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Intertemporal discounting and tenurial contracts

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Abstract

A simple two-period model, without uncertainty and strategic complementarity in the labor market or asymmetric information of any kind, is employed to study the nature of contracts signed between a landlord and a tenant in a rural economy. The paper provides conditions under which different types of contracts may prevail. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The study of tenurial contracts in a rural economy has gained much attention in development economics. In particular, existence of share tenancy has been a source of puzzlement and has thereby provoked a large literature.[2] We refer to Singh (1989) for an excellent survey on this topic. The trouble began with the Marshallian analysis by which one could only conclude that landlords prefer fixed-rent contracts to share tenancy. It was then conjectured that some form of exogenous uncertainty may explain share tenancy. Later work proved that just one

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[2] For example, see Cheung (1968), Bardhan and Srinivasan (1971), and Stiglitz (1974).

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source of uncertainty was not sufficient. Newbery (1977) showed that uncertainty in the labor market was also needed. There had been other attempts to explain share tenancy by introducing asymmetric information and differential managerial skills. However, all such attempts were unsuccessful. Hallagan (1978), Allen (1982), and Basu (1984) had shown that what is needed is at least two factors of production for which quality is uncertain and the information among buyers and sellers is asymmetric.

In a more recent work, Basu (1992) argues that in poor areas, it is natural to have each contract underwritten by an implicit limited-liability clause which allows a person to forego paying rent under extreme crop failures. As a consequence, share tenancy may arise in order to mitigate the tension between the landlord and the tenant, as the former prefers safe projects while the latter prefers risky ones. Sengupta (1997) extends Basu's work by showing that limited liability and moral hazard in effort provides a richer theory of share contracts when the tenant also has discretion in the choice of the project. Muthoo (1998) shows that tenurial contracts are sensitive to the tenant's privately known farming skill. In particular, tenants with "low" skills work for wage contracts, those with "high" skills work for fixed-rent contracts, and tenants with "moderate" skills work for share contracts. Basu (1993) speculates that in an oligopsonistic labor market with strategic complementarity, an explanation for share tenancy may exist. With empirical evidence for positive wage response of unskilled labor supply from Bardhan (1979) and Rosenzweig (1984), Ray (1999) builds upon Basu's speculation and shows that share tenancy can be explained as a strategic delegation on part of the landlords if the reservation utility of the landlord is greater than that of the tenant.

The contribution of this paper to the existing literature is to provide a simple two-period framework to study different types of tenurial contracts and to provide a new explanation for the existence of each one of them. We abstain from uncertainty and strategic complementarity in the unskilled labor market and assume away asymmetric information of any kind. In our environment, there is only one landlord who owns a piece of land. The present quality of the land is inversely related to the level of production in the past. The landlord employs a tenant for the two periods by offering him a contract in each period. The employed tenant is a monopsonist and hires unskilled labor from the labor market by paying a minimum wage which is exogenously given.

We define three types of contracts, namely, *wage-input* contracts, *share* contracts, and *fixed-rent* contracts. In a wage-input contract, the landlord employs a tenant, offers a wage, and specifies the amount of unskilled labor that the tenant has to hire. In all other types of contracts, the landlord leaves production decisions to the tenant but specifies a fraction of the total foodgrain output that the tenant retains and a land rent that the tenant pays him. The last two types of contracts (linear contracts) will henceforth be referred to as *tenurial contracts*. In a share contract, this fraction is strictly between zero and one while in a fixed-rent contract, this fraction is equal to one.[3] Mention must also be made here that our model does not allow us to readily include wage contracts as in Muthoo (1998). In order to define such contracts, he needs to assume that there is always "gains from trade" between the landlord and the tenant and therefore even with zero effort on part of the tenant, foodgrain output exceeds the sum of the reservation utilities of both parties. In our model, the tenant's effort level will play no role, and production will depend on the level of employment of unskilled labor. It may then seem absurd to assume that foodgrain production is positive with zero level of unskilled labor. We will therefore need to change the optimization problem of the landlord to define wage-input contracts from the standard one which will be used to define tenancy contracts.

The paper provides conditions under which each type of contract may prevail in equilibrium. The driving forces behind our results are: (i) the dependence of the quality of land[4] in period 2 on the level of employment of unskilled labor in period 1, and (ii) the difference in the way the landlord and the tenant discount future profits. In particular, the paper shows that in rural economies where the quality of land deteriorates with previous use and there are *input-specification* costs, (a) if tenants have a positive probability of leaving the land in the future, and/or they are relatively poor and therefore do not care enough for future profits, share tenancy is more likely to be the prevalent institution; (b) if the future is valued equally by the landlord and the tenant, a fixed-rent contract is the likely outcome; and (c) if the tenant values the future more than the landlord, and offering wage-input contracts does not involve high input-specification costs, a wage-input contract is the dominant institution; otherwise, that is when such input-specification costs are high, a fixed-rent contract is offered.

The rest of the paper is structured as follows. Section 2 describes the model. Section 3 compares between share contracts and fixed-rent contracts. Section 4 specifies the framework to allow for wage-input contracts, provides a discussion on the costs associated with them, and compares such contracts with the other two types of tenurial contracts studied in Section 3. The paper concludes in Section 5.

[3] As Sengupta (1997) points out, Shetty's (1988) model actually shows that contracts where the share is greater than one may be theoretically optimal. However, such contracts seem unrealistic in practice and we therefore rule them out. For an extensive discussion on this issue, we refer the reader to Ray and Singh (1998). We thank an anonymous referee for drawing our attention on this issue.

[4] There is empirical literature (e.g. Allen and Lueck, 1992; Ackerberg and Botticini, 2000) which acknowledges and tests the hypothesis that certain types of contracts provide the tenant with incentives to exploit and abuse the soil or other valuable assets. Our modelling framework may encompass this possibility but is not limited to it. The role of sharecropping, in our paper, may also come through a different channel than in the above mentioned papers. For example, the tenant may have the same horizon in the farm as the landlord [and therefore no incentive to damage the landlord's assets] but due to different preferences over future income he may produce more (or less) today than what the landlord would have liked and consequently a share contract may mitigate these differences.

2. The model

Consider a rural economy where credit is difficult to avail. There is a landlord who is a monopsonist in the labor market and operates in a two-period time horizon, where each period is denoted by $t = 1,2$. Let s_t be the quality of his land and L_t be the amount of unskilled labor employed in period t. We assume that $s_1 > 0$ is given exogenously while s_2 diminishes with L_1, the level of employment of unskilled labor in period 1. Thus, $s_2 = s_2(L_1)$, for which we assume that $\mathrm{d}s_2(L_1)/\mathrm{d}L_1 < 0$, $\mathrm{d}^2 s_2(L_1)/\mathrm{d}^2 L_1 < 0$ and $|\mathrm{d}s_2(L_1)/\mathrm{d}L_1| < M$, for some $0 < M < \infty$. These assumptions on the function $s_2(\cdot)$ imply that although the rate of decrease in the quality of land increases, the rate itself is bounded. Let Q_t be the (non-storable) foodgrain output in period t. We assume that foodgrain prices are the same and equal to one in both periods. The production function that uses unskilled labor to produce Q_t is given by $Q_t = s_t f(L_t)$. Foodgrain output increases with the quality of land. We assume that for each period t, the function $f(\cdot)$ is twice continuously differentiable, increasing, strictly concave and $f(0) = 0$.

In order to carry out farming, the landlord has to employ a tenant[5] by offering him contracts in each period which we assume to be binding.[6] A tenurial contract in period t is a pair (α_t, β_t) with $\alpha_t \in [0,1]$ and $\beta_t \in \mathbf{R}$, having the following interpretation. The employed tenant retains α_t fraction of the total foodgrain output in period t and pays a lump-sum amount equal to β_t. A pair (α_t, β_t) is a share contract if $\alpha_t \in (0,1)$, while it is a fixed-rent contract if $\alpha_t = 1$. The last type of contract is a wage-input contract, which is a pair $(\gamma_t, \tilde{L}_t)$ where $\gamma_t \in \mathbf{R}_+$ is the wage that the tenant receives in period $t = 1,2$, while $\tilde{L}_t$ is the input specification that mentions how many unskilled laborers the landlord wants the tenant to hire in period t. We will study wage-input contracts separately in Section 4 as it will require a different analytical framework.

We assume that any amount of unskilled labor can be hired at the existing minimum wage $\overline{w}$, which may be thought of as the reservation utility of laborers and assumed to be positive. In other words, we assume that the supply function for unskilled labor is horizontal.

The decision problem in this environment is as follows. At the beginning of period 1, the landlord offers the contract (α_1, β_1) and at the beginning of period 2 he offers the contract (α_2, β_2), to maximize the discounted sum of his profits over the two periods. The employed tenant hires as many unskilled laborers as he

[5] We assume that the landlord has very low managerial skills and therefore, own cultivation is ruled out. Our model therefore includes the case of *absentee-landlordism*, an institution which is highly prevalent in many rural economies, where almost all farming activities are under the control of the farm tenant.

[6] The contracts we consider is a sequence of two short-term contracts rather than a long-term one. Hence, the "binding" refers to the terms of each contract as opposed to the enforceability of the long-term relationship.

wishes in the two periods at the existing minimum wage $\overline{w}$ in order to maximize the discounted sum of his profits. Let $\delta_L \in [0,1]$ and $\delta_T \in [0,1]$ be the subjective intertemporal discount factors of the landlord and the tenant, respectively.

The tenant in period 1 foresees the effect of his employment decision in that period on the quality of land in period 2. Therefore, at the beginning of the planning horizon, he confronts the profit function

$$\pi_1 = [\alpha_1 s_1 f(L_1) - \overline{w}L_1 - \beta_1] + \delta_T[\alpha_2 s_2(L_1) f(L_2) - \overline{w}L_2 - \beta_2].$$

We assume that π_1 is strictly concave in L_1 and L_2 for any $\alpha_1 \geq 0$ and $\alpha_2 > 0$.[7]

The tenant knows that at the beginning of period 2, he will face the problem

$$\max_{L_2} \pi_2 = \alpha_2 s_2(L_1) f(L_2) - \overline{w}L_2 - \beta_2. \tag{1}$$

Let L_2^* be the solution of the maximization problem as in Eq. (1). Notice that this solution will depend on L_1 and the parameters of the problem. Thus, while solving his period 1 problem, the tenant keeps account of this dependence and chooses L_1 to solve the problem

$$\max_{L_1} \pi_1 = [\alpha_1 s_1 f(L_1) - \overline{w}L_1 - \beta_1] + \delta_T[\alpha_2 s_2(L_1) f(L_2^*) - \overline{w}L_2^* - \beta_2]. \tag{2}$$

Let L_1^* be the solution of the maximization problem as in Eq. (2).

The reservation utility of the tenant in each period is equal to $r > 0$, which is thought of as subsistence consumption. Therefore, we assume that at the end of each period, the landlord must give the employed tenant at least r. Keeping this and the maximization problem of the tenant in mind, the landlord solves the problem

$$\max_{((\alpha_1,\beta_1),(\alpha_2,\beta_2))} \Pi = [(1-\alpha_1) s_1 f(L_1^*) + \beta_1] + \delta_L(1-\alpha_2) s_2(L_1^*) f(L_2^*) + \beta_2], \tag{3}$$

subject to

$$\text{(i) } \alpha_1 s_1 f(L_1^*) - \overline{w}L_1^* - \beta_1 \geq r \tag{4}$$

$$\text{(ii) } \alpha_2 s_2(L_1^*) f(L_2^*) - \overline{w}L_2^* - \beta_2 \geq r. \tag{5}$$

In Section 3, we solve for the equilibrium tenurial contracts that the landlord will offer in each period.

[7] In the Section 3, we will show that the equilibrium value of α_2 is always 1, independent of the strict concavity of π_1. Below, we offer a parametric example where π_1 is strictly concave in $L_1$and L_2. Consider a production function of the form: $f(L) = L^a$, with $0 < a < 1$ and $s_2(L_1) = s_1 - bL_1^2$, which satisfy the assumptions ofour model. It can be easily checked that π_1 is strictly concave in L_1 and L_2 for any $L_2 \geq 0$ and any $L_1 \in [0, \sqrt{s_1(1-a)/b(1+a)})$. Now observe that the highest level of employment in period 1 is when $\alpha_1 = 1$ and $\delta_T = 0$ [full incentives and no future for the tenant, see also Eq. (7)]. In this case $L_1^* = (s_1 a/w)^{1/1-a}$. One can easily find conditions on the parameters (s_1, a, w, b) such that $L_1^* \in [0, \sqrt{s_1(1-a)/b(1+a)})$ which guarantees the concavity of π_1 in therelevant region of the choice variables.

3. Equilibrium tenurial contracts

In this section, we will not consider the possibility of the landlord offering a wage-input contract. Instead, we will restrict ourselves to the tenurial contracts mentioned before. We begin with the problem of the tenant.

Let us consider the first-order conditions of the tenant's problem. The first-order condition of the tenant in period 2, which is derived from Eq. (1), is

$$\alpha_2 s_2 \frac{\partial f(L_2)}{\partial L_2} - \overline{w} = 0. \tag{6}$$

The first-order condition of the tenant in period 1, which is derived from Eq. (2), is

$$\left[\alpha_1 s_1 \frac{\partial f(L_1)}{\partial L_1} + \delta_{\mathrm{T}} \alpha_2 \frac{\partial s_2(L_1)}{\partial L_1} f(L_2^*) - \overline{w}\right] + \delta_{\mathrm{T}} \frac{\partial L_2^*}{\partial s_2} \frac{\partial s_2(L_1)}{\partial L_1}\left[\alpha_2 s_2(L_1) \frac{\partial f(L_2^*)}{\partial L_2} - \overline{w}\right] = 0,$$

which when combined with Eq. (6), yields

$$\alpha_1 s_1 \frac{\partial f(L_1)}{\partial L_1} + \delta_{\mathrm{T}} \alpha_2 \frac{\partial s_2(L_1)}{\partial L_1} f(L_2^*) - \overline{w} = 0. \tag{7}$$

The landlord chooses the pair of contracts $((\alpha_1, \beta_1), (\alpha_2, \beta_2))$ to maximize the discounted sum of his profits over the two periods. First note that for any pair of equilibrium contracts, the participation constraints as given by Eqs. (4) and (5) will hold with equality, thereby ensuring the fact that the landlord extracts all the surplus from the tenant he employs. Substituting Eqs. (4) and (5) in Eq. (3), the landlord's profit maximization problem can be rewritten as

$$\max_{\alpha_1, \alpha_2} \left[s_1 f(L_1^*) - \overline{w} L_1^* - r\right] + \delta_{\mathrm{L}}\left[s_2(L_1^*) f(L_2^*) - \overline{w} L_2^* - r\right]. \tag{8}$$

The first-order conditions of the landlord's maximization problem with respect to α_1 and α_2 are

$$\frac{\partial L_1^*}{\partial \alpha_1}\left[s_1 \frac{\partial f(L_1^*)}{\partial L_1^*} - \overline{w} + \delta_{\mathrm{L}} \frac{\partial s_2(L_1^*)}{\partial L_1^*}\left(f(L_2^*) + s_2(L_1^*) \frac{\partial f(L_2^*)}{\partial L_2^*} \frac{\partial L_2^*}{\partial s_2} - \overline{w} \frac{\partial L_2^*}{\partial s_2}\right)\right] = 0, \tag{9}$$

and

$$\frac{\partial L_2^*}{\partial \alpha_2}\left[s_2(L_1^*) \frac{\partial f(L_2^*)}{\partial L_2^*} - \overline{w}\right] = 0. \tag{10}$$

Before proving the main result in this section, we prove the following lemma which states that under our assumptions, any tenurial contract with a higher α_t induces a higher level of employment of unskilled labor in period $t = 1, 2$.

Lemma 1. *The tenant's equilibrium levels of employment of unskilled labor in period 1 and period 2 increase monotonically in α_1 and α_2, respectively, that is, $\partial L_t^* / \partial \alpha_t > 0$, $t = 1, 2$.*

Proof. See Appendix A. ■

We now state and prove the main result in this section.

Proposition 2. *The equilibrium tenurial contracts in the two periods are as follows:*

(i) in period 2, the landlord offers a fixed-rent contract, that is, $\alpha_2 = 1$;
(ii) whenever the discount factor of the tenant is less than that of the landlord, the landlord offers a share contract in period 1, that is, whenever $\delta_T < \delta_L$, we have $\alpha_1 \in (0,1)$;
(iii) whenever the discount factor of the tenant is greater than or equal to that of the landlord, the landlord offers a fixed-rent contract in period 1, that is, whenever $\delta_T \geq \delta_L$, we have $\alpha_1 = 1$.

Proof. Substituting Eq. (10) in Eq. (9), we first rewrite the first-order conditions of profit maximization for the landlord. The two conditions are given by Eq. (10) and

$$\frac{\partial L_1^*}{\partial \alpha_1}\left[s_1 \frac{\partial f(\cdot)}{\partial L_1^*} + \delta_L \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*) - \overline{w} \right] = 0. \tag{11}$$

(i) Since by Lemma 1, we have $\partial L_2^* / \partial \alpha_2 > 0$, Eqs. (10) and (6) can hold simultaneously if and only if $\alpha_2 = 1$. This proves that the second period contract is a fixed-rent.
To prove the rest of the proposition, we proceed as follows. Rewrite Eq. (7) as

$$\begin{aligned} & s_1 \frac{\partial f(\cdot)}{\partial L_1^*} + \delta_L \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*) - \overline{w} \\ & \quad = (1-\alpha_1) s_1 \frac{\partial f(\cdot)}{\partial L_1^*} + (\delta_L - \delta_T \alpha_2) \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*). \end{aligned} \tag{12}$$

Substituting the R.H.S. of Eq. (12) in Eq. (11) and setting $\alpha_2 = 1$, we get

$$\frac{\partial L_1^*}{\partial \alpha_1}\left((1-\alpha_1) s_1 \frac{\partial f(\cdot)}{\partial L_1^*} + (\delta_L - \delta_T) \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*) \right) = 0. \tag{13}$$

From Eq. (7), we have

$$s_1 \frac{\partial f(\cdot)}{\partial L_1^*} = \frac{\overline{w} - \delta_{\mathrm{T}} \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*)}{\alpha_1}. \tag{14}$$

Then, from Eqs. (13) and (14), we have

$$\frac{\partial L_1^*}{\partial \alpha_1}\left[\frac{1-\alpha_1}{\alpha_1}\left(\overline{w} - \delta_{\mathrm{T}} \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*)\right) + (\delta_{\mathrm{L}} - \delta_{\mathrm{T}}) \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*)\right] = 0. \tag{15}$$

Since by Lemma 1, $(\partial L_1^* / \partial \alpha_1) > 0$, it must be that

$$\left[\frac{1-\alpha_1}{\alpha_1}\left(\overline{w} - \delta_{\mathrm{T}} \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*)\right) + (\delta_{\mathrm{L}} - \delta_{\mathrm{T}}) \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*)\right] = 0. \tag{16}$$

Notice also that from our assumptions, $(\overline{w} - \delta_{\mathrm{T}}(\partial s_2(L_1^*))/(\partial L_1^*) f(L_2^*)) > 0$ and $(\partial s_2(L_1^*))/(\partial L_1^*) f(L_2^*) < 0$.
(ii) To show that $\alpha_1 > 0$, suppose on the contrary that $\alpha_1 = 0$. Then the L.H.S. of Eq. (16) is $+\infty$ and therefore the landlord finds it profitable to increase α_1. Finally, to show that $\alpha_1 < 1$, suppose on the contrary that $\alpha_1 = 1$. Then, the L.H.S. of Eq. (16) becomes equal to

$$(\delta_{\mathrm{L}} - \delta_{\mathrm{T}}) \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*)$$

which, with $\delta_{\mathrm{T}} < \delta_{\mathrm{L}}$, from our assumptions is negative and therefore the landlord can increase his profits by reducing α_1. Therefore, it must be that $0 < \alpha_1 < 1$.
(iii) With $\delta_{\mathrm{T}} = \delta_{\mathrm{L}}$, Eq. (16) reduces to

$$\frac{1-\alpha_1}{\alpha_1}\left(\overline{w} - \delta_{\mathrm{T}} \frac{\partial s_2(L_1^*)}{\partial L_1^*} f(L_2^*)\right) = 0.$$

which can hold if and only if $\alpha_1 = 1$.
With $\delta_{\mathrm{T}} > \delta_{\mathrm{L}}$, we have $(\delta_{\mathrm{L}} - \delta_{\mathrm{T}})(\partial s_2(L_1^*)/\partial L_1^*) f(L_2^*) > 0$ and therefore for Eq. (16) to hold, it must be that $((1-\alpha_1)/\alpha_1)[\overline{w} - \delta_{\mathrm{LT}}(\partial s_2(L_1^*)/\partial L_1^*) f(L_2^*)] < 0$ which is possible if and only if $\alpha_1 > 1$. However, with the restriction that $\alpha_1 \in [0,1]$, this implies that $\alpha_1 = 1$. ■

The intuition behind the above result is as follows. Suppose that the discount factor of the tenant is less than that of the landlord. The landlord knows that if in period 1 he offers a fixed-rent contract, the tenant will have all the incentive to produce as much as he wants and care less (than the landlord) about the quality of the land the day after, since the tenant's valuation of future profit is less than that of the landlord. Therefore, the landlord would like to keep a balance in the quality of his land between the two periods and ensure himself with a good harvest at the end of period 2. The only way he can achieve this is by reducing the incentive of the tenant in period 1. A share contract serves this purpose. It takes away some incentive from the tenant in period 1 by which he is forced to reduce foodgrain production in period 1, thereby enabling the landlord to maintain the desired quality of land.

The reverse works when the discount factor of the tenant is higher than that of the landlord. In this situation, the landlord knows that the tenant will tend to produce less in period 1 and try to save the quality of land for period 2. The landlord has to therefore give the tenant an extra incentive to produce more in period 1. A simple fixed-rent does not balance the needs of the landlord with those of the tenant because under fixed-rent contracts, the profit incentive of the tenant is the same as that of the landlord under own cultivation. Therefore, the only way the landlord can increase this incentive beyond what a fixed-rent contract does, is by offering a contract that rewards the tenant by more than the additional output he produces. A contract with $\alpha > 1$ would ideally do this job. However, for various reasons as discussed in Ray and Singh (1998), such contracts may not be enforceable in practice. Given this, the next best available tenurial alternative is a fixed-rent contract.

In period 2, since there is no future, the landlord does not care about the quality of land thereafter, and consequently, cares only about maximizing current profits. An $\alpha = 1$ ensures productive efficiency, i.e., the surplus which is distributed between the tenant and the landlord is maximized. Notice that any $\alpha < 1$ reduces the tenant's incentives to maximize the surplus and therefore the profits accruing to the landlord are reduced as well. Finally, by the nature of the optimization problem we have addressed, any solution is Pareto efficient.

In Section 4, we will lay the framework for defining wage-input contracts which we have ignored so far. We end this section with the following remarks.

Remark 1. Imagine a situation where the landlord employs a tenant in each period but the tenant employed in period 1 has a positive probability of leaving the farm in period 2. This could be for various reasons like migrating to the urban areas in search of a better job and so on. Then, even if the discount factors of the landlord and the tenant are equal (say δ), share tenancy will exist in period 1. To see this, replace δ_T by $(1-p)\delta$, where p will then be the probability with which the tenant hired in period 1 will not be available in period 2. Thus, in a less developed economy with high rates of rural–urban migration, share tenancy may turn out to be a dominant tenurial institution.

Remark 2. The discount factors are behavioral parameters which determine the pattern of intertemporal preferences of the landlord and the tenant. There is no reason for one discount factor to exceed another. One may argue that a relatively "poor" tenant usually values consumption more in the present than in the future, as opposed to a landlord who is typically wealthy. Therefore, it may be reasonable to assume that $\delta_T < \delta_L$. As a consequence, in very poor agrarian economies with few but wealthy landlords, share tenancy is more likely to prevail. On the other hand, δ_T may as well be greater than δ_L. For example, the tenant may foresee the marriage of his daughter in period 2. If he faces the problems of dowry along with ceremonial expenses, he may have a greater valuation of the future than that of the landlord. In such cases, the landlord will prefer a fixed-rent contract in period 1 to a share contract. In Section 4, we will see that whenever $\delta_T > \delta_L$, a wage contract may also become a dominant institution.

Remark 3. One may also view the role of share tenancy in our framework from a slightly different perspective. Suppose (α_1^*, β_1^*) and $(1, \beta_2^*)$ are the equilibrium contracts obtained when $\delta_T < \delta_L$. Now consider the following pair of fixed-rent contracts $(1, \beta_1^{**})$ and $(1, \beta_2^*)$ with

$$\beta_1^{**} = (1-\alpha_1^*) s_1 f\big(L_1^*(\alpha_1^*)\big) + \beta_1^* .$$

This contract can be thought of as an "output-rental" contract where the landlord asks the tenant to deliver pre-specified levels of output over the two periods equal to the landlord's profits under the contracts (α_1^*, β_1^*) and $(1, \beta_2^*)$, respectively.[8] Clearly, if the landlord can enforce this contract over the two periods, then share tenancy loses its bite in our model. However, in the absence of perfect *enforceability of a long term relationship*[9] between the two players, an output-rental contract is not viable. Under an output-rental contract, a tenant has an incentive to over-produce in period 1 and then defect after period 1 is over, since he will not be able to meet his obligation in period 2 and earn his reservation income.[10] Such a contract is more difficult to enforce than a share contract which eliminates the incentives for such a defection. The landlord, in order to sustain a long-term relationship with the tenant under an output-rental contract, must impose some punishment if the tenant defects. Such punishments may entail the landlord spreading the news among other existing landlords of the tenant being less trustworthy. Clearly an output-rental contract will not be chosen by the landlord

[8] We thank an anonymous referee for bringing this type of contracts to our attention.

[9] Enforceability of the current contract is easier than enforcing a long term relationship.

[10] With a share contract the marginal product of labor in period 1 equals the wage rate, while under an output-rental contract $MPL_1 > w$ [see Eq. (7)]. Therefore, an output-rental contract creates incentives for deviation which are absent in a share contract.

since it involves the "punishment costs" while at the same time it does not add anything to the landlord's profits. An interesting observation following this discussion is that share contracts may play a central role in dynamic environments by mitigating the tension between defection and over-production on part of the tenant, and optimality in the eyes of the landlord.

4. Wage contracts

A typical wage contract as in Sengupta (1997) does not fit readily in the model studied above. One may define a wage contract by setting $\alpha_t = 0$ and letting $\beta_t < 0$. The problem with this kind of a contract in our model is that such a specification is not complete, as it does not directly specify the output that the tenant has to return to the landlord. The landlord has to therefore specify the output of foodgrains that he would demand at the end of the period. Since the technology is known to both parties, this in turn is analogous to the landlord specifying the amount of unskilled labor that the tenant has to hire. In the model studied by Muthoo (1998), tenants choose effort levels which affect production and in order to define a wage contract, one needs to assume that zero effort on part of the tenant produces at least the reservation utility of the landlord and the tenant. In our model, we could resort to such an assumption, except that since production is only affected by the level of employment of unskilled labor, such an assumption may not make much of a sense. Therefore, we resort to wage-input contracts.

4.1. Wage-input contracts and input-specification costs

We relate wage contracts to the following. The landlord pays the tenant a fixed wage[11] and asks for the level of output to be delivered in each of the two periods which maximizes the landlord's profits. Such a contract is a wage-output contract. Alternatively, (this is the definition of a wage contract in our paper) the landlord may go a step further and specify also the input combination that the tenant should employ. Such a contract is a *wage-input* contract. We view the tenant in a wage contract as a step in the hierarchy above the unskilled workers. He is responsible for hiring workers, capital and whatever else is needed for production and

[11] We rule out certain types of contingent contracts, like wage schedule contracts, because in our environment with perfect monitoring, no asymmetric information and no uncertainty, they do not enhance efficiency (over the tenurial contracts) and they are costly to implement. This cost comes from the need to tie the wage to any possible contingency. Therefore, we chose to focus on "fixed-wage" contracts as they are relatively simple. In any case, it is not our aim to compare the overall profitability of wage-contingent contracts with fixed-wage contracts.

supervision over the unskilled workers, while the landlord is responsible for paying all input costs. In exchange, the tenant receives a fixed payment (like a manager drawing salaries).

There are at least three basic problems with the wage-output contracts.

(A) A mere output specification may lead to over employment in the following sense. Suppose the landlord asks the tenant to deliver 10 bushels of foodgrains in both periods. Without any further instructions, the tenant may not minimize costs of production. Assume that the tenant needs two unskilled workers to meet the landlord's demand. Under the reasonable assumption of free disposal (produce and burn), the tenant, to gain popularity in the village and/or to help family members and friends, or even sometimes under the pressure of labor unions, may find it beneficial to over-employ.

(B) There may also be some fixed or sunk costs which do not affect the level of production but influence the landlords profits. A wage-tenant, operating under a wage-output contract, may not have enough incentives to minimize such costs. Also, there may be some inputs which are fixed but their costs may depend on the effort that the tenant exerts when he negotiates the deal with the supplier of these inputs. For example, suppose that for efficient production the tenant has to hire some unskilled laborers as well as some fixed capital (say one tractor). The tractor may be leased or purchased, but the tenant may not have enough incentives to negotiate the best deal (lowest price).

(C) Usually, within one cropping period, there exist a lean and a peak season. In a lean season, the farm needs few laborers who perform various tasks in order to increase the productivity of the land in the peak (harvest) season where more laborers are hired. The overall cost of labor within one cropping period depends critically on the decision about how to allocate and pay the workers in the lean and the peak seasons, and raises the issue of optimally choosing between permanent and casual labor. A simple output specification clearly does not take care of this issue.

There are at least two solutions to the above mentioned problems.

(1) The landlord may specify exactly the input mix (such as in a wage-input contract) and give clear instructions to the tenant about how to organize farming. That completes the contract and since there are no enforceability issues in our framework (at least in any given period) the tenant operates efficiently. But this solution involves some specification costs, no matter how small.

(2) The other solution is to give the tenant a certain percentage of the profits, so that the tenant has an incentive to operate efficiently. But then such contracts are essentially the share or the fixed rent contracts.

The message from the above discussion is that in any "wage-like" contract the landlord has to get more involved in farm management one way or another, as opposed to the share or fixed-rent contracts where the tenant will do the "right things" without much supervision. Such an involvement is costly, no matter how small. Therefore, let Θ denote any work, supervision or specification that the landlord undertakes while offering a wage contract and let $\varepsilon(\Theta)$ be the cost associated with Θ. Notice that tenurial contracts are costless in this sense.

Formally, a wage-input contract offered over the two periods is a tuple

$$\left((\gamma_1,\tilde{L}_1),(\gamma_2,\tilde{L}_2);\Theta\right),$$

which promises the tenant a wage equal to γ_t in period $t = 1, 2$, specifies the levels of employment of unskilled labor $\tilde{L}_1$ and $\tilde{L}_2$ in periods 1 and 2, respectively, and is associated with any form of supervision or specification given by Θ, such that the landlord receives $s_t f(\tilde{L}_t) - \overline{w}\tilde{L}_t - \gamma_t$ in period t and over the two periods incurs a specification cost equal to $\varepsilon(\Theta)$.

4.2. Optimality of wage-input contracts

Since the reservation utility of the tenant in each period is equal to r, the landlord sets $\gamma_t = r$ for $t = 1, 2$. Then, $(\tilde{L}_1, \tilde{L}_2)$ is the solution to the problem

$$\max_{L_1,L_2}\left[s_1 f(L_1) - \overline{w}L_1 - r\right] + \delta_L\left[s_2(L_1) f(L_2) - \overline{w}L_2 - r\right] - \varepsilon(\Theta). \quad (17)$$

The following lemma states that if the input-specification cost is zero, then for any pair of tenurial contracts studied in Section 3, there exists a pair of wage-input contracts that yields the same profit to the landlord. The proof is straightforward and is therefore not provided.

Lemma 3. *Let $((\alpha_1, \beta_1), (\alpha_2, \beta_2))$ be any arbitrary pair of tenurial contracts, and let $\Pi(\alpha_t, \beta_t)$ be the discounted sum of profits of the landlord over the two periods when he employs a tenant by offering $((\alpha_1, \beta_1), (\alpha_2, \beta_2))$. Then, there exists a pair of wage-input contracts $((\gamma_1, \tilde{L}_1), (\gamma_2, \tilde{L}_2), \varepsilon(\Theta)$ such that if $\varepsilon(\Theta) = 0$, we have*

$$\Pi\left(\gamma_t,\tilde{L}_t\right) \geq \Pi(\alpha_t,\beta_t),$$

where $\Pi(\gamma_t, \tilde{L}_t)$ is the discounted sum of profits of the landlord over the two periods when he employs a tenant by offering $((\gamma_1, \tilde{L}_1),(\gamma_2, \tilde{L}_2); \Theta)$.

The idea behind the above lemma can be easily seen as follows. We have seen that the problem of choosing equilibrium tenurial contracts of the form $((\alpha_1, \beta_1), (\alpha_2, \beta_2))$ reduces to choosing (α_1, α_2) which in turn uniquely determines the levels of unskilled laborers in the two periods. Thus, in case of wage-input contracts, the landlord can always directly specify these choices of the tenant. In this sense, whatever profit the landlord can achieve with tenurial contracts, he can as well achieve with wage-input contracts under the assumption that there are no input specification costs.

The following theorem shows that whenever the discount factor of the tenant is less than or equal to that of the landlord, even an arbitrarily small input specification cost makes the landlord prefer tenurial contracts. However, if the tenant discounts the future a lot more that what the landlord does, and the input specification costs are small, wage-input contracts tend to become more popular.

Theorem 4. *(i) For any $\varepsilon(\Theta) > 0$, the landlord offers a fixed-rent contract in period 2.*
In period 1, the following holds:
*(ii) If $\delta_T \leq \delta_L$, then for any $\varepsilon(\Theta) > 0$, the landlord prefers a **tenurial** contract to a wage-input contract. Consequently, he offers a **share** contract if $\delta_T < \delta_L$, and a fixed-rent contract if $\delta_T = \delta_L$;*
*(iii) if $\delta_T > \delta_L$, and $\varepsilon(\Theta)$ is sufficiently small, the landlord prefers a **wage-input** contract to tenurial contracts. Conversely, if $\varepsilon(\Theta)$ is sufficiently large, the landlord prefers **tenurial** contracts to wage-input contracts. Consequently, he offers a fixed-rent contract.*

Proof. The first-order condition of the landlord's maximization problem under a wage-input contract, as in Eq. (17), with respect to L_2 is

$$s_2(L_1)\frac{\partial f(L_2)}{\partial L_2} - \overline{w} = 0. \tag{18}$$

(i) Using Eqs. (6) and (18), it is easy to see that a fixed-rent contract in period 2 yields the same profit to the landlord as the optimal period 2 wage-input contract. Therefore, with $\varepsilon(\Theta) > 0$, the landlord offers a fixed-rent contract in period 2.
To prove the rest of the theorem, we will use $\alpha_2 = 1$ whenever needed, and proceed as follows. Let $\tilde{L}_2$ be the solution of Eq. (18). Notice that $\tilde{L}_2$ depends on L_1. Using Eq. (18), the first-order condition of the landlord's problem as in Eq. (17) with respect to L_1 is

$$s_1\frac{\partial f(L_1)}{\partial L_1} + \delta_L\frac{\partial s_2(L_1)}{\partial L_1} f\big(\tilde{L}_2\big) - \overline{w} = 0. \tag{19}$$

Let $\tilde{L}_1$ be the solution of Eq. (19). Thus, $(\tilde{L}_1, \tilde{L}_2)$ will constitute the input specifications in a wage-input contract.
Let L_1^* and L_2^* be the optimal choice of employment of unskilled labor by the tenant under any type of a tenurial contract of the form $((\alpha_1, \beta_1), (\alpha_2, \beta_2))$. These are solutions to Eqs. (6) and (7). Let us recall Eq. (7) which will be used in the proof. It is given by

$$\alpha_1 s_1\frac{\partial f(L_1)}{\partial L_1} + \delta_T\frac{\partial s_2(L_1)}{\partial L_1} f(L_2^*) - \overline{w} = 0.$$

Consider the (α_1, L_1) plane. Since $\tilde{L}_1$ does not involve α_1, it is a horizontal line with y-intercept equal to $\tilde{L}_1$. Notice that at $\alpha_1 = 0$, we have $L_1^* = 0$. This is easy to see from the profit function of the tenant since we assume $\overline{w} > 0$. Thus, the function $L1^{*}(\alpha_1)$ starts at zero when $\alpha_1 = 0$. Since $\tilde{L}_1 > 0$, $L1^{*}(\alpha_1)$ starts below the constant function $\tilde{L}_1(\alpha_1) = \tilde{L}_1$. At $\alpha_1 = 1$, Eqs. (7) and (19) differ only in the discount factors of the tenant and the landlord. Taking total differential of Eq. (7) with respect to L_1^* and δ_T at $\alpha_1 = 1$, and setting $\alpha_2 = 1$, we have

$$\frac{\mathrm{d}L_1^*}{\mathrm{d}\delta_T} = -\frac{f(L_2^*)\partial s_2(L_1^*)/\partial L_1^*}{B}, \tag{20}$$

where,

$$B = s_1\frac{\partial^2 f(L_1)}{\partial L_1^2} + \delta_T\frac{\partial^2 s_2(L_1)}{\partial L_1^2} f(L_2^*) - \delta_T\frac{\left(\dfrac{\partial s_2(L_1)}{\partial L_1}\dfrac{\partial f(L_2^*)}{\partial L_2^*}\right)^2}{s_2(L_1)\dfrac{\partial^2 f(L_2^*)}{\partial L_2^*}}$$

which is negative since the expression A as defined in Eq. (A4) (in Appendix A) is negative for any $\alpha_1 \geq 0$ and $\alpha_2 > 0$. Thus, from Eq. (20), we have $dL_1^*/d\delta_T < 0$.
(ii) Suppose $\delta_T < \delta_L$. Then, we have $L_1^*(\alpha_1 = 1) > \tilde{L}_1$. By continuity of L_1^* in α_1, there must exist an α_1^* strictly greater than zero and strictly less than 1 such that $L_1^*(\alpha_1^*) = \tilde{L}_1$. This would imply that there exists a share contract for period 1 which ensures the same profit to the landlord as his equilibrium wage-input contract without any input-specification cost. Thus, for any $\varepsilon(\Theta) > 0$, the landlord strictly prefers this share contract to his best wage-input contract.
Now suppose $\delta_T = \delta_L$. In that case, we have $L_1^*(\alpha_1 = 1) = \tilde{L}_1$. This implies that the fixed-rent contract for period 1 ensures the same profit to the landlord as his equilibrium wage-input contract without any input-specification cost. Thus, for any $\varepsilon(\Theta) > 0$, the landlord strictly prefers this fixed-rent contract to his best wage-input contract.
(iii) Since $\delta_T > \delta_L$, we have $L_1^*(\alpha_1 = 1) < \tilde{L}_1$. Therefore, if there exists an α_1^* such that $L_1^*(\alpha_1^*) = \tilde{L}_1$, it must be that $\alpha_1^* > 1$. This follows from the proof of part (iii) of Proposition 2. Assume for the moment that contracts with a share greater than one are feasible. We first need to show the existence of such an α_1^*. Replacing the second term of Eq. (7) by $-\delta_T MK$, where $K = f(L_2^*(L_1 = 0, \alpha_2 = 1))$, we get

$$\alpha_1 s_1\frac{\partial f(L_1)}{\partial L_1} - \delta_T MK - \overline{w} = 0. \tag{7$'$}$$

Let $\hat{L}_1$ be the solution of Eq. (7′) which is given by

$$\hat{L}_1 = f'^{-1}\left(\frac{\overline{w} + \delta_T MK}{\alpha_1 s_1}\right),$$

where $f'^{-1}(\cdot)$ is the inverse of the first derivative of the function $f(\cdot)$. Clearly, for any $\alpha_1 \geq 0$, we have $\hat{L}_1 \leq L_1^*(\alpha_1)$. Now, consider the problem of the landlord in case of wage-input contracts. The highest possible value of $\tilde{L}_1$ is obtained when $\delta_L = 0$, and is given by

$$\tilde{L}_1 = f'^{-1}\left(\frac{\overline{w}}{s_1}\right).$$

Note that $f'^{-1}(\cdot)$ is a strictly decreasing function. Thus, $\tilde{L}_1 = \hat{L}_1$ if and only if

$$\alpha_1 = \overline{\alpha} \equiv 1 + \frac{\delta_T MK}{\overline{w}}.$$

Clearly, $1 < \overline{\alpha} < \infty$, and is by construction an upper bound for α_1^* such that the input specification of the landlord in period 1 coincides with the employment decision of the tenant at α_1^*. This therefore ensures the existence of α_1^* as required. However, since we restrict α_1^* to be between 0 and 1, and in Proposition 2 we show that whenever $\delta_T > \delta_L$, the constrained best tenurial contract available to the landlord is a fixed-rent one, it is clear that tenurial contracts are not optimal if $\varepsilon(\Theta)$ is sufficiently small. Only when $\varepsilon(\Theta)$ is large, the landlord is better off offering the fixed-rent contract. ■

The intuition behind the above result goes as follows. Whenever the discount factor of the tenant is less than or equal to that of the landlord, a tenurial contract achieves whatever profit a wage-input contract can, without input specification costs. Therefore, with even a small input specification cost, the landlord finds it optimal to go for tenurial contracts. However, if the discount factor of the tenant exceeds that of the landlord, since the first-best tenurial contracts with shares greater than one are not feasible, the constrained tenurial contracts (which are then typically in the form of fixed-rents) lead to inefficiencies and hurt the landlord's profits. Therefore, if the input specification costs of offering wage-input contracts are small, the landlord is better off to incur such minimal costs and hire the tenant by offering a wage-input contract.

5. Conclusion

We employ a simple two-period model without uncertainty, asymmetric information, or strategic complementarity to study different types of contracts that are written between landlords and tenants in rural economies. We show that in the terminal period, the tenant and the landlord agree on a fixed-rent contract. In the first period, if the landlord happens to be more patient than the tenant, a share contract prevails in equilibrium; while if the landlord and the tenant discount the future equally, a fixed-rent contract is signed. On the other hand, if the intertemporal discount factor of the tenant is higher than that of the landlord, we show that wage-input contracts may be offered if input specification costs associated with such contracts are not too high.

Although we do not provide the formal proofs, it seems intuitively clear that as long as our assumptions hold, the results obtained apply at least to any finite number of periods. It is quite easy to see that the terminal period contract will necessarily be a fixed-rent one. However, as long as there is a future, differences in intertemporal preferences will, in spirit, drive our results. What will then be the exact sequence of the types of contracts over the planning horizon may be an interesting and challenging question that remains unanswered. Mention must also be made of contracts, which are not offered each period, but are rather signed for a given length of periods. In such contracts, the landlord announces a triple (α, β, T), where α will be the fraction of foodgrain output that the tenant retains in each of the T periods, and pays an annual rent equal to β. In such a case, we may sustain share tenancy at each period if $\delta_T < \delta_L$. There are some interesting issues that may be addressed in this regard, one such being the optimality of the length of contracts. Finally, our model may have some interesting implications for the modelling framework employed by Ray (1999). For example, Ray shows that in an environment where tenants compete for labor, share tenancy prevails in equilibrium under certain conditions as mentioned in the introduction. However, this may not be true in a multi-period environment where tenants and landlords perceive the future differently. Surely, the terminal period will sustain share tenancy in that case, but other forms of contracts may be sustained in the non-terminal periods depending upon the degree of competition for labor and the pattern of intertemporal preferences.

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Appendix A. Proof of Lemma 1

Consider the first-order condition of the tenant as in Eq. (7). Totally differentiating with respect to L_1 and α_1, we get

$$\frac{\partial L_1}{\partial \alpha_1} = -\frac{s_1 \dfrac{\partial f(L_1)}{\partial L_1}}{\alpha_1 s_1 \dfrac{\partial^2 f(L_1)}{\partial L_1^2} + \delta_T \alpha_2 \dfrac{\partial^2 s_2(L_1)}{\partial L_1^2} f(L_2^*) + \delta_T \alpha_2 \left(\dfrac{\partial s_2(L_1)}{\partial L_1} \right)^2 \dfrac{\partial f(L_2^*)}{\partial L_2^*} \dfrac{\partial L_2^*}{\partial s_2}}. \tag{A1}$$

Taking total differential of Eq. (6) with respect to L_2^* and s_2, we have

$$\frac{\partial L_2^*}{\partial s_2} = -\frac{\dfrac{\partial f(L_2^*)}{\partial L_2^*}}{s_2 \dfrac{\partial^2 f(L_2^*)}{\partial L_2^{*2}}}. \tag{A2}$$

Substituting Eq. (A2) in Eq. (A1), we get

$$\frac{\partial L_1}{\partial \alpha_1} = -\frac{s_1 \dfrac{\partial f(L_1)}{\partial L_1}}{A}, \tag{A3}$$

where

$$A = \alpha_1 s_1 \frac{\partial^2 f(L_1)}{\partial_1^2} + \delta_T \alpha_2 \frac{\partial^2 s_2(L_1)}{\partial L_1^2} f(L_2^*) - \delta_T \alpha_2 \frac{\left(\dfrac{\partial s_2(L_1)}{\partial L_1} \dfrac{\partial f(L_2^*)}{\partial L_2^*} \right)^2}{s_2(L_1) \dfrac{\partial^2 f(L_2^*)}{\partial L_2^{*2}}}. \tag{A4}$$

Since π_1 is strictly concave in L_1 and L_2 for any $\alpha_1 \geq 0$ and $\alpha_2 > 0$, the Hessian matrix is negative definite. Therefore, we have

$$\begin{vmatrix} \partial^2 \pi_1 / \partial L_1^2 & \partial^2 \pi_1 / \partial L_1 \partial L_2 \\ \partial^2 \pi_1 / \partial L_2 \partial L_1 & \partial^2 \pi_1 / \partial L_2^2 \end{vmatrix} > 0, \tag{A5}$$

where

$$\partial^2 \pi_1 / \partial L_1^2 = \alpha_1 s_1 \frac{\partial^2 f(L_1)}{\partial L_1^2} + \delta_T \alpha_2 \frac{\partial^2 s_2(L_1)}{\partial L_1^2} f(L_2),$$

$$\partial^2 \pi_1 / \partial L_1 \partial L_2 = \partial^2 \pi_1 / \partial L_2 \partial L_1 = \delta_T \alpha_2 \frac{\partial s_2(L_1)}{\partial L_1} \frac{\partial f(L_2)}{\partial L_2}, \text{ and}$$

$$\partial^2 \pi_1 / \partial L_2^2 = \delta_T \alpha_2 s_2(L_1) \frac{\partial^2 f(L_2)}{\partial L_2^2}$$

Using the above expressions, Eq. (A5) implies that

$$\delta_T \alpha_2 \frac{\partial s_2(L_1)}{\partial L_1} \frac{\partial f(L_2)}{\partial L_2} \left[\alpha_1 s_1 \frac{\partial^2 f(L_1)}{\partial L_1^2} + \delta_T \alpha_2 \frac{\partial^2 s_2(L_1)}{\partial L_1^2} f(L_2) \right] - \left(\delta_T \alpha_2 s_2(L_1) \frac{\partial^2 f(L_2)}{\partial L_2^2} \right)^2 > 0,$$

from which we get

$$\alpha_1 s_1 \frac{\partial^2 f(L_1)}{\partial_1^2} + \delta_T \alpha_2 \frac{\partial^2 s_2(L_1)}{\partial L_1^2} f(L_2) < \delta_T \alpha_2 \frac{\left(\frac{\partial s_2(L_1)}{\partial L_1} \frac{\partial f(L_2)}{\partial L_2} \right)^2}{s_2(L_1) \frac{\partial^2 f(L_2)}{\partial L_2^2}}. \tag{A6}$$

Since Eq. (A6) implies that the expression A as defined in Eq. (A4) is less than zero, we have $\partial L_1^* / \partial \alpha_1 > 0$.

Similarly, totally differentiating Eq. (6) with respect to α_2 and L_2^*, we get

$$\frac{\partial L_2^*}{\partial \alpha_2} = -\frac{\partial f(L_2^*)/\partial L_2^*}{\alpha_2 \partial^2 f(L_2^*)/\partial L_2^{*\,2}},$$

which from our assumptions is clearly positive.

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