



Market segmentation and collusive behavior

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Abstract

The recent literature on oligopolistic third-degree price discrimination has been primarily concerned with rival firms' incentives to acquire customer-specific information and the consequences of such information on firm profitability and welfare. This literature has taken mostly a static view of the interaction between competing firms. In contrast, in this paper we investigate the impact of customer-specific information on the likelihood of tacit collusion in a dynamic game of repeated interaction. This issue is very important because competitive price discrimination usually leads to a cutthroat price competition (prisoners' dilemma) among firms. Firms, therefore, may seek ways to soften competition and sustain higher prices. Our main result is that collusion becomes more difficult as the firms' ability to segment consumers improves.

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1. Introduction

We examine the interplay between market segmentation and the likelihood of tacit collusion. The recent rapid growth of the Internet as a medium of communication and commerce, combined with the development of sophisticated software tools, are to a large extent responsible for producing a new kind of information: databases with detailed records about consumers' preferences. Such data are gleaned from a customer's transactions with a firm and public records

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and are used to assemble a detailed picture of consumer preferences. Firms can utilize this information to segment consumers into different groups and charge each group a different price (third-degree price discrimination). Dell, for example, follows this practice.² Dynamic pricing, a business strategy in which prices are varied frequently by channel, product, customer and time, is another prominent example.³ Price discrimination can also be facilitated by intelligent agents that insert personalized targeted discounts on pop-up windows on a consumer's computer screen.⁴ As the Information Technology (IT) advances the quantity and quality of customer data increase and the resulting segmentation becomes increasingly refined.

The recent literature on oligopolistic third-degree price discrimination⁵ has been primarily concerned with rival firms' incentives to acquire customer-specific information and the consequences of such information on firm profitability and welfare.⁶ But this literature has taken mostly a static view of the interaction between competing firms.⁷ In contrast, in this paper we investigate the impact of customer-specific information in a dynamic game of repeated interaction. This allows us to examine the feasibility of collusive strategies, an issue that has not been investigated before in the context of a third-degree oligopolistic price discrimination model. This issue is very important because competitive price discrimination usually leads to a cutthroat price competition (prisoner's dilemma) among firms. Firms, therefore, may seek ways to soften competition and sustain higher prices.

We employ Hotelling's duopoly model of horizontal product differentiation (Hotelling, 1929), where firms interact repeatedly in an infinite horizon setting. Firms attempt to sustain monopoly profits via the *grim-trigger* strategy. We assume that in each period of the infinitely repeated game firms have access to information of a certain (exogenously given) quality which allows them to segment consumers into different groups and charge each group a different price.⁸ As in Liu and Serfes (2004), this segmentation is imperfect and it is modeled as a partition of the characteristics' space.⁹ Moreover, consumer information of higher quality is modeled as a refinement of the partition. In other words, a refinement allows firms to reduce the "noise" around each consumer's true preferences. The limit of the refinement process is perfect information. We perform a number of interesting comparative statics with respect to the quality of information.

² According to the June 8, 2001 *Wall Street Journal*: "One day recently, the Dell Latitude L400 ultralight laptop was listed at \$2307 on the company's Web page catering to small businesses. On the Web page for sales to health-care companies, the same machine was listed at \$2228, or 3% less. For state and local governments, it was priced at \$2072.04, or 10% less than the price for small businesses."

³ See, "The New Realities of Dynamic Pricing," by A. Kambil and V. Agrawal, <http://www.accenture.com/xdoc/en/ideas/outlook/7.2001/pricing.pdf>.

⁴ See Pancras and Sudhir (2005) and Ghose and Huang (2005) for more examples of targeted pricing based on information about consumer preferences. Both papers study the "effectiveness" of price and quality customization in competitive settings.

⁵ For example, Anderson and de Palma (1988), Thisse and Vives (1988), Bester and Petrakis (1996), Chen (1997), Corts (1998), Fudenberg and Tirole (2000), Shaffer and Zhang (2000) and Liu and Serfes (2004).

⁶ See Stole (2003) for a comprehensive survey.

⁷ Exceptions include the papers by Chen (1997) and Fudenberg and Tirole (2000) that study two-period models.

⁸ The segmentation can also be thought of as being geographic. For example, the U.S. Supreme Court recently agreed to hear the case of *Twombly vs. Bell Atlantic*. This is a class-action lawsuit filed on behalf of all people with telephones. The suit claims that, since telephone deregulation, the regional Bells have conspired to "stay out of each other's geographic markets" so as not to compete against each other, see <http://www.techlawjournal.com/topstories/2005/20051003b.asp>.

⁹ Liu and Serfes (2005a,b) have also adopted this modeling framework to explore the impact of consumer information of a certain accuracy on product quality and product variety.

If information acquisition and pricing decisions are made simultaneously (and the cost of acquiring and/or using the information is sufficiently low), in a one-shot game, the availability of consumer information leads to lower profits than the profits under a uniform pricing rule. This is due to the best response asymmetry (Corts, 1998), which intensifies the competition between the firms. Hence, firms have very strong incentives to coordinate their strategies in an attempt to increase their profitability. Firms can collude on two dimensions. One is on information acquisition and the second is on price(s). Given this, there are three *simple* collusive schemes. In the first one firms segment the market and charge a monopoly price in each consumer segment (*monopoly discriminatory pricing*). When this type of collusion is not sustainable, firms may seek an agreement that limits the usage of consumer information (since competitive price discrimination, as we mentioned above, leads to very low profits). In the second scheme firms agree not to acquire information (i.e., not to engage in any segmentation) and charge monopoly uniform prices (*monopoly uniform pricing*). Finally, in the third scheme, firms agree not to acquire information, but they charge Nash uniform prices (*competitive uniform pricing*). For each collusive scheme we find the minimum discount factor (as a function of the quality of consumer information) that is needed to sustain monopoly profits. We are mainly interested in the following two questions: a) which collusive scheme is easier to sustain in equilibrium? and b) how does the easiness of collusion change with the firms' ability to segment consumers, i.e., as the partition gets refined?

As expected, the first collusive scheme yields the highest profits and the third scheme the lowest among the three different collusive schemes. The first collusive scheme allows firms to extract consumer surplus without having to bear the adverse effects of intensified price competition that the pricing flexibility brings when strategies are left uncoordinated. Furthermore, the first collusive scheme is easier to sustain in equilibrium than the second one. This implies that the second scheme, where firms charge monopoly uniform prices, is dominated by the first one and therefore it will never be an equilibrium. When the quality of consumer information is low, then the third collusive scheme is easier to sustain in equilibrium than the first one. The reverse is true when consumer information of high quality is available. The most important result is that the minimum discount factors are increasing functions of the quality of consumer information. It then follows that collusion becomes more difficult as the firms' ability to segment consumers increases. This result continues to hold when we allow firms to collude imperfectly (provided that the discount factor is not too low), or when the punishment phase is more severe than Nash reversion (see Section 5).

In a one-shot game, consumer welfare is the highest when firms compete using the available information. In this case, consumer surplus is inverse U-shaped with respect to the quality of consumer information. This implies that information of moderate quality yields the highest benefit to the consumers. The consumer welfare results change when we move from static to dynamic interaction. For low discount factors collusion is not sustainable and consumer surplus is inverse U-shaped as discussed above.¹⁰ The worst outcome for consumers is when the discount factor is high and firms charge discriminatory monopoly prices. Given that such collusion is sustainable, better consumer information allows firms to extract more consumer surplus. This implies that consumer surplus is a monotonically decreasing function of the quality of information. However, better information also makes collusion more difficult and may lead to the breakdown of collusion. Consumer surplus would increase in this case.

¹⁰ When firms collude imperfectly, then some level of collusion is always sustainable. Better information makes collusion more difficult and consumer welfare is increasing with respect to the quality of information.

Collusion in an infinitely repeated framework with horizontally differentiated products has been studied by, among others, Chang (1992), Friedman and Thisse (1993), Häckner (1995) and Schultz (2005). The first three papers are mainly interested in addressing the issue of product selection (i.e., firms' locations). Chang (1992) allows firms to redesign their products intertemporally at a fixed cost. He shows that flexible product design makes collusion more difficult to sustain, since if collusion breaks down firms can redesign their products to mitigate price competition. Häckner (1995) shows that the degree of product differentiation is in inverse relationship with the discount factor, providing that redesigning costs are negligible. Firms move closer to each other as they become more patient. Friedman and Thisse (1993) assume that firms choose their locations once at the beginning of the game and then they choose prices in an infinite horizon setting. They show that firms will choose to locate at the center of the interval. By doing this, firms commit to the most severe punishments profits, which allows them to expand the set of discount factors that make collusion possible.¹¹ Schultz (2005) investigates the effect of increased market transparency on the consumer side of the market on the firms' ability to sustain collusion. Our model may also be viewed as one where firms interact in many markets (segments). Bernheim and Whinston (1990) show that multimarket contact may make collusion easier. In contrast, we show that as the number of segments increases (i.e., more markets) collusion becomes more difficult [we offer a more extensive discussion on the comparison between our results and those in Friedman and Thisse (1993) and Bernheim and Whinston (1990) in Section 4].

Our results have antitrust and policy implications. When antitrust agencies (e.g. Department of Justice or FTC) review a merger case they follow either the unilateral effects (UE) or the coordinated effects (CE) approach (see Sibley and Heyer, 2003). Under the UE approach it is assumed that firms compete in a static fashion, whereas under the CE approach the assumption is that the interaction is repeated and some collusion is sustainable. When a market fits the CE approach the critical question is whether a proposed merger will relax constraints on coordination. A lengthy checklist has been proposed as leading to coordination.¹² The results we derive can also be viewed as adding an item to that checklist. Our model is mainly applicable to markets where firms have significant amounts of consumer information, product differentiation is primarily on a horizontal dimension and firms can use their information to make targeted offers/discounts (third-degree price discrimination). Alternatively, the segmentation may be geographic such that firms divide the big market into smaller submarkets to avoid head-on competition. In such markets we predict that when the segmentation becomes more refined (which may be due to a merger of firms who pool their customer data) collusion will be less likely, all else equal (see Section 4 where this discussion is continued).

The rest of the paper is organized as follows. The main model is presented in Section 2 and the equilibrium analysis is presented in Section 3. In Section 4, we compare the three simple collusive schemes and we lay out a number of antitrust and policy implications that emerge from this model. We also discuss consumer welfare results. We extend the model in Section 5, where we allow for imperfect collusion and for punishments that are more severe than the Nash

¹¹ Lederer and Hurter (1986) introduce a static model where the characteristics' space is two-dimensional and firms are able to price discriminate perfectly. In contrast to Friedman and Thisse (1993), firms will never locate next to each other.

¹² The list, which is taken from Sibley and Heyer (2003), includes the following items: (1) High concentration, (2) Barriers to entry, (3) Homogeneous products, (4) Homogeneous costs, (5) Absence of a maverick firm, (6) Transparent firm strategies and payoffs, (7) History of coordination, (8) Low market elasticity of demand and (9) Many small consumers, making frequent purchases. See also Tirole (1988, Chapter 6) for a similar list of factors that facilitate or hinder collusion.

punishments. We conclude in Section 6. The proofs of the two propositions can be found in the Appendix.

2. The description of the model

There are two firms, denoted by A and B and indexed by i , who sell non-durable brands and are located at the two endpoints of a Hotelling interval of unit length. A continuum of consumers, of unit mass, is uniformly distributed on the interval. Firms interact in an infinitely repeated game framework. Time is denoted by, $t=0, 1, 2, \dots$

The stage game. The stage game is described as follows. Each consumer derives a benefit equal to V if he buys a product from either one of the firms. We may view each firm's action as a price function $p_i: [0, 1] \rightarrow \mathbb{R}_+$, where $p_A(x)$ and $p_B(x)$ are the prices that firms A and B charge, respectively, to consumer located at point x . Both firms' marginal costs are normalized to zero. In addition, each consumer incurs a linear unit transportation cost denoted by $c > 0$. Therefore, a consumer who is located at point $x \in [0, 1]$ and buys from firm A enjoys a surplus of $V - cx - p_A(x)$. Likewise, if he buys from firm B his surplus is $V - c(1-x) - p_B(x)$. Each consumer buys the product which gives him the highest positive surplus. We assume that V is sufficiently high, ensuring that each consumer will buy (covered market).

Firms can develop or acquire a database which helps them to segment the consumers into distinct groups based upon each consumer's relative brand loyalty. We assume that the information partitions the $[0, 1]$ interval into n sub-intervals (indexed by $m, m=1, \dots, n$) of equal length. In this case, a firm can charge different prices ($p_{im}, i=A, B$ and $m=1, \dots, n$) to different segments of consumers, through the use of targeted coupons. In practice, firms can obtain such information from a number of different sources, such as: i) directly through repeated past transactions with the customers, ii) via a telemarketing or direct-mail survey, iii) from credit card reports, or iv) from a marketing firm [see Pancras and Sudhir (2005) and Shaffer and Zhang (2000) for a more extensive discussion and more references on this issue].¹³ Arbitrage between consumers is not feasible.

We further assume that $n=2^k$, where k takes on all integer values, $k=0, 1, 2, 3, 4, \dots$. Hence, n will parameterize the precision of consumer information, with higher n 's being associated with higher information precision. Moreover, $n=1$ corresponds to no price discrimination and $n=\infty$ to perfect price discrimination. Note that n does *not* take on all integer values, but rather $n=1, 2, 4, 8, 16, \dots$ (i.e., information refinement). We assume that information of precision n is available to both firms at a sufficiently low price and that the current state of technology dictates n which the firms take as exogenously given. Firms choose whether to acquire information (or whether to use it if they already have it) and their prices simultaneously in each stage of the infinitely repeated game. Since the pricing and information acquisition decisions are made simultaneously and the cost of information is assumed to be sufficiently low, it is clear that in a one-shot game a firm will always have an incentive to acquire information and use it.

A remark on the information structure. The kind of information we have in mind is about consumer characteristics (e.g. gender, age, income group, purchase history, etc.). This information, after it has been processed and analyzed, helps the firms to segment the consumers

¹³ One example is the Abacus Catalog Alliance, a database that contains transactional data with detailed information on consumer and business-to-business purchasing and spending behavior. It is a blind alliance of 1800 merchants offering shared data representing over 90 million households and is the largest proprietary database of consumer transactions used for target marketing purposes (see <http://www.doubleclick.com/us/>).

into different groups. Firms can now price according to each group’s willingness to pay for the different brands. More data about consumers (and/or more sophisticated techniques employed to analyze these data) lead to a finer segmentation. This is consistent with the way most practitioners and empirical researchers view market segments (e.g. Besanko et al., 2003 and Rossi et al., 1996). We have made two simplifying assumptions which are nevertheless necessary in order to reduce the complexity of the model: i) the size of all segments are equal and ii) the distribution is uniform. Moreover, in practice, a firm’s strategy regarding customer information consists of at least two main elements: i) whether to collect detailed information and if so ii) how much to invest in such a process. More firm resources directed towards this goal should result in consumer databases of higher quality. More importantly, the state of the existing technology imposes an exogenous bound on the quality. The model would become intractable if we attempted to endogenize the quality of information. Therefore, for tractability, we assume that the quality of information is exogenously given. This allows us to focus completely on the first strategic element, by implicitly assuming that the existing technology – which is beyond a firm’s control – is entirely responsible for the quality of a customer database and a firm can only choose whether to acquire such a database or not.

The infinitely repeated game. Firms collude in the infinitely repeated game via a *grim trigger* strategy described as follows. We assume perfect monitoring. Firms start by charging collusive prices, $p_i^C(x)$, $i=A, B$, (to be defined in the next section). Continue charging $\{p_A^C(x), p_B^C(x)\}$ if neither firm has deviated in a previous stage. However, if either firm deviates in period t , then both firms revert forever to the Nash equilibrium starting in period $t+1$ (Friedman, 1971). We should emphasize at this point that the collusive prices that a firm charges may very well be all the same, i.e., $p_i^C(x)=\text{constant}$, for all $x \in [0, 1]$. This implies that the firm has not acquired information and charges a uniform price to consumers. Hence, we also allow firms to collude on information acquisition decisions.

We denote by $\pi_i^C(n)$, $\pi_i^D(n)$ and $\pi_i^N(n)$ the one-shot collusive, deviation and Nash profits of firm i respectively. The common discount factor is denoted by δ . Hence, the collusive prices constitute a subgame perfect equilibrium (SPE) of the infinitely repeated game if and only if,

$$\sum_{t=0}^{\infty} \delta^t \pi_i^C(n) \geq \pi_i^D(n) + \sum_{t=1}^{\infty} \delta^t \pi_i^N(n) \Leftrightarrow \delta \geq \delta(n) \equiv \frac{\pi_i^D(n) - \pi_i^C(n)}{\pi_i^D(n) - \pi_i^N(n)}, \quad i = A, B. \quad (1)$$

Next we derive the one-shot Nash profits that firms revert to after a deviation takes place.

2.1. The punishment (Nash) profits

Since the cost of information is sufficiently low and information acquisition and pricing decisions are made simultaneously it is clear that both firms will choose to acquire information and engage in price discrimination. Both firms know in which of the n segments each consumer is located and therefore they are able to charge different prices for different segments. The interval $[0, 1]$ is equally divided into n segments, each one having length equal to $1/n$. Segment m can be expressed as the interval $\left[\frac{(m-1)}{n}, \frac{m}{n}\right]$, where m is an integer between 1 and n (see Fig. 1).

In segment m , firm A and B charge prices p_{Am}^N and p_{Bm}^N , the demands of their products are,

$$d_{Am} = \frac{p_{Bm}^N - p_{Am}^N + c}{2c} - \frac{m-1}{n} \quad \text{and} \quad d_{Bm} = \frac{m}{n} - \frac{p_{Bm}^N - p_{Am}^N + c}{2c},$$

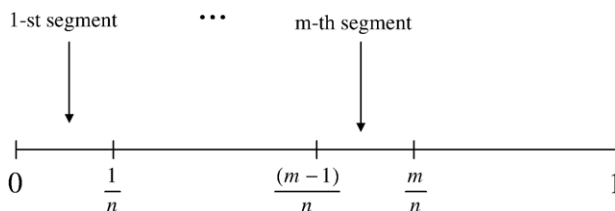


Fig. 1. Partition of the unit interval.

with d_{Am} and d_{Bm} in $[0, 1/n]$ and their profits are

$$\pi_{Am}^N(p_{Am}^N, p_{Bm}^N) = p_{Am}^N d_{Am}, \quad \text{and} \quad \pi_{Bm}^N(p_{Am}^N, p_{Bm}^N) = p_{Bm}^N d_{Bm}.$$

Firm i 's, problem is,

$$\max_{p_{im}^N \geq 0} \pi_{im}^N(p_{Am}^N, p_{Bm}^N), \quad \text{for each } m, m = 1, \dots, n, \quad \text{and } i = A, B.$$

The ability of both firms to treat each segment independent of the other ones allows us to solve for the equilibrium in each subinterval separately and then aggregate over all subintervals to find the equilibrium profits, denoted by $\pi_A^N(n)$ and $\pi_B^N(n)$, as a function of the information quality. The next proposition summarizes the solution to the above problem.

Proposition 1. *For each $n \geq 2$, there exist two thresholds (integers) m_1 and m_2 (with $0 \leq m_1 < m_2 \leq n + 1$) where,*

$$m_1 = \frac{n}{2} - 1 \quad \text{and} \quad m_2 = \frac{n}{2} + 2,$$

such that:

- i) [This case is valid only when $m_1 \geq 1$]. Firm A's equilibrium demand is equal to $1/n$ in all segments from 1 to m_1 , i.e., firm A is a constrained monopolist in these segments. Firm B's equilibrium demand in these segments is zero.
- ii) Both firms sell positive quantities in the segments from $m_1 + 1$ to $m_2 - 1$.
- iii) [This case is valid only when $m_2 \leq n$]. Firm B's equilibrium demand is equal to $1/n$ in all segments from m_2 to n , i.e., firm B is a constrained monopolist in these segments.

Finally, the equilibrium profits of each firm as a function of n are:

$$\pi_i^N(n) = \frac{(9n^2 - 18n + 40)c}{36n^2}, \quad i = A, B. \tag{2}$$

Proof. See the Appendix.¹⁴ □

Suppose $n=8$. Then $m_1=3$ and $m_2=6$, implying that firms A and B are constrained monopolists in segments 1, 2, 3 and 6, 7, 8, respectively. In segments 4 and 5 both firms sell

¹⁴ The proof is the same with the Proof of Proposition 1 in Liu and Serfes (2004). For the sake of completeness, we provide the proof in the Appendix.

positive quantities. The prices that firm *A* charges, starting from segment 1 are: $p_{A1}^N = 3c/4$, $p_{A2}^N = c/2$, $p_{A3}^N = c/4$, $p_{A4}^N = c/6$, $p_{A5}^N = c/12$ and $p_{A6}^N = p_{A7}^N = p_{A8}^N = 0$. Firm *B*'s prices are symmetric with the highest price in segment 8 and a price equal to zero in segment 1. The equilibrium profits are, $\pi_A^N = \pi_B^N = .2049c$.

There are two opposing effects at play that determine the Nash profits: an intensified competition and a surplus extraction effect. The pricing flexibility, facilitated by the consumer information, allows firms to extract more surplus from the consumers and at the same time it intensifies the competition. When the quality of consumer information is low the second effect dominates the first, while for high information precision the first effect becomes more dominant. Therefore, the Nash profits are U-shaped as a function of the quality of information. Moreover, profits are always below the Nash profits under a uniform competitive pricing rule, i.e., $\pi_i^N(n) < c/2$, for all $n \geq 2$ (see Fig. 2). Thus, firms have strong incentives to eliminate the intensified competition effect. This can be achieved either by acquiring information and coordinating their pricing strategies, or by not acquiring information at all.

3. Analysis

We search for an outcome where collusion is sustainable as a SPE. We will focus our analysis on symmetric collusion. This can be justified because firms are ex ante symmetric. There are three simple collusive schemes that we will examine: i) (collusive scheme 1) firms acquire consumer information of an exogenous quality n and charge monopoly discriminatory prices, ii) (collusive scheme 2) firms do not acquire information and charge monopoly uniform prices and iii) (collusive scheme 3) firms do not acquire information and compete with uniform prices. The first collusive scheme is the one that yields the highest profits to the firms. Firms are able to extract consumer surplus, while at the same time they eliminate the competition. In the second and third collusive schemes firms agree not to acquire (or not to use) customer-specific information, but they may collude or not on their pricing decisions. For each collusive scheme, we find the minimum value of the discount factor δ (as a function of the quality of information n) above which collusion is sustainable as a SPE of the infinitely repeated game. We are mainly interested in the

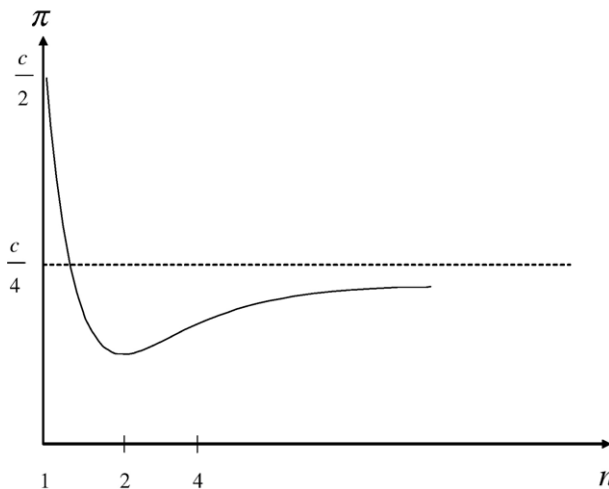


Fig. 2. One-shot (Nash) profits.

following two questions: a) which collusive scheme is easier to sustain in equilibrium? and b) how does the easiness of collusion change with the firms’ ability to segment consumers?¹⁵

The three schemes that we examine in this section are characterized by their simplicity. In schemes 1 and 2 firms split the market and each firm charges monopoly price(s) in its own territory. For instance, if firms agreed to charge less than monopoly prices (in order to sustain partial collusion), then each firm would have to charge prices in the rival’s segments even when these prices lead to zero sales. This is needed to curb the incentives for deviation, but it makes such a scheme much more difficult (and more costly) to be carried out by the firms than scheme 1 (for more on this see Section 5.1). Moreover, in scheme 3 the agreement is simply not to engage in price discrimination, while pricing is competitive. This is an appealing scheme especially when consumer information is costly. When collusion breaks down firms simply revert to Nash play. These types of schemes can be easily implemented by the firms and therefore they can serve as benchmark cases. This does not mean that more elaborate schemes do not exist. In Section 5, we allow firms to coordinate on less than monopoly profits and we allow for punishments that are more severe than Nash reversion. The main results remain unchanged.

3.1. Collusive scheme 1: firms acquire information and charge monopoly discriminatory prices (monopoly discriminatory pricing)

Firms behave as a multi-product monopolist. The collusive scheme is described as follows. Firm *A* will sell to consumers who are located in $[0, 1/2]$ and firm *B* will sell to consumers in $[1/2, 1]$. Due to symmetry, we focus on firm *B*. It is easy to show that firm *B*’s optimal pricing strategy is to leave the consumers who are located at the beginning of each segment with zero surplus.¹⁶ These consumers are located at $x = \frac{m-1}{n}$, where $m=(n/2)+1, \dots, n$ (see Fig. 1). The collusive prices are given by,

$$V - p_{Bm}^C - c \left(1 - \frac{m-1}{n}\right) = 0 \Rightarrow p_{Bm}^C = \frac{Vn - cn + cm - c}{n}, \quad m = \frac{n}{2} + 1, \dots, n. \quad (3)$$

The prices of firm *A* can be derived similarly. We can envision a situation where each firm posts one price (that is available to *all* consumers in the market) and then offers targeted discounts to each consumer group in its own market. The posted price for each firm is the highest price among the segment prices. The highest price for firm *A* is in segment 1 and for firm *B* in segment *n*, i.e., $p_{A1}^C = p_{Bn}^C = (Vn - c)/n$. Those who pay a firm’s posted price belong in the group of consumers with the strongest preferences for the firm’s good. The consumer with the weakest preference in that group receives zero surplus. Hence, all other consumer groups will receive a negative surplus if they pay the firm’s posted price. Therefore, each firm in each segment competes only with the zero outside option.

The monopoly collusive profits are given by,

$$\pi_B^C(n) = \sum_{m=\frac{n}{2}+1}^n \frac{p_{Bm}^C}{n} = \frac{4Vn - cn - 2c}{8n} = \pi_A^C(n). \quad (4)$$

¹⁵ In what follows we assume that firms can segment the consumers into at least two groups, i.e., $n \geq 2$.

¹⁶ The sufficient condition for this to be true is $V \geq \frac{(n+2)c}{2n}$. It turns out that this constraint is never binding. The next threshold we derive about *V* in this subsection, denoted by \bar{V}_1 , which guarantees that if *V* exceeds that threshold a deviating firm captures the entire market is always greater than $\frac{(n+2)c}{2n}$.

Let us now assume that firm *A* deviates from the collusive agreement. Denote firm *A*'s prices after deviation by p_{Am}^{dev} , $m=(n/2)+1, \dots, n$. Firm *A*'s prices in the segments that are located in the first half of the interval are the same with those before deviation, i.e., $p_{Am}^C = p_{Am}^{dev}$, $m=1, \dots, n/2$. The indifferent consumer in segment *m* is given by,

$$V - cx_m - p_{Am}^{dev} = V - c(1 - x_m) - p_{Bm}^C \Rightarrow x_m = \frac{Vn + c(m - 1) - np_{Am}^{dev}}{2nc}.$$

The price that maximizes firm *A*'s profits, $\pi_{Am}^{dev} = p_{Am}^{dev} (x_m - \frac{m-1}{n})$, in segment *m* (assuming that both firms have positive demands) is given by,

$$p_{Am}^{dev} = \frac{Vn - c(m - 1)}{2n}.$$

Both firms have strictly positive demands in segment *m* located in $[1/2, 1]$, i.e., $x_m \in (\frac{m-1}{n}, \frac{m}{n})$ provided that $V < \frac{(m+3)c}{n}$. This implies that if $V \geq \frac{(m+3)c}{n}$ then firm *B*'s market share in segment *m* is zero, following firm *A*'s deviation from the collusive agreement. The highest threshold is obtained when $m=n$, i.e., the last segment, and is given by $\frac{(n+3)c}{n}$. Therefore, if $V \geq \frac{(n+3)c}{n}$ then firm *B*'s market share is zero in all segments. In order to simplify the analysis and obtain neat results we assume that $V \geq \bar{V}_1 = \frac{(n+3)c}{n}$.¹⁷ Given that firm *B* is driven out of each segment, firm *A*'s segment price is given by,

$$V - c\frac{m}{n} - p_{Am}^{dev} = V - c\left(1 - \frac{m}{n}\right) - p_{Bm}^C \Rightarrow p_{Am}^{dev} = \frac{Vn - c(m + 1)}{n}.$$

The profits after deviation are given by,

$$\pi_A^{dev}(n) = \pi_A^C(n) + \sum_{m=\frac{n}{2}+1}^n \frac{p_{Am}^{dev}}{n} = \frac{2Vn - c(n + 2)}{2n}. \tag{5}$$

Using (1) together with (2), (4) and (5), we can conclude that the monopoly prices (3) are sustainable as a SPE if and only if,

$$\delta \geq \delta_1(n) \equiv \frac{\pi_i^D(n) - \pi_i^C(n)}{\pi_i^D(n) - \pi_i^N(n)} = \frac{9n(4Vn - 6c - 3cn)}{2(36Vn^2 - 18cn - 27cn^2 - 40c)}, \quad i = A, B. \tag{6}$$

It turns out (details are omitted) that δ_1 is an increasing function of the quality of information *n*. In other words, as the ability of firms to segment consumers increases, collusion becomes more difficult to sustain. For example, when $n=4$ and $V=1.75c$, then $\delta_1=.388$. Moreover, $\lim_{n \rightarrow \infty} \delta_1(n) = 1/2$. In the limit, firms have perfect information about each consumer's preferences and practice perfect price discrimination. In this case firms compete à la Bertrand and as it is well known monopoly profits can be sustained if and only if the discount factor exceeds 1/2.

The intuition about why collusion becomes more difficult as the quality of consumer information improves is as follows. The minimum discount factor above which collusion is

¹⁷ The threshold \bar{V}_1 is monotonically decreasing in *n*. It starts at $5c/2$ when $n=2$ and tends to c as $n \rightarrow \infty$. Without an assumption that *V* is high, we would have to keep track of which consumer segments firm *B* has a positive market share and of which it is out of the market after firm *A*'s deviation. To complicate matters further, this would depend on the quality of information *n*, leading to murky results.

sustainable can be expressed as follows, $\delta_1(n) \equiv \frac{\pi_i^D(n) - \pi_i^C(n)}{[\pi_i^D(n) - \pi_i^C(n)] + [\pi_i^C(n) - \pi_i^N(n)]}$. Both the benefit from cheating $\pi_i^D(n) - \pi_i^C(n)$ and the loss from punishment $\pi_i^C(n) - \pi_i^N(n)$ are increasing with n . Let us look at the benefit from cheating first. When firms collude they split the market in half and each firm charges monopoly discriminatory prices in its own market. Clearly, as information improves, these profits increase. Moreover, when a firm deviates it maintains its profits in its own market and steals customers from the rival firm’s market. The profits that the deviating firm makes from its rival’s market also increase with the quality of information, since better information allows the deviating firm to target consumers more effectively. Hence, as consumer information improves the profits after deviation increase faster than the collusive profits, i.e., $\pi_i^D(n) - \pi_i^C(n)$ increases with n . Now let us turn to the loss from punishment. Better information allows firms to target consumers more effectively. The collusive profits, however, increase faster than the Nash profits simply because the Nash profits are dragged down by the competitive effect, whereas under collusion each firm is unchallenged by its rival. Therefore, $\pi_i^C(n) - \pi_i^N(n)$ increases as n increases. Furthermore, the gain from cheating $\pi_i^D(n) - \pi_i^C(n)$ increases relatively faster than the loss from punishment $\pi_i^C(n) - \pi_i^N(n)$, as consumer information improves. Better information leaves each firm increasingly more vulnerable to opportunistic behavior from its rival. Two things happen when consumer information improves. Firms can identify consumers with more accuracy and as a consequence prices in the collusive agreement increase. A deviating firm now can take advantage of both the better information and the higher collusive prices. This makes a deviation increasingly more profitable. Hence, firms can continue to sustain monopoly profits as consumer information improves only if the discount factor goes up.

3.2. Collusive scheme 2: firms agree not to acquire information and charge a monopoly uniform price (monopoly uniform pricing)

The collusive scheme in this case is described as follows. Firms agree not to acquire information and moreover they charge uniform monopoly prices. As in collusive scheme 1, we assume that firms split the $[0, 1]$ interval. If V is higher than c , it is each firm’s optimal strategy to leave the consumer who is located at $1/2$ with zero surplus. Therefore, the collusive prices and profits are given by,

$$p_A^C = p_B^C = V - \frac{c}{2} \quad \text{and} \quad \pi_A^C = \pi_B^C = \frac{V}{2} - \frac{c}{4}. \tag{7}$$

Now we assume that firm A deviates from the collusive agreement. The best deviation for firm A is to acquire information and charge each consumer segment in $[0, 1]$ a different price. By following the logic that we outlined in collusive scheme 1 above, we can show that firm B ’s market share is zero, after firm A ’s deviation, if $V \geq \bar{V}_2 = \frac{c(3n+4)}{2n}$. (Note that $\bar{V}_2 > c$). The maximum profits when firm A deviates are given by,

$$\pi_A^{dev}(n) = \frac{2Vn - c(n+2)}{2n}. \tag{8}$$

The collusive scheme is sustainable if and only if,

$$\delta \geq \delta_2(n) \equiv \frac{\pi_i^D(n) - \pi_i^C(n)}{\pi_i^D(n) - \pi_i^N(n)} = \frac{9n(2Vn - 4c - cn)}{(36Vn^2 - 18cn - 27cn^2 - 40c)}, \quad i = A, B. \tag{9}$$

As the threshold discount factor in collusive scheme 1, $\delta_2(n)$ is an increasing function of the quality of information n (again the details are omitted). The intuition is similar to the one about the monotonicity of $\delta_1(n)$ that we offered in the previous subsection. The profits after deviation are the same in collusive schemes 1 and 2 [see (5) and (8)]. Since the collusive profits under scheme 1 are greater than the profits under scheme 2, it follows easily that collusive scheme 1 is easier to be sustained in equilibrium than scheme 2, i.e., $\delta_2(n) > \delta_1(n)$. Thus, it is more difficult to sustain a uniform monopoly price than discriminatory monopoly prices.

3.3. Collusive scheme 3: firms agree not to acquire information and compete with uniform prices (competitive uniform pricing)

In this collusive scheme firms agree not to acquire information and they compete with uniform prices. It is easy to show that the prices and profits under this collusive scheme are given by,

$$p_A^C = p_B^C = c \quad \text{and} \quad \pi_A^C = \pi_B^C = \frac{c}{2}. \tag{10}$$

This outcome is the standard Hotelling equilibrium. It is well-known that for the market to be covered V must exceed $\bar{V}_3 = 3c/2$.

We now assume that firm A deviates. The best deviation for firm A is to acquire information and charge each consumer segment in $[0, 1]$ a different price. The indifferent consumer in segment m is given by,

$$V - cx_m - p_{Am}^{\text{dev}} = V - c(1 - x_m) - p_B^C \Rightarrow x_m = \frac{2c - p_{Am}^{\text{dev}}}{2c}.$$

Assuming that $x_m < m/n$, the price that maximizes firm A 's profits, $\pi_{Am}^{\text{dev}} = p_{Am}^{\text{dev}}(x_m - \frac{m-1}{n})$, in segment m is given by,

$$p_{Am}^{\text{dev}} = \frac{(n - m + 1)c}{n}.$$

Using p_{Am}^{dev} we can show that the indifferent consumer in segment m is located at,

$$x_m = \frac{n + m - 1}{2n}.$$

It follows that $x_m < m/n$, only if $m > n - 1$. In other words, firms share the segment demand only in the last segment, $m = n$. In all other segments the deviating firm captures the entire segment demand. Hence, the optimal deviation prices and profits are given by,

$$p_{Am}^{\text{dev}} = \begin{cases} \frac{2c(n - m)}{n}, & \text{if } m = 1, \dots, n - 1 \\ \frac{(n - m + 1)c}{n}, & \text{if } m = n \end{cases}, \quad \text{and} \quad \pi_A^{\text{dev}}(n) = \frac{c(2n^2 - 2n + 1)}{2n^2}. \tag{11}$$

The collusive scheme is sustainable if and only if,

$$\delta \geq \delta_3(n) \equiv \frac{\pi_i^D(n) - \pi_i^C(n)}{\pi_i^D(n) - \pi_i^N(n)} = \frac{18(n^2 - 2n + 1)}{27n^2 - 18n - 22}, \quad i = A, B. \tag{12}$$

It can be readily verified that $\delta_3(n)$ is a monotonically increasing function of n and $\lim_{n \rightarrow \infty} \delta_3(n) = 2/3$. When firms have access to perfect information, this collusive scheme is sustainable if and only if $\delta \geq 2/3$. The intuition is similar to the one about the monotonicity of $\delta_1(n)$ that we offered in the subsection that examines the first collusive scheme.

Next, we compare the profitability and how easy it is for the firms to sustain collusion under each one of the three different collusive schemes.

4. Comparison of the three simple collusive schemes

First of all collusive scheme 2, where firms collude on a uniform price, is dominated by scheme 1 where firms collude on discriminatory prices. The latter scheme yields higher collusive profits than the former and moreover it is easier to sustain as a SPE, i.e., $\delta_1(n) < \delta_2(n)$ for all n . Therefore, we will compare collusive scheme 1 with collusive scheme 3. In scheme 3 firms agree not to acquire information and compete with uniform prices. The collusive profits in this case are obviously lower than the ones from scheme 1. Now we compare the likelihood of collusion by examining the ranking of the threshold discount factors, $\delta_1(n)$ and $\delta_3(n)$.

Assumption 1. *It is assumed that V is sufficiently high. More specifically,*

$$V > \max\{\bar{V}_1, \bar{V}_2, \bar{V}_3\},$$

where $\bar{V}_1 = \frac{(n+3c)}{n}$, $\bar{V}_2 = \frac{c(3n+4)}{2n}$, $\bar{V}_3 = \frac{3c}{2}$.

A reservation value above \bar{V}_3 ensures that the market, in scheme 3, is covered before any deviation takes place. This is the standard threshold in Hotelling-type models. Moreover, this ensures that the market is also covered, before any deviation, in schemes 1 and 2. Thresholds \bar{V}_1 and \bar{V}_2 guarantee that a deviating firm, in schemes 1 and 2, respectively, captures the entire market. In scheme 3 a deviating firm never captures the entire market, as we have already indicated in a previous section. As we explained in Footnote 17, Assumption 1 greatly simplifies the analysis. It can be easily verified that \bar{V}_1 is above \bar{V}_3 only for low n 's (i.e., $n=2$ and 4) and \bar{V}_2 is very close to \bar{V}_3 for high n 's, since $\lim_{n \rightarrow \infty} \bar{V}_2 = 3c/2$. Therefore, Assumption 1 is not much more restrictive than the standard covered market assumption in Hotelling-type models. (In Footnote 19, we investigate the consequences of relaxing this assumption.)

The next proposition summarizes the result (Assumption 1 holds).

Proposition 2. *When the quality of consumer information is low ($n=2$ or 4), then it is easier for the firms to sustain competitive uniform prices (collusive scheme 3) than monopoly discriminatory prices (collusive scheme 1), i.e., $\delta_1(n) > \delta_3(n)$, provided that $V > \bar{V}_4 = 297c/56 \approx 5.3c$. When the quality of consumer information is high ($n > 4$), the reverse is true, i.e., $\delta_3(n) > \delta_1(n)$, for any V . Moreover, collusion becomes more difficult, under both collusive schemes, as the quality of consumer information improves, i.e., $\delta_3(n)$ and $\delta_1(n)$ are monotonically increasing functions of n .¹⁸*

Proof. See the Appendix. \square

The intuition is as follows. In collusive schemes 1 and 3 the profits in the punishment phase are the same. The collusive and the deviation profits in scheme 1 are higher than those in scheme 3.

¹⁸ When $V < \bar{V}_4$ but greater than $29c/14$, then scheme 3 is more difficult to sustain than scheme 1 also for $n=4$ (see the Proof of Proposition 2). Note, from Assumption 1, that V cannot fall below $29c/14$, when $n=2$. Higher V makes scheme 1 more appealing because the collusive profits in that scheme do depend on V .

We see that collusive scheme 1 has an advantage (relative to scheme 3) in terms of profits and a disadvantage in terms of deviation profits. The advantage regarding the collusive profits is clear. The disadvantage regarding the deviation profits is explained as follows. The collusive discriminatory prices are higher on average than the price in collusive scheme 3. Thus, a firm that deviates from scheme 1 takes advantage of the high collusive prices and makes higher profits than the profits from a deviation under scheme 3. When the quality of consumer information is low the disadvantage of scheme 1 exceeds its advantage. Firms cannot target consumers very effectively and therefore the collusive profits under scheme 1 are not much higher than those under scheme 3. That is why collusive scheme 1 is more difficult to sustain in equilibrium than scheme 3. As the quality of consumer information increases both collusive schemes become more difficult to be sustained. Moreover, $\delta_3(n)$ increases faster than $\delta_1(n)$. The reason is simple. The collusive profits under scheme 3 do not vary with n , since firms charge a uniform price. This makes collusion under scheme 3 increasingly more difficult relative to collusion under scheme 1 as consumer information improves. Therefore, the function $\delta_3(n)$ intersects the function $\delta_1(n)$ once and from below.¹⁹

Remark. The result of Proposition 2 should be interpreted with caution. In particular, we predict that when n is low collusion with uniform competitive prices is easier than collusion with monopoly discriminatory prices. Nevertheless, if we allow for imperfect collusion (as we do in Section 5) then this distinction becomes less sharp. Indeed, for discount factors in $[\delta_3(n), \delta_1(n)]$ some imperfect collusion is possible. More specifically, what matters then is the level of imperfectly collusive profits that firms wish to sustain. This implies that firms may very well use discriminatory pricing, but sustain low profits (if δ is low and cannot sustain monopoly profits), suggesting that with imperfect collusion uniform non-discriminatory pricing may not be observed even when the discount factor is low.²⁰ Therefore, Proposition 2 is based on the special form of collusion that is analyzed in Section 3.

Next, we summarize our results (see also Fig. 3).

- Low quality of customer-specific information, i.e., $n=2$ or 4. If $\delta > \delta_1(n) > \delta_3(n)$, then firms charge collusive discriminatory prices, see (3). If $\delta_3(n) < \delta < \delta_1(n)$, then firms collude on not to acquire information and compete with uniform prices. Prices in this case are given by (10). Finally, if $\delta < \delta_3(n)$, then collusion breaks down. Firms acquire information and compete with discriminatory prices. Profits are given by (2). Firms in this case are in a prisoner's dilemma.
- High quality of customer-specific information, i.e., $n > 4$. If $\delta > \delta_1(n)$, then firms charge collusive discriminatory prices, see (3). Finally, if $\delta < \delta_1(n)$, then collusion breaks down. Firms acquire information and compete with discriminatory prices. Profits are given by (2). Firms in this case are in a prisoner's dilemma.

It is clear from Fig. 3 that collusion becomes more difficult as the firms' ability to segment consumers improves. Since segments may also be thought of as distinct geographical regions, one can also view a refinement of the partition of the characteristics' space as an increase in the

¹⁹ We also examined the effect of low reservation values V on the sustainability of collusion. In particular, we investigated the feasibility of collusion for all V 's when $n=2$ and we compared it with $n=1$ (it is very difficult to extend it for all n 's). We were able to show that, in all three schemes that we consider in Section 3, collusion is more difficult when $n=2$. This is consistent with our main result.

²⁰ However, one drawback of discriminatory pricing is that it may be more "costly" to the firms than uniform pricing. So firms may prefer uniform pricing to discriminatory pricing if both yield the same profits.

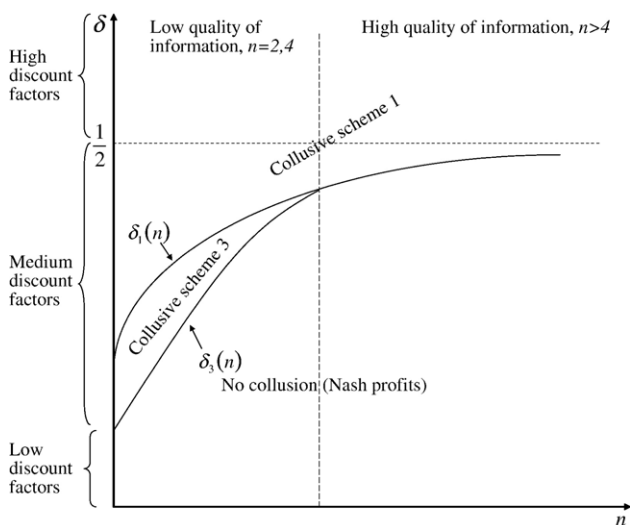


Fig. 3. Partition of the (δ, n) space.

number of markets. Therefore, our result (which states that finer segmentation hinders collusion) is in contrast to [Bernheim and Whinston \(1990\)](#) who demonstrate that multimarket contact may make collusion easier. In [Bernheim and Whinston \(1990\)](#), multimarket contact allows firms to pool the incentive compatibility constraints. Therefore, it is quite possible that, when markets are viewed in isolation, one market has a binding constraint but the constraint in another market is slack. When markets are combined and the constraints are pooled firms can utilize the slackness in one market in order to increase the level of collusion in the other market where the constraint binds. In our model things are working differently. First, unlike the model of [Bernheim and Whinston \(1990\)](#), the size of the big market (i.e., the unit interval) is fixed and by more markets what we really mean is more submarkets. Second, our model is of horizontal differentiation, whereas [Bernheim and Whinston \(1990\)](#) are primarily concerned with the case of homogeneous goods.²¹ The explicit assumption of horizontal differentiation has implications on how firms compete when collusion breaks down and more importantly on how they collude. Since consumers have a most preferred product, it makes sense in the collusive phase that firms will split the entire market and each firm only serves the markets that belong in its own “territory”. An increase in the number of markets makes collusive profits higher (because firms can now target markets more effectively), but the profits from deviation increase disproportionately. The segmentation of the market during collusion leaves the firms very vulnerable to opportunistic behavior from the rival. That is why an increase in the number of markets does not make collusion easier. This effect also plays a role in the difference between our result and that in [Friedman and Thisse \(1993\)](#), where firms commit to the most severe punishment profits by locating next to each other in order to expand the set of discount factors that make collusion possible. Although both the firms’ choice to locate next to each other (as in [Friedman and Thisse, 1993](#)) and the acquisition of better (finer) consumer information (as in our paper) tend to intensify competition in the event collusion breaks down, they lead to markedly different predictions regarding the likelihood of

²¹ In Section 7, they briefly look at the case of differentiated products, but their model is significantly different from ours.

collusion. In our paper better information makes collusion less likely, whereas in [Friedman and Thisse \(1993\)](#) the opposite happens when the degree of product differentiation is minimized. The difference, as we argued above, is that better information increase the incentives for deviation disproportionately, which actually makes collusion more difficult. On the contrary, when product differentiation is minimized, the incentives for cheating are not as strong.

4.1. Consumer welfare

Since aggregate demand is fixed and all consumers buy from the closest firm, the collusive outcomes are always efficient.²² The interesting exercise is to investigate how the consumer surplus gets affected by the presence of consumer information of various levels of quality. In a static framework, consumer surplus is given by,²³

$$CS(n) = V - \frac{c(27n^2 - 36n + 88)}{36n^2}.$$

Consumer surplus is inverse U-shaped with respect to the quality of consumer information n and always higher than the surplus under a uniform competitive pricing rule. It can be easily verified that consumer surplus is maximized at $n=4$. This says that in a static framework consumer information of “moderate” quality yields the highest benefit for the consumers. However, this may change when firms interact repeatedly. If the discount factor is low, then no collusion is sustainable and consumer surplus is inverse U-shaped as a function of n . If the discount factor exceeds $1/2$, then consumer surplus monotonically decreases with n . This is because collusion is always sustainable and firms can extract more consumer surplus as their ability to refine the consumer segmentation improves. For intermediate discount factors, monopoly collusion is sustainable for low n 's and breaks down when n is high. As δ increases n should increase for collusion to break down. Once monopoly collusion breaks down consumer welfare jumps up.²⁴

4.2. Antitrust and policy implications

The antitrust and policy implications that emerge from our analysis are as follows. From [Fig. 3](#) it is clear that collusion is less likely when n is high and δ is low and more likely when n is low and δ is high. Markets with many small consumers that make frequent purchases (implying high discount factors)²⁵ and where firms cannot engage in a very effective consumer segmentation are more susceptible to collusion. On the other hand, markets with few buyers that make infrequent purchases (implying low discount factors) and where firms can target consumers more accurately are less likely to support collusive behavior. Thus, the easiness with which firms can segment the market should be added to the list of factors that hinder collusion.

²² However, under imperfect collusion, this may not be the case. Interestingly, when market sharing in the collusive phase is allowed, some consumers do not buy from the closest firm. This minimizes the incentives for deviation (for more on this, see Section 5.1).

²³ In a static framework firms compete using the available information of quality n (as in the punishment phase of the infinitely repeated game). For a derivation see [Liu and Serfes \(2004, Section 4.2\)](#).

²⁴ The consumer welfare predictions have the same flavor when we allow for imperfect collusion. Although, in that case, collusion does not break down as the segmentation becomes finer, consumer welfare increases (see Section 5.1).

²⁵ We refer the reader to the [U.S. Department of Justice and the Federal Trade Commission \(1992\)](#) Horizontal Merger Guidelines and to [Sibley and Heyer \(2003\)](#) for a discussion on the factors that influence the discount factors and consequently the easiness of collusion.

Furthermore, in markets with low discount factors consumer surplus is maximized when the quality of information is “moderate”. This is because collusion is not sustainable and, as we indicated in the above subsection, consumer surplus is an inverse U-shape with respect to the quality of consumer information. This implies that a regulation that limits (but does not prohibit) the use of consumer information is the best for the consumers. In markets that are characterized by medium discount factors consumer information of higher quality makes collusion less likely and therefore leads to higher consumer surplus. Hence, regulatory authorities should encourage and promote the collection and application of consumer information. Finally, in markets with high discount factors better information is not going to make collusion less likely (when $\delta \geq 1/2$, see Fig. 3), but it will lower consumer surplus. This is because better information allows firms to extract more consumer surplus when they collude successfully. Therefore, regulatory authorities should discourage the collection and application of consumer information.

5. Extensions

We extend the paper in two directions. First, we look at imperfect collusion and second we analyze the effect of more severe than Nash punishments.

5.1. Imperfect collusion

In the previous sections we were concerned with the sustainability of monopoly profits given the quality of consumer information n . Firms can also sustain less than monopoly profits. In this subsection, given δ and n , we find the maximum level of profits that can be sustained in a SPE with Nash punishments, assuming that firms do not share any segment demand in the collusive phase.²⁶ We maintain the simplifying assumption that a deviating firm captures the entire market. Let us look at collusive scheme 1, where firms possess consumer information of quality n but now charge less than monopoly prices. Firm A makes sales only in $[0, 1/2]$, while firm B makes sales only in $[1/2, 1]$. Although firms have zero market share in the rival’s territory, they nevertheless charge positive prices in each segment of the partition. For example, firm B charges positive prices in each segment in $[0, 1/2]$. These prices ensure that the marginal consumer in each segment of the partition is indifferent between the two firms. This does not change the collusive profits, but it curbs the incentives for deviation. Without this mechanism imperfect collusion would actually be more difficult than monopoly collusion, since the incentives for deviation would be large (details are omitted).²⁷ A sequence of imperfectly collusive prices is denoted by

²⁶ A more general approach would be as follows. Fix a target level of imperfectly collusive profits and find the location of the marginal consumers in *each* segment to *minimize* the deviation profits. The solution to this minimization problem may entail sharing (where firm A makes sales in firm B ’s territory and vice versa). This problem, however, is quite difficult due to the general number of segments. Our approach in this section is simpler, while it supports partial collusion. Nevertheless, it is *not* the best for the firms. By allowing for segment sharing in the collusive arrangement, imperfect collusion becomes easier (this does not affect perfect collusion). We were able to prove this result by assuming that $n=2$. In particular, we showed (assuming that collusive profits are not too low) that firms need a lower discount factor when they share segments than when they do not. To check the robustness of our main result, we then compared the easiness of collusion between $n=1$ and $n=2$, when segment sharing is allowed. We showed that collusion is more difficult to sustain when $n=2$ than when $n=1$, which is consistent with our main result.

²⁷ We did not have to make a similar assumption when we examined collusion on monopoly profits. By the very fact that monopoly profits are the highest, a deviating firm cannot increase its profits in its own territory. As a result, a restraint, in the form of rival prices in a firm’s own market, is not needed to make deviation profits lower.

p_{im}^{ImC} , $m=1, \dots, n$ and $i=A, B$. Let π_i^{ImC} , $i=A, B$ denote the level of profits that firms wish to sustain, where $\pi_i^{ImC}(n) \leq \pi_i^C(n)$, for all n , and $\pi_i^C(n)$ are the monopoly profits [see (4)]. The minimum discount factor that is required to sustain $\pi_i^{ImC}(n)$ is given by,

$$\sum_{t=0}^{\infty} \delta^t \pi_i^{ImC}(n) = \pi_i^D(n) + \sum_{t=1}^{\infty} \delta^t \pi_i^N(n). \tag{13}$$

The major difficulty here arises from the fact that there exist many different imperfectly collusive price sequences that yield the same level of profits. In order to find the minimum discount factor (for a given level of profits) we have to determine the optimal distribution of prices which, in general, is a very difficult task. We did not face this problem when we examined monopoly collusion, simply because there is only *one* price sequence that generates the monopoly profits. Nevertheless, given the assumptions we make we are able to solve this problem. The solution is not complete (in the sense that will become clear later) but it provides further evidence that the main result of the previous sections is robust.

We show that as long as $p_{Bm}^{ImC} > \frac{(2m+2-n)c}{n}$, for $m=(n/2)+1, \dots, n$ (and symmetrically defined for $m=1, \dots, n/2$, then only the summation of prices matters and not how they are distributed (further details are omitted). This solves the problem of determining the optimal distribution of prices. The above condition ensures that a deviating firm captures the entire $[0, 1]$ market. For this to happen, prices, before deviation, must be above a lower bound and that is why p_{Bm}^{ImC} must exceed $\frac{(2m+2-n)c}{n}$ for segments in $[1/2, 1]$ (assuming that firm *A* deviates). Furthermore, firm *A* sets a price in the segments in $[1/2, 1]$ so that the marginal consumer in each element of the partition is indifferent between the two firms (analogously, firm *B* sets a price in the $[0, 1/2]$ segments). Then, by summing up the lower bound for the segment prices multiplied by firm *B*'s market share in each segment we derive the minimum level of profits that is allowed by this method, i.e., $\frac{1}{c} \sum_{m=\frac{n}{2}+1}^{m=n} \frac{(2m+2-n)c}{n} = \frac{3c}{2n} + \frac{c}{4}$. Firms now can attempt to sustain any level of profits higher than $\frac{3c}{2n} + \frac{c}{4}$. Observe that our method does not allow firms to sustain profits that are less than $\frac{3c}{2n} + \frac{c}{4}$, since that would violate our assumptions which in turn would make the distribution of prices relevant. Nonetheless, our assumptions are not very restrictive, since a wide range of imperfectly collusive profits can be sustained and moreover that range expands as n increases. For example, when $n=8$ our method works for profits that are greater than $.4375c$, whereas the Nash profits [from (2)] are $.20486c$. On the other hand, the monopoly profits are $\frac{V}{2} - \frac{10c}{64}$, with $V \geq \bar{V}_1 = 1.375c$ [see (4)]. So, if $V=3c$, then monopoly profits are $1.34375c$. Any level of profits in $(.4375c, 1.34375c]$ satisfies our assumptions, while a profit level in $[\bar{.20486c}, .4375c]$ does not. When $n=16$, firms can attempt to sustain any level of profits above $.34375c$ and the Nash profits are $.223c$. This suggests that our method is not applicable only for a small range of low profits, or for low discount factors. In addition, the range of profits that fall outside the acceptable range shrinks as n increases. In the limit, our method applies to any level of profits above Nash profits since $\lim_{n \rightarrow \infty} \frac{3c}{2n} + \frac{c}{4} = \frac{c}{4}$. Second, from (13) we can derive the minimum discount factor that is needed to sustain $\pi^{ImC}(n) = \bar{\pi}$, where $\bar{\pi} \in (\frac{3c}{2n} + \frac{c}{4}, \pi^C(n)]$ (subscript is dropped due to symmetry). This is given by,

$$\delta^{ImC}(n|\bar{\pi}) = \frac{9n(4n\bar{\pi} - cn - 2c)}{2(36n^2\bar{\pi} - 9cn^2 - 20c)}.$$

The following can be easily verified. As expected, the minimum discount factor that is required to sustain imperfect collusion is lower than the discount factor that is required to sustain

monopoly collusion, i.e., $\delta^{\text{ImC}}(n|\bar{\pi}) \leq \delta_1(n)$, and when the imperfectly collusive profits $\bar{\pi}$ are equal to the monopoly profits π^C the two discount factors are the same, i.e., $\delta^{\text{ImC}}(n|\bar{\pi} = \pi^C(n)) = \delta_1(n)$. Finally, and most importantly, $\delta^{\text{ImC}}(n|\bar{\pi})$ is increasing in n , with $\lim_{n \rightarrow \infty} \delta^{\text{ImC}}(n|\bar{\pi}) = 1/2$ which implies that collusion is more difficult, as the information becomes better. In that case firms will have to sustain lower collusive profits. This is the same pattern with the one that $\delta_1(n)$, from scheme 1, exhibits. The main conclusion is that our prediction, that better information makes collusion more difficult, continues to hold even when firms sustain less than monopoly profits, provided that the discount factor is not too low.²⁸ The intuition is similar to the intuition we offered about why collusive scheme 1 becomes more difficult to sustain as the quality of information improves.

Therefore, it follows that if the discount factor is at a level at which firms can sustain imperfect collusion (i.e., $\delta < 1/2$, but not too low), then an increase in n will lead to higher consumer surplus. Of course, if the discount factor is high (i.e., greater than $1/2$), then, as we discussed in Section 4.1, monopoly profits are sustainable and better information lowers consumer surplus.

5.2. More severe punishments: stick and carrot

Globally optimal punishments are usually intractable, see [Abreu \(1986\)](#) and [Bernheim and Whinston \(1990\)](#). Therefore, we restrict our attention to symmetric punishment paths of the following form. Following [Abreu \(1986\)](#), we assume that if a deviation from the collusive path (monopoly profits) occurs firms will enter the punishment phase for T periods. During the punishment phase both firms charge zero prices (marginal cost). After T periods pass firms move back to the collusive phase. If a firm deviates during the punishment phase, then the punishment phase is prolonged by one more period. We denote the one-shot deviation profits from the punishment phase by $\pi^{\text{Dp}}(n)$, which are the profits when the deviator plays a static best response to a zero price. We now have two types of deviations: one is from the collusive path and it is denoted by D and the other is from the punishment path and it is denoted by D_p . We denote by $\Pi(n)$ the present value of each firm's profits (given n) at period t when the punishment phase begins assuming that neither firm deviates during the punishment phase, i.e.,

$$\Pi(n) = 0 + \frac{\delta^T}{1 - \delta} \pi^C(n). \tag{14}$$

There will be no deviation from the collusive agreement if and only if,

$$\frac{\pi^C(n)}{1 - \delta} \geq \pi^D(n) + \delta \Pi(n) \Rightarrow [\text{using (14)}] \frac{1 - \delta^{(T+1)}}{1 - \delta} \geq \frac{\pi^D(n)}{\pi^C(n)}, \tag{IC_Cooperation}$$

where $\pi^C(n)$ is given by (4) and $\pi^D(n)$ is given by (5). This is the standard incentive compatibility constraint, see also (1). But now we need one more incentive compatibility constraint to ensure that neither firm will deviate from the punishment path. There will be no deviation from the punishment phase if and only if,

$$\Pi(n) \geq \pi^{\text{Dp}}(n) + \delta \Pi(n) \Rightarrow \Pi(n) \geq \frac{\pi^{\text{Dp}}(n)}{1 - \delta} \Rightarrow [\text{using (14)}] \delta^T \geq \frac{\pi^{\text{Dp}}(n)}{\pi^C(n)}, \tag{IC_Punishment}$$

where $\pi^{\text{Dp}}(n) = \frac{c(n^2 - 2n + 2)}{4n^2}$ (we omit details regarding its derivation).

²⁸ If the discount factor is very low (i.e., close to zero), then our method is not applicable and we do not know how imperfectly collusive profits vary with n .

Our goal is twofold. First, we would like to find a $\Pi(n)$ that satisfies both (IC_Cooperation) and (IC_Punishment) and at the same time the minimum discount factor δ^* that is required to sustain monopoly profits is less than the minimum discount factor with Nash punishment, i.e., $\delta_1(n)$, see (6). Actually, given that the constraints are satisfied, we want to minimize δ^* . Second, and given a more severe than Nash punishment, we would like to compare the minimum discount factor under more severe punishments with $\delta_1(n)$.

Observe that both incentive compatibility constraints should be satisfied with equality. To see this, suppose by way of contradiction that this is not true. In particular, δ^* is minimum, but (IC_Punishment) is satisfied with strict inequality, while (IC_Cooperation) binds. Firms can then increase T without violating (IC_Punishment). Now observe that $\frac{1-\delta^{T+1}}{1-\delta}$ [i.e., the left-hand side of (IC_Cooperation)] is increasing in δ and T . Hence, an increase in T will relax (IC_Cooperation) which implies that δ^* can be lowered further, a contradiction to the assumption that δ^* is minimum. We can arrive at a similar contradiction if we assume that (IC_Cooperation) is not binding, or both constraints are not binding.

Therefore, the minimum discount factor (together with T) that is needed to support monopoly profits is the solution to the two incentive compatibility constraints and is given by,²⁹

$$\delta^*(n) = \frac{n(4Vn - 3cn - 6c)}{2(4Vn^2 - 3cn^2 - 2cn - 2c)}.$$

It can be easily verified that monopoly collusion with a stick and carrot punishment is easier than with Nash reversion, i.e., $\delta^*(n) < \delta_1(n)$. Moreover, $\delta^*(n)$ is an increasing function of n (as $\delta_1(n)$ is). This indicates that our main result is robust to modeling changes that involve more severe punishments. Although a lower discount factor is needed to sustain monopoly profits with more severe punishments than with Nash reversion, nevertheless both discount factors are increasing functions of the quality of information n .

6. Conclusion

We study an infinitely repeated Hotelling duopoly game, where in each period firms have access to customer-specific information of a certain quality. The information allows the firms to segment consumers into groups and target each group with a different price. Consumer information is modeled as a partition of the characteristics space. A refinement of the partition signifies consumer information of a higher quality. It is a well-known result in the literature of oligopolistic third-degree price discrimination, under best-response asymmetry, that the static game is a prisoner's dilemma. Firms engage in price discrimination and make lower profits than the profits under a uniform pricing rule. The reason is that the intensified competition effect dominates the surplus extraction effect. Hence, in a repeated framework firms have strong incentives to coordinate their actions to eliminate the intensified competition effect.

We investigate the sustainability of three *simple* collusive schemes: i) firms charge monopoly discriminatory prices, ii) firms charge uniform monopoly prices and iii) firms charge competitive uniform prices. The first collusive scheme yields the highest profits and the third the lowest. The second collusive scheme is dominated by the first one (since it turns out that it is more

²⁹ We should note at this point that T is treated as a non-integer. Not surprisingly, if we force T to be an integer, then a solution may not exist. In other words, there may not exist an integer T that satisfies both incentive compatibility constraints and at the same time yields punishment profits that are more severe than Nash. Therefore, our solution should be viewed as an approximation.

difficult to sustain and yields lower profits) and therefore it will never be an equilibrium. When the firms' ability to segment consumers is limited (i.e., low quality of information), the third collusive scheme is easier to sustain in equilibrium than the first one. This implies that firms segment the market and price discriminate for either high (monopolistic price discrimination) or low (competitive price discrimination) discount factors and charge a uniform competitive price for medium discount factors (see Fig. 3). For high levels of information quality the third collusive scheme is dominated by the first one, since it is more difficult to sustain and yields lower profits. In this case, firms always practice price discrimination. For high discount factors it is monopolistic price discrimination and for low discount factors (when collusion breaks down) it is competitive price discrimination. The main result is that collusion becomes increasingly more difficult as the quality of information improves. This conclusion continues to hold when we assume imperfect collusion or more severe than Nash punishments.

Our modeling framework has a number of limitations and therefore our main result, that collusion becomes more difficult as firms' ability to segment consumers increases, may be sensitive to these assumptions. Some limitations are inherent to the Hotelling model in general (e.g. symmetric demands, uniform distribution, covered market, inelastic demand), while others are more specific to our model (e.g. equal-sized segments).³⁰ Moreover, other forms of collusion and in particular a one where firms share the segment demands in the collusive phase have not been fully explored.

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Appendix A

Proof of Proposition 1. We conjecture the following structure. There exist two integers m_1 and m_2 with $0 \leq m_1 < m_2 < n + 1$, such that: i) [*left segments*] firm A is a constrained monopolist in all segments from 1 to m_1 (if $m_1 = 0$, then firm A is never a constrained monopolist), ii) [*middle segments*] in all segments from $m_1 + 1$ to $m_2 - 1$ the two firms sell positive quantities and iii) [*right segments*] in all segments from m_2 to n firm B is a constrained monopolist (again, if $m_2 = n + 1$ firm 2 is never a constrained monopolist). Next, we set out to prove that this structure indeed holds.

◆ Both firms charge strictly positive prices (middle segments).

Ignoring the nonnegativity constraints and setting $\frac{\partial \pi_m^N}{\partial p_m} = 0$, $i = A, B$, we obtain following solutions for the prices,

$$p_{Am}^N = \frac{c(n - 2m + 4)}{3n} \quad \text{and} \quad p_{Bm}^N = \frac{c(2m - n + 2)}{3n}.$$

³⁰ The results about monopoly collusion remain qualitatively unaffected when instead of a linear we use a quadratic transportation cost.

Using these prices we obtain the demands,

$$d_{Am} = \frac{-2m + n + 4}{6n} \quad \text{and} \quad d_{Bm} = \frac{2m - n + 2}{6n}.$$

We can see that d_{Am} is decreasing in m , and d_{Bm} is increasing in m . This means that firm A may decide to charge a zero price and give the entire segment demand to firm B for segments that are in firm B 's territory. Analogously, for firm B in segments that are in firm A 's territory. For segments in the middle of the interval both firms charge positive prices. Observe that $d_{Bm} = \frac{2m - n + 2}{6n} \leq 0$ for any $m \leq \frac{n}{2} - 1$ and $d_{Am} = \frac{-2m + n + 4}{6n} \leq 0$ for any $m \geq \frac{n}{2} + 2$. Now define $m_1(n)$ to be the largest integer that is less than or equal to $\frac{n}{2} - 1$, and $m_2(n)$ to be the smallest integer that is greater than or equal to $\frac{n}{2} + 2$. Obviously,

$$m_1 = \frac{n}{2} - 1 \quad \text{and} \quad m_2 = \frac{n}{2} + 2.$$

This will be used later in the proof. Hence, for any $m = m_1 + 1, \dots, m_2 - 1$, both firms charge strictly positive prices and have strictly positive segment demands.

- ◆ Firm A charges strictly positive prices while firm B charges a zero price (left segments). Following the analysis above, this case is valid for $m \leq m_1$. Then $d_{Bm} \leq 0$: This implies that $d_{Bm} = 0$ and $d_{Am} = 1/n$. This further implies that $p_{Bm}^N = 0$, and p_{Am}^N is the solution to $d_{Am}(p_{Bm}^N = 0) = 1/n$, which yields $p_{Am}^N = \frac{c(n - 2m)}{n}$.
- ◆ Firm B charges strictly positive prices while firm A charges a zero price (right segments). This case is valid for $m \geq m_2$. This case is symmetric to case 2. Firm B 's prices in these segments are: $p_{Bm}^N = \frac{c(2m - n - 2)}{n}$.

Therefore, firms' profits for each n are,

$$\pi_A^N(n) = \sum_{m=1}^{m_1} \frac{c(n - 2m)}{n^2} + \sum_{m=m_1+1}^{m_2-1} \frac{c(2m - n - 4)^2}{18n^2},$$

$$\pi_B^N(n) = \sum_{m=m_1+1}^{m_2-1} \frac{c(2m - n + 2)^2}{18n^2} + \sum_{m=m_2}^n \frac{c(2m - n - 2)}{n^2}.$$

By performing the summation and using ($m_1 = \frac{n}{2} - 1$ and $m_2 = \frac{n}{2} + 2$) we obtain,

$$\pi_i^N(n) = \frac{(9n^2 - 18n + 40)c}{36n^2}, \quad i = A, B. \quad \square$$

Proof of Proposition 2. The difference between $\delta_1(n)$ and $\delta_3(n)$ is expressed as follows,

$$\Delta(n) = \frac{9((36n^4 - 216n^3 + 232n^2)V + 252cn^3 - 298cn^2 + 116cn - 27cn^4 - 160c)}{2(36Vn^2 - 18cn - 27cn^2 - 40c)(27n^2 - 18n - 22)}.$$

We differentiate $\Delta(n)$ with respect to V to obtain,

$$\frac{d\Delta(n)}{dV} = \frac{79cn^2(9n - 10)}{(36Vn^2 - 18cn - 27cn^2 - 40c)^2} > 0, \quad \text{for all } n \geq 2.$$

First we show that $\Delta(n) < 0$ for all $n \geq 8$. We let V go to infinity. This yields,

$$\lim_{V \rightarrow \infty} \Delta(n) = -\frac{9n^2 - 54n + 58}{2(27n^2 - 18n - 22)}.$$

The above expression is negative if $n > 3 + \sqrt{\frac{23}{3}} \approx 4.6$. Since $\Delta(n)$ is monotonically increasing in V , this would imply that $\Delta(n) < 0$ for all $n \geq 8$ (recall that the next value of n after 4 is 8) and all $V > 0$.

What remains now is to check the sign of $\Delta(n)$ for $n=2$ and 4. It turns out that $\Delta(n=2) > 0$ if and only if $V > \bar{V}_4 = 29c/14 \approx 2.071c$. Moreover, $\Delta(n=4) > 0$ if and only if $V > \bar{V}_4 = 297c/56 \approx 5.3c$.

Therefore, V must be greater than the maximum \bar{V}_4 which is $297c/56 \approx 5.3c$.

Finally, the monotonicity of $\delta_1(n)$ and $\delta_3(n)$ can be proved easily and the steps are not shown here. \square

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