta_2) = (h, h)$. Upon observing $s = \emptyset$, each agent assumes the other agent is of skill 2/3 with probability 2/5. Given this if they exert an effort equal to 10, their expected payoff is $\frac{3}{5}(10 - \frac{20}{3}) + \frac{2}{5}\left(-\frac{20}{3}\right) = -\frac{2}{3} < 0$ while if they exert an effort equal to 15, their expected payoff is zero. Thus whenever there is a high skilled agent, given the above strategies the best response effort from him is 15. Therefore the payoff of the principal from this strategy is exactly what he would have obtained with the strategy $\sigma = (0, 0, 1)$ with $\mu(\ell) < h/\ell$ without any further updating on part of the agents. Clearly the principal cannot improve upon these payoffs. Thus incomplete contracting occurs with positive probability[14] in equilibrium and achieves first best discriminatory outcomes.

[14] As we stated in theorem 4 a strategy where the principal offers an incomplete contract in each state with probability 1 is also an equilibrium. This can be easily checked since if $\sigma'' = (0, 0, 1)$ the updated belief of the high type agent that the other agent is of low type given that he has observed an incomplete contract is $1/2$ which is less than $h/\ell = 2/3$.

5 Conclusion

Observed incompleteness of contractual agreements may occur out of various reasons and becomes a difficult venture to capture all of them in a general setting. In this paper we show that in case of labor contracts, the flow of information regarding the skills of the work-force amongst the employers and the employee may also play a major role. In this respect, widespread incompleteness of labor contracts in nondiscriminatory environments stems out of the fact that the employers may sometimes find it beneficial to hide information regarding the profile of existing skills in their work force by strategically choosing to remain vague while writing their contracts. This induces each employee to engage in some form of an indirect competition in order to earn future employment, thereby enabling the employer to successfully induce the highest efforts from each one of them. More interestingly, in our environment where the employer has perfect information about the true skills of his employees and where discriminatory contracts are prohibited by law or otherwise, we show that incomplete contracts also achieve the first best discriminatory outcomes in our setting. Consequently, each employee, given his true skill, exerts the highest possible effort and as a result, the employer extracts the entire surplus from each one of them without being discriminatory at all.

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