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Risk sharing vs. incentives: Contract design under two-sided heterogeneity

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Abstract

We study the matching patterns between heterogeneous principals and agents in a principal agent model. The resulting equilibrium relationship between risk and incentives could be negative, positive or U-shaped. These results may provide an explanation for the absence of systematic empirical support for the standard risk model. © 2005 Elsevier B.V. All rights reserved.

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1. Introduction

Standard principal agent models predict a negative relationship between risk and performance pay (e.g. [Holmstrom and Milgrom, 1987](#)). This prediction has generated a voluminous empirical literature across many fields and markets (see [Prendergast, 2002](#) for a comprehensive survey) which attempts to verify the validity of the risk sharing model. Nevertheless, only a small fraction of these applications have indeed confirmed the inverse relationship, while the majority of them has either discovered a positive or an insignificant association between risk and the power of the contract. The conclusion then

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is that risk does not seem to play a major role in contract design. One theme appears to be common among the vast majority of the empirical papers: although the data come from a market which consists of heterogeneous principals and agents, the analysis ignores the possibility of endogenous matching between principals and agents and tests the predictions which emerge from an isolated principal agent pair.¹ Our central message in this paper is that *risk* may be *important* in contract design *even if* the data unveil a *positive* relationship between risk and incentives.

We embed an isolated principal agent pair into a two-sided heterogeneous principal agent market to study the equilibrium matching patterns. Principals are heterogeneous with respect to the riskiness of their asset and agents with respect to their degree of risk aversion. We find sufficient conditions for the existence of a positively assortative matching (PAM, i.e., agents with higher degrees of risk aversion are matched with higher risk principals) and a negatively assortative matching (NAM, i.e., agents with higher degrees of risk aversion are matched with lower risk principals). This generates predictions which come closer to the empirical regularities, without departing from the standard risk sharing model.² Under a NAM, the equilibrium relationship between risk and incentives may be *positive*. The intuition is as follows. An increase in risk lowers the incentives provided that the degree of risk aversion is held fixed. This is the standard *negative effect*. On the other hand, the increase in risk lowers the degree of risk aversion (due to the NAM) which puts upward pressure on incentives. This is a *positive effect* which has been overlooked. The overall effect of risk on incentives depends on the relative strength of these two effects. If the matching curve is steep enough, then the positive effect outweighs the negative and the relationship between risk and incentives is positive.

The possibility of a PAM is also quite interesting. [Akerberg and Botticini \(2002\)](#) find that tenants with relatively low wealth (and hence high degree of risk aversion) were employed to cultivate vines, while the less volatile cereal was contracted out to wealthier tenants (low risk averse tenants). This positive relationship between the degree of risk aversion of a tenant and the riskiness of a crop (i.e., a PAM) seems at first sight counterintuitive ([Akerberg and Botticini, 2002](#), p. 579). It is nevertheless perfectly consistent with equilibrium behavior.

2. The model

A market consists of a continuum of risk neutral principals and risk averse agents. Principals, who are indexed by p , are identified by the variance of their asset, σ^2 , and are uniformly distributed on the interval $[\sigma_L^2, \sigma_H^2]$. Agents, who are indexed by a , are identified by their degree of absolute risk aversion, r , and are uniformly distributed on the interval $[r_L, r_H]$. One principal has to match with exactly one agent, and vice versa, to produce output. Denote a given pair by (p, a) . This is a standard moral hazard problem (e.g. [Holmstrom and Milgrom, 1987](#)). The production function for this specific pair is

¹ A notable exception is the empirical paper by [Akerberg and Botticini \(2002\)](#). They argue that in the presence of endogenous matching the degree of risk aversion is correlated with the riskiness of the asset and if the former cannot be measured accurately (it is not observed by the econometrician) its error is correlated with the principal's characteristics causing the coefficients to be biased. Using a historical data set on agricultural contracts from Renaissance Tuscany, they find strong evidence for endogenous matching between landlords and tenants and that risk sharing is an important determinant of contract choice.

² A number of papers deviate from the risk sharing model in an attempt to explain the positive relationship between risk and incentives (e.g. [Prendergast, 2002](#)).

$y_{p,a} = e_a + \epsilon_p$, where e_a denotes the agent's (non-contractible) effort and ϵ_p is normally distributed (and independently of the other principals' error distributions) with mean zero and variance (risk) σ_p^2 . Effort is costly, with its cost equal to $ce_a^2/2$ (same across agents). The agent's utility function is $V = 1 - \exp[-r_a(w_{p,a} - ce_a^2/2)]$, where $w_{p,a}$ is the agent's compensation. All players have a zero reservation utility. The specification adopted in this paper, while not very general, is the most commonly used in the literature (see Prendergast, 2002, p. 1076, for a discussion on this issue). In this setting, it is well-known that the optimal compensation scheme is linear in output, i.e., $w_{p,a} = \alpha_{p,a}y_{p,a} + \beta_{p,a}$, where α denotes the level of incentives and β the fixed salary. Moreover, the optimal power of the contract is given by,

$$\alpha_{p,a}(r_a, \sigma_p^2) = \frac{1}{1 + cr_a\sigma_p^2}. \quad (1)$$

Now let us turn to the issue of two-sided matching between principals and agents. The only coalitions that matter are of size two (who can share their surplus in any way they wish, i.e., transferable utility) and moreover there are no externalities across coalitions. Therefore, any reasonable solution concept should maximize aggregate surplus (see Legros and Newman, 2002 for a similar approach).

3. Efficient matching

The next proposition summarizes the efficient matching patterns. The profit function exhibits non-monotonic marginal products and as a consequence general matching patterns are not easily characterized (for a discussion on this issue see Shimer and Smith, 2000, p. 346).³ Therefore, we focus on monotone (assortative) matching. Moreover, this kind of matching has a special intuitive appeal and yields straightforward testable implications.

Proposition 1 (*Sufficient conditions for assortative matching*).

- (i) (NAM). If $\sigma_H^2 r_H \leq 1/c$, then the efficient matching is negatively assortative, i.e., low risk averse agents are matched with high risk principals and vice versa.
- (ii) (PAM). If $\sigma_L^2 r_L \leq 1/c$, then the efficient matching is positively assortative, i.e., low risk averse agents are matched with low risk principals and vice versa.

Proof. The expected profits, after we plug (1) into the principal's expected profit function, are given by,

$$\Pi_{p,a} = \frac{1}{2c(1 + cr_a\sigma_p^2)}. \quad (2)$$

³ For example, Kremer and Maskin (1996) introduce a production function of the form $f(x, y) = \max\{x^2y, xy^2\}$, which exhibits non-monotonic marginal products.

The cross partial derivative is given by,

$$\frac{\partial \Pi_{p,a}}{\partial r_a \partial \sigma_p^2} = \frac{c r_a \sigma_p^2 - 1}{2(1 + c r_a \sigma_p^2)^3}. \quad (3)$$

The above derivative is positive if $r_a \sigma_p^2 \geq 1/c$ and negative if $r_a \sigma_p^2 \leq 1/c$. Hence, if $r_L \sigma_L^2 \geq 1/c$, then the cross partial is guaranteed to be positive for all r 's and σ^2 's in the support of the distributions and the profit function is supermodular. It follows then (see Becker, 1973) that a PAM is efficient. Similarly, if $\sigma_H^2 r_H \leq 1/c$, the profit function is submodular and we obtain a NAM. \square

If the product of the high ends of the supports of the types distributions is relatively low, then we obtain a NAM. In other words, in markets where the degrees of risk aversion are low and/or the variances of the assets are low, efficiency dictates that agents with low degrees of risk aversion should match with principals who own high risk projects and vice versa. A principal who owns an asset which is subject to high volatility benefits more if he hires an agent who is more tolerant towards risk. Conversely, if the product of the low ends of the supports of the types distributions is relatively high, then we obtain a PAM. This means that in markets where the degrees of risk aversion are high and/or the asset variances are high, agents with low degrees of risk aversion are matched with principals who own low risk projects and vice versa.

Let us look at the following illustrative example. Suppose there are a number of different islands and people who live on these islands want to buy umbrellas (of various degrees of durability or quality) to protect themselves from the rain. The intensity of rain varies from one island to another. Islands in this story are the principals and umbrellas are the agents. The intuition behind the results can be captured by answering the question, “which island will bid the most for the most durable umbrella?” For the NAM case we consider a situation where the overall intensity of the rain is relatively low (i.e., low risk). Then the island with the highest intensity of rain will receive the highest benefit from having the “best umbrella”. This is a negatively assortative matching between intensity of rain and umbrella qualities across the islands, i.e., more rain-higher umbrella quality.

For the PAM case we consider a situation where the overall intensity of rain is high (i.e., high risk). The island that benefits the most from the highest quality umbrella is now the one with the lowest rain intensity. The island with the highest intensity of rain does not benefit much, because its residents will get wet no matter what and therefore there is no reason to bid aggressively for the highest quality umbrella. This is a positively assortative matching between intensity of rain and umbrella qualities across the islands, i.e., more rain-lower umbrella quality.

4. Relationship between risk and incentives

Now we investigate the equilibrium relationship between risk and incentives. First, we assume a NAM. Since each principal is matched with exactly one agent and vice versa, it must be the case that,

$$\int_{\sigma_L^2}^{\sigma^2} \frac{1}{\sigma_H^2 - \sigma_L^2} dx = \int_r^{r_H} \frac{1}{r_H - r_L} dy. \quad (4)$$

This is a “measure consistency” condition which says that the mass of principals is equal to that of agents (see Legros and Newman, 2002). Eq. (4) yields the following equilibrium relationship between the degree of risk aversion and variance (i.e., a matching function),

$$r(\sigma^2) = k - b\sigma^2, \tag{5}$$

where $k = \frac{r_H\sigma_H^2 - r_L\sigma_L^2}{\sigma_H^2 - \sigma_L^2} > 0$ and $b = \frac{r_H - r_L}{\sigma_H^2 - \sigma_L^2} > 0$. Now plug (5) into (1) to obtain the equilibrium relationship between risk and incentives. (We suppress the dependence of incentives on p and a .) This yields the following expression,

$$\alpha(\sigma^2) = \frac{\sigma_H^2 - \sigma_L^2}{\sigma_H^2 - \sigma_L^2 + c(r_H\sigma_H^2 - r_L\sigma_L^2)\sigma^2 - c(r_H - r_L)\sigma^4}. \tag{6}$$

Differentiating (6) with respect to σ^2 we obtain,

$$\begin{aligned} \frac{d\alpha(\sigma^2)}{d\sigma^2} &= \underbrace{\frac{\partial\alpha(r, \sigma^2)}{\partial\sigma^2}}_{\text{Negative effect}} + \underbrace{\frac{\partial\alpha(r, \sigma^2)}{\partial r} \frac{dr(\sigma^2)}{d\sigma^2}}_{\text{Positive effect}} \\ &= \frac{c(\sigma_H^2 - \sigma_L^2)(-r_H\sigma_H^2 + r_L\sigma_L^2 + 2r_H\sigma^2 - 2r_L\sigma^2)}{[\sigma_H^2 - \sigma_L^2 + c(r_H\sigma_H^2 - r_L\sigma_L^2)\sigma^2 - c(r_H - r_L)\sigma^4]^2}. \end{aligned} \tag{7}$$

There are two opposing effects on incentives as risk increases. First, an increase in risk lowers the incentives if the degree of risk aversion is kept unchanged (negative effect). Second, an increase in risk lowers the degree of risk aversion (due to the NAM) which increases the incentives (positive effect).

Now assume a PAM. In this case both of the effects that we mention above are negative and the resulting relationship between risk and incentives is always negative. The next proposition summarizes the equilibrium relationship between risk and incentives.

Proposition 2 (Relationship between risk and incentives). *First, suppose that $r_H\sigma_H^2 \leq 1/c$ so that we have a NAM. Then, the resulting equilibrium relationship between risk and incentives is,*

- (i) Positive, if $\sigma_L^2 \geq \frac{r_H\sigma_H^2}{2r_H - r_L}$.
- (ii) Negative, if $r_H \leq 2r_L$, and $\sigma_H^2 \geq \frac{\sigma_L^2 r_L}{2r_L - r_H}$.
- (iii) U-shaped, if one of the following two conditions is satisfied:
 1. (a) $r_H < 2r_L$, (b) $\sigma_H^2 < \frac{\sigma_L^2 r_L}{2r_L - r_H}$ and (c) $\sigma_L^2 < \frac{r_H\sigma_H^2}{2r_H - r_L}$,
 2. (a) $r_H < 2r_L$ and (b) $\sigma_L^2 < \frac{r_H\sigma_H^2}{2r_H - r_L}$.

Second, suppose that $r_L\sigma_L^2 \geq 1/c$ so that we have a PAM. Then, the resulting equilibrium relationship between risk and incentives is always negative.

Proof. The proof when we have a PAM is obvious. Therefore, we assume that we have a NAM. It can be readily verified that $\frac{d\alpha}{d\sigma^2} > 0$ (see (7)) if and only if $\sigma^2 > \lambda = \frac{r_H\sigma_H^2 - r_L\sigma_L^2}{2(r_H - r_L)}$. Next, we find conditions for λ to be in $[\sigma_L^2, \sigma_H^2]$. The following can be easily verified.

First, $\lambda < \sigma_H^2$ if and only if $r_H > 2r_L$. Second, $\lambda < \sigma_H^2$ if and only if $r_H < 2r_L$ and $\sigma_H^2 < \frac{\sigma_L^2 r_L}{2r_L - r_H}$. Finally, $\lambda > \sigma_L^2$ if and only if $\sigma_L^2 < \frac{r_H \sigma_H^2}{2r_H - r_L}$. The rest of the proof follows easily. \square

Suppose that we have a NAM. If the distribution of risk across principals is “tight” relative to the distribution of the degrees of risk aversion across agents, then the equilibrium relationship between risk and incentives is positive. This is because a relatively “tight” support of the risk distribution yields a steep matching curve, which in turn implies that the positive effect outweighs the negative. Hence, a *positive* relationship between performance pay and the power of the contract is *compatible* with the risk sharing model.

On the other hand, if the distribution of the degrees of risk aversion across agents is “tight” relative to the distribution of risk across principals, then the equilibrium relationship between risk and incentives is negative. In this case the matching curve is not too steep and the negative effect dominates the positive. Finally, the relationship between risk and incentives is U-shaped when neither distribution is too “tight”.

5. Empirical implications

Below, we summarize the main empirical implications that emerge from our analysis.

1. A NAM is a *necessary* condition for the existence of a positive relationship between risk and incentives. We should expect a NAM in markets populated by agents with low degrees of risk aversion and/or by principals with low risk assets.
2. A NAM and a steep matching curve is a *sufficient* condition for the existence of a positive relationship between risk and incentives. This should take place in markets where the distribution of risk is “tight” relative to the distribution of the degrees of risk aversion.
3. A PAM is a *sufficient* condition for the existence of a negative relationship between risk and incentives. We should expect a PAM in markets populated by agents with high degrees of risk aversion and/or by principals with high risk assets.

The results in [Akerberg and Botticini \(2002\)](#) are consistent with the last case. In particular, using agricultural data from Renaissance Tuscany, they find a PAM and a negative relationship between risk and incentives.

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