



## Retailer price distributions and promotional activities<sup>☆</sup>

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### ABSTRACT

An interesting feature of the data is that the price distribution in the absence of promotional activities first order stochastically dominates that under display or feature advertising. The theoretical model we introduce can yield an equilibrium that is consistent with the above observations.

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### 1. Introduction

Researchers working with scanner data have noted a somewhat counter-intuitive relationship between promotional activities, such as feature advertising (a flyer sent by a retailer to consumers) and end-of-aisle displays in the supermarket, and retail prices. These types of promotional activities and advertisement, in general, are often treated, particularly in the empirical literature, as parallel “demand shifters.” In this context, most in-store displays and feature advertisements should, a priori, result in higher predicted retail prices. However, it has been observed that prices usually, but not always, fall during these periods of promotional activity. For example, [Hendel and Nevo \(2006\)](#) find a positive correlation between both in-store display and feature advertising and retail price reductions.<sup>2</sup> On average, then, the observed price of a product is lower under such promotional activities.

Thinking about the mean/average is only part of the picture, however. As [Figs. 1 and 2](#) illustrate, looking at observed distributions under a

promotional activity and no such activity reveal an interesting detail: the price distribution for non-displayed or non-featured products *first order stochastically dominate* the corresponding price distribution for displayed or featured products.<sup>3</sup> This first order stochastic dominance observation is a much stronger prediction than merely asserting that the mean price under a promotional activity is lower.

In this paper, we formulate a model of retail promotion that is a variant of [Varian's \(1980\)](#) and [Narasimhan's \(1988\)](#) models and similar to [Iyer et al. \(2005\)](#).<sup>4</sup>

For ease of exposition, in the remaining of the paper, we limit our discussion of retailer promotional activities to in-store display. The intuition and modeling assumptions, however, also apply to feature advertising.

Our model accounts for two key observable features in the data: i) items are put on display only a fraction of the time and ii) prices vary significantly even within a short period of time. Our primary goal is to specify a model that might allow us to determine which conditions result in the first order stochastic dominance we observe in the data.

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<sup>2</sup> In some related work, [Nevo and Wolfram \(2002\)](#) find the somewhat counter-intuitive result that shelf prices are lower during periods when coupons are offered.

<sup>3</sup> We use weekly store-level data on sales, price, and display that were collected by ACNielsen in Springfield, Missouri. The data, which come from 9 retail stores over a 102 week period (from 1986 to 1988) were collected through retail checkout scanners. A Kolmogorov–Smirnov test rejects the hypothesis that the promotion/non-promotion price distributions are equal in three of the four diagrams. While the feature/non-feature price distributions for Surf laundry detergent appear to display first order stochastic dominance, the Kolmogorov–Smirnov test fails to reject the null that the distributions are equal.

<sup>4</sup> [Iyer et al. \(2005\)](#) are addressing different questions. In particular, they are mainly interested in the profitability of targeted advertising. The main similarity between our paper and their paper is that we utilize their advertising framework to model in-store display.

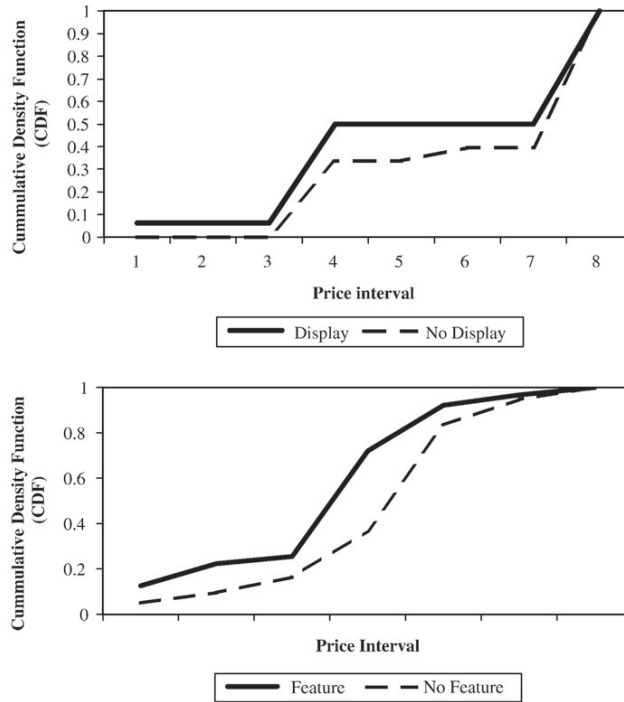


Fig. 1. Price distributions for Hunt's 32 oz ketchup.

Accurately understanding the role of display, and its relationship with price, is important in empirical models, as well. As far as we know, this is the first theoretical paper that formally examines the interplay between display and prices.

Display has two effects. First, it boosts demand (positive effect) and second it makes demand more elastic (negative effect) by attracting more comparison shoppers relative to loyal customers. If display attracts many more comparison shoppers, then the negative effect dominates the positive and the price distribution under no display first order stochastically dominates the distribution under display.

The bigger question that our paper addresses is: why firms cut prices in periods of high demand? There are several explanations offered in the literature. These include: loss leadership (Chevalier et al. (2003)), changes in price elasticities of demand (Bils (1989)), informative advertising (Warner and Barsky (1995)), collusion (Rotemberg and Saloner (1986)), and excess inventory. While these papers offer a number of convincing arguments, we believe that they cannot explain the consistent price difference (observed across many categories) between display and non-display. Nor are they able to explain the presence of first order stochastic dominance.

An important difference between our paper and the rest of that literature is that demand creation is endogenous in our model (through display), while the other papers treat demand surges as exogenous. Another difference is that our equilibrium is in mixed strategies (as opposed to pure) which we believe fits our retail data much better.

The paper that is most similar to our model is Johnson and Myatt (2006). Johnson and Myatt (2006) propose a framework for analyzing transformations of demand. Some transformations lead to demand shifts while others induce demand rotations. In particular, demand rotations are equivalent to the dispersion of consumer valuations. An increase in dispersion leads to a clockwise rotation of the demand curve. Our model, however, does not readily conform to their framework. One key difference is that in our model all consumers always have the same willingness to pay (no dispersion whatsoever). Demand rotates, in our framework, not because dispersion has changed but because more comparison shoppers become aware of the product when it is put on display.

## 2. The model

The market consists of two firms,  $i=1,2$ . Each firm produces its product at a zero marginal cost. The consumer market is comprised of

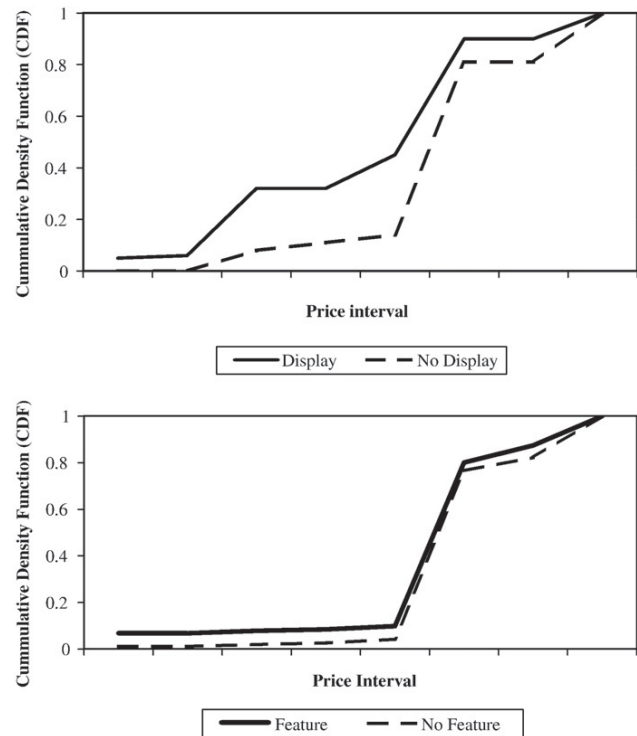


Fig. 2. Price distributions for Surf laundry detergent.

a unit mass of consumers with unit demands and reservation price  $r$ . Each firm has a segment of consumers who, if they buy, they buy only from that firm as long as the price is below the reservation price (loyal consumers,  $\ell$ ). The proportion of these consumers per firm is  $\alpha \leq \frac{1}{2}$ . The remaining consumers are comparison shoppers (switchers,  $s$ ) and buy from the firm that charges the lower price, if they are aware of both product offerings. If they are aware of only one product offering, they buy as long as the price is below  $r$ . If they are not aware of any product offering, then they do not buy. The size of the comparison shoppers is  $1 - 2\alpha$ .

Aside from choosing a price, each firm also decides whether to display (in-store) its product or not. (As we pointed out in the Introduction, the model also applies to feature advertising, after a simple relabeling). If a firm puts its item on display ( $D$ ) then all of its loyal customers and the comparison shoppers are aware of the product offering and the price. Otherwise (i.e., no display,  $ND$ ), only a fraction  $\phi_\ell > 0$  of the loyal consumers and a fraction  $\phi_s > 0$  of the switchers is aware of the existence of the product. Therefore, display increases the number of consumers who consider purchasing the product. We assume that display entails a fixed cost  $f > 0$ . Firms choose simultaneously whether to put the product on display and the price.

A pure strategy price equilibrium does not exist (e.g. Varian (1980)). Denote by  $\beta_i$  the probability with which firm  $i$  puts its item on display. The price distribution (cdf) of firm  $i$  will be conditional on whether that firm has its product on display or not. Denote that distribution by  $F_i^D(p)$  if the product is on display and by  $F_i^{ND}(p)$  if the product is not on display. The supports of the distributions are:  $[z_D, r]$  and  $[z_{ND}, r]$  (note that the lower end of the supports depends on whether the product is on display). We search for a symmetric mixed strategy equilibrium, SMSE (so we will drop the subscript  $i$ ), where firms also play mixed display strategies.

If both firms display with probability 1, then each firm can guarantee for itself, in a mixed strategy price equilibrium, a profit equal to  $\alpha r - f$  by charging  $p = r$  and selling to its loyal customers. If  $\alpha r - f$  is greater than  $\phi_\ell \alpha r$  (the guaranteed profit under no display) then firms will indeed display with probability 1. In other words firms display with probability 1 if the cost of display is not very high,  $f \leq \alpha r(1 - \phi_\ell)$ . If, on the other hand,  $f \geq \alpha r(1 - \phi_\ell)$  then the display equilibrium may involve mixed strategies. The profit of a firm, if it displays and charges a price  $p$ , is,

$$\begin{aligned} \pi^D(p) &= \alpha p + \beta [1 - F^D(p)] p(1 - 2\alpha) + (1 - \beta) [1 - F^{ND}(p)] \\ & p(1 - 2\alpha) + (1 - \beta) F^{ND}(p) p(1 - \phi_s)(1 - 2\alpha) - f. \end{aligned} \quad (1)$$

If the firm does not display and charges a price  $p$  its profit is,

$$\pi^{ND}(p) = \alpha \phi_\ell p + \beta [1 - F^D(p)] p \phi_s (1 - 2\alpha) + (1 - \beta) [1 - F^{ND}(p)] p \phi_s (1 - 2\alpha). \quad (2)$$

### 3. Analysis and results

In this section, we search for a SMSE and we examine the relationship of the equilibrium prices between display and no display.

Define,

$$X \equiv \left\{ \begin{array}{l} (\alpha, \phi_s, \phi_\ell, f, r) \in [0, 1]^3 \times \mathbb{R}_+^2 : \phi_\ell \geq \lambda_1 \equiv 1 - \frac{f}{r\alpha} \text{ and} \\ \phi_s \leq \lambda_2 \equiv \frac{1 - \alpha}{1 - 2\alpha} - \frac{\alpha \phi_\ell}{1 - 2\alpha} - \frac{f}{r(1 - 2\alpha)} \text{ and } \phi_s \leq \lambda_3 \equiv \frac{\alpha \phi_\ell r}{\alpha r \phi_\ell + f} \end{array} \right\}$$

Fig. 3 depicts the three constraints and the set  $X$  is the grey area in that figure. The next proposition summarizes the SMSE.

**Proposition 1.** (Existence of a SMSE)

If  $(\alpha, \phi_s, \phi_\ell, f, r) \in X$ , then there exists a symmetric mixed strategy equilibrium (SMSE) which can be described as follows,

- Each firm displays with probability,

$$\beta^* = 1 - \frac{f - r\alpha(1 - \phi_\ell)}{r(1 - \phi_s)(1 - 2\alpha)}. \quad (3)$$

- When a firm displays ( $D$ ) it draws its prices from the following distribution function,

$$F^{D*}(p) = \frac{r(p - z_D)}{p(r - z_D)}, \quad (4)$$

with support  $[z_D \equiv \frac{f}{(1 - \alpha) - \phi_s(1 - 2\alpha) - \alpha \phi_\ell}, r]$ .

- When a firm does not display ( $ND$ ) it draws its prices from the following distribution function,

$$F^{ND*}(p) = \frac{r(p - z_{ND})}{p(r - z_{ND})}. \quad (5)$$

with support  $[z_{ND} \equiv \frac{\alpha r \phi_\ell (1 - \phi_s) - f \phi_s}{\alpha(\phi_\ell - \phi_s)}, r]$ .

**Proof.** See Appendix.  $\square$

As we argue in the Appendix, there exist parameter constellations such that the set  $X$  is nonempty, see also Fig. 3 where a representative case is depicted.

For a SMSE to exist it must be the case that  $\phi_\ell \geq \phi_s$ . In other words, display must always attract more comparison shoppers than loyal customers.

The next proposition is concerned with the comparison of prices between display and no display.

**Proposition 2.** (Display can lead to lower prices)

Let  $(\alpha, r, f, \phi_s, \phi_\ell) \in X$ . Then, there exists a threshold given by,

$$\bar{\phi}_s = \frac{\alpha \phi_\ell (-r + \alpha \phi_\ell r + f + \alpha r)}{2\alpha^2 \phi_\ell r - \alpha \phi_\ell r + 2\alpha f - f}$$

such that,

- If  $\phi_s > \bar{\phi}_s$  the distribution under display  $F^{D*}$  first order stochastically dominates the distribution under no display  $F^{ND*}$ .
- If  $\phi_s < \bar{\phi}_s$  the distribution under no display  $F^{ND*}$  first order stochastically dominates the distribution under display  $F^{D*}$ .

**Proof.** See Appendix.  $\square$

Proposition 2 says that display leads to stochastically lower prices than the no display prices—and a lower average price—if relatively few switchers are aware of the product offering before display, i.e., low  $\phi_s$ , see Fig. 3. There are two opposing effects due to display. First, display shifts the demand curve out, which leads, ceteris paribus, to higher prices (positive effect). Second, display, in a SMSE, attracts relatively more comparison shoppers than loyal customers. This makes demand more elastic which leads, ceteris paribus, to lower prices (negative

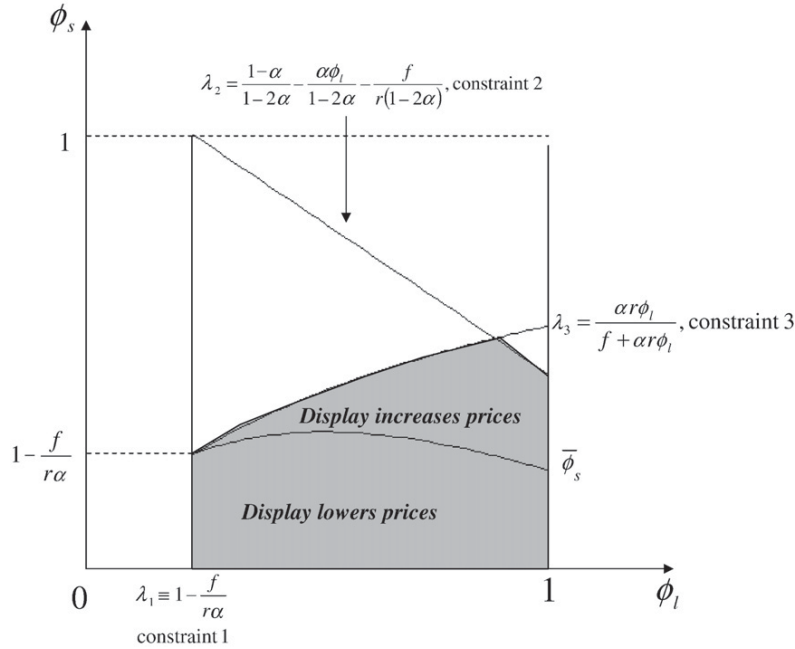


Fig. 3. Region of parameter values that guarantee the existence of a SMSE (grey area).

effect). When the negative effect dominates the positive then prices fall (stochastically) after the item is put on display. This happens when display attracts many more comparison shoppers than loyal customers, i.e., when  $\phi_s$  is very low. This assumption seems quite realistic. Otherwise, the negative effect is dominated by the positive and display leads to higher prices.<sup>5</sup>

**A Proof of Propositions 1 and 2.** When the firm displays the guaranteed profit from charging the reservation price  $r$  is  $\pi^D(p=r)$ , see (1), and it is equal to,

$$\pi^D(p=r) = \alpha r + (1-\beta)r(1-\phi_s)(1-2\alpha) - f.$$

When the firm does not display its guaranteed profit from charging the reservation price, see (2), is,

$$\pi^{ND}(p=r) = \alpha\phi_r r.$$

In a mixed strategy equilibrium the firm must be indifferent between display and no display. Setting  $\pi^D(p=r) = \pi^{ND}(p=r)$  and solving with respect to  $\beta$  we derive the equilibrium probability with which firms display,

$$\beta^* = 1 - \frac{f - r\alpha(1 - \phi_r)}{r(1 - \phi_s)(1 - 2\alpha)}.$$

It turns out that,

$$\beta^* \in (0, 1) \Leftrightarrow f \in (r\alpha(1 - \phi_r), r[(1 - \alpha) - \phi_s(1 - 2\alpha) - \alpha\phi_r]). \quad (6)$$

<sup>5</sup> While our empirical examples in Figs. 1 and 2 show the opposite case (display or feature advertising lowers price), the theoretical model predicts that this finding is true only for certain parameter values. We would, therefore, expect to observe promotional activities leading to higher prices if we had sufficient data for a wide variety of products.

This suggests that firms will randomize with respect to their choice to put the item on display if the cost of display  $f$  is neither too high nor too low. Otherwise, firms will either display with probability 1 or they will display with probability zero.

By substituting  $\beta^*$  into Eqs. (1) and (2), setting each profit function equal to the guaranteed profits, which are  $\alpha\phi_r r$ , and solving with respect to  $F^D(p)$  and  $F^{ND}(p)$  we can derive the candidate equilibrium distribution functions. These are given by,

$$F^{D*}(p) = \frac{r(-p\phi_s + 2p\phi_s\alpha - \alpha p - \alpha p\phi_r - f + p)}{p[r[(1 - \alpha) - \phi_s(1 - 2\alpha) - \alpha\phi_r] - f]}$$

$$F^{ND*}(p) = \frac{r(p\phi_s\alpha - \alpha p\phi_r - \alpha\phi_r\phi_s r + \alpha\phi_r r - f\phi_s)}{p\phi_s(r\alpha(1 - \phi_r) - f)}.$$

Now we want to ensure that  $F^{D*}(p)$  and  $F^{ND*}(p)$  are cumulative distribution functions. We must have:  $F^{D*}(p=z_D)=0$ ,  $F^{D*}(p=r)=1$  and  $\frac{dF^{D*}}{dp} \geq 0$  (and similar properties for  $F^{ND*}(p)$ ).

First, it can be readily verified that  $F^{D*}(p=r) = F^{ND*}(p=r) = 1$ . The lower bounds of the supports are derived by solving  $F^{ND*}(p=z_{ND})=0$  and  $F^{D*}(p=z_D)=0$  with respect to  $z_{ND}$  and  $z_D$ . This gives us,

$$z_D \equiv \frac{f}{(1 - \alpha) - \phi_s(1 - 2\alpha) - \alpha\phi_r} \quad (7)$$

$$z_{ND} \equiv \frac{\alpha\phi_r(1 - \phi_s) - f\phi_s}{\alpha(\phi_r - \phi_s)}. \quad (8)$$

Using Eqs. (7) and (8) and after some simplifications we obtain,

$$F^{D*}(p) = \frac{r(p - z_D)}{p(r - z_D)} \text{ and } F^{ND*}(p) = \frac{r(p - z_{ND})}{p(r - z_{ND})}.$$

To summarize, we have derived the following constraints, which must be satisfied in order for a SMSE to exist.

1. Constraint 1:  $\phi_r \geq \lambda_1 \equiv 1 - \frac{f}{r\alpha}$ . This ensures that  $\beta^* \leq 1$ , see Eq. (6).

2. Constraint 2:  $\phi_s \leq \lambda_2 = \frac{1-\alpha}{1-2\alpha} - \frac{\alpha\phi_r}{1-2\alpha} - \frac{f}{r(1-2\alpha)}$ . This ensures that  $\beta^* \geq 0$ , see Eq. (6).
3. Constraint 3:  $\phi_s \leq \lambda_3 = \frac{\alpha\phi_r r}{\alpha r \phi_r + f}$ . This ensures that  $z_{ND} \geq 0$ . Constraint 1 ensures that  $z_{ND} \leq r$ , provided that  $\phi_s > 0$ . If  $\phi_s = 0$ , then  $z_{ND} = r$ . From constraints 1 and 2 it also follows that  $z_D \leq r$ . Finally,  $z_D$  is always non-negative.

The following question now arises: is there a non-empty set of parameter values that satisfy all three constraints? This would prove that  $X \neq \emptyset$  (see Proposition 1). We have shown that this is indeed the case (details are straightforward and are omitted).

Next, we would like to study the properties of the equilibrium distribution functions. We set  $F^{D^*}(p) = F^{ND^*}(p)$  and we solve with respect to  $\phi_s$ . (Since the distribution functions have the same functional form this amounts to setting  $z_D = z_{ND}$  and solving for  $\phi_s$ ). This yields the following threshold,

$$\bar{\phi}_s(\phi_r) = \frac{\alpha\phi_r(-r + \alpha\phi_r r + f + \alpha r)}{2\alpha^2\phi_r r - \alpha\phi_r r + 2\alpha f - f}.$$

When  $\phi_s = \bar{\phi}_s$ , then  $z_D = z_{ND} = \frac{\alpha\phi_r + f}{1-\alpha}$ . If  $\phi_s > \bar{\phi}_s$ , then  $F^{D^*}(p) > F^{ND^*}(p)$  and therefore  $F^{ND^*}(p)$  first order stochastically dominates  $F^{D^*}(p)$ . In this case display leads to stochastically lower prices, and, of

course, a lower average price. Otherwise, display leads to stochastically higher prices and a higher average price.

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