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Semicollusion vs. Full Collusion: The Role of Demand Uncertainty and Product Substitutability

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We examine the profitability of two different cartel organizational forms: *full collusion,* under which firms collude on both price and quality, and *semicollusion,* under which firms collude on price only. We show that, in the presence of demand uncertainty that cannot be contracted upon in the cartel agreement, firms may be better off limiting their collusive agreement to price only. However, a positive relationship between demand uncertainty and the relative profitability of semicollusion exists *only* for low levels of demand substitutability. The converse is true for high levels of demand substitutability. Therefore, if demand substitutability is sufficiently high, *no* level of demand uncertainty will make semicollusion the optimal organizational form. In contrast, semicollusion is guaranteed to be optimal for a sufficiently low level of demand substitutability. The market structure described is motivated by and closely parallels that of shipping cartels.

Keywords: semicollusion, market uncertainty, cartels, shipping conferences.

JEL classification: L12, D43.

1 Introduction

In an industry in which firms compete only in prices (or quantities), industry profits will increase if firms collude rather than engage in competition. One would also expect this to be true if the firms have more than one strategic variable at their disposal. If, for instance, firms could choose product quality and price, one would expect collusion with respect to both strategic variables to yield higher profits than collusion with respect to only one of them.

It is often observed, however, that cartels choose not to collude on all aspects of their business *even when* this is possible and legal. Dick (1992) examines a number of semicollusive arrangements of the Japanese export cartels and how these affect their performance. He provides numerous instances of when the firms voluntarily restrict the scope of their collusive agreements. Sicotte (1996) shows that during the era when shipping cartels could operate unencumbered by government restrictions, approximately half of the shipping cartels fixed only price, forgoing restrictions on the quality of service.[1] Other examples of legal cartels and their practices can be found in Nakazawa and Weiss (1989), and Audretsch (1989).

Deltas, Serfes, and Sicotte (1999) investigate the factors that led to the choice of a semicollusive versus a fully collusive agreement in American shipping cartels in the pre-WWI era. They find that one of these factors is the length of the route with shorter routes more likely to be semicollusive than longer routes. One of the characteristics of cartels operating on short routes is their vulnerability to opportunistic entry by the so-called "tramps." These are ships, typically smaller and slower, that would transport cargo when demand conditions make it profitable and would operate without any regular schedule. Tramps were less frequently observed in longer routes because operating in these routes required bigger and faster ships and a higher level of commitment. A consequence of the stronger competition by tramps on short routes is that any increase in service quality of cartel members operating in those routes, such as more frequent schedules and faster service, would have different effects compared to long routes. Suppose cartel members were free to noncooperatively choose their quality of service. Then, a unilateral increase in service quality by a single member of a cartel would have two main effects: (i) it would divert some traffic from other cartel members, and (ii) it would increase total traffic for the cartel members by diverting traffic from tramps. For short routes the second effect would be relatively bigger than the first compared to longer routes. As we show below, this differential across markets effect is a determining factor in the optimal organization of the cartels.

In this paper, we propose a stylized model that incorporates four key features of the shipping cartels. First, competition between firms is not limited to price only, but also includes other factors, such as service

1 Restrictions on service were clearly feasible since the remaining cartels did, in fact, impose them.

quality. Second, the effects of a deviation from a cooperative equilibrium differ across markets, as illustrated above. Third, market demand can not be predicted with certainty at the time a cartel agreement is concluded. Fourth, a collusive agreement cannot be adjusted instantaneously and cannot be contingent on the realization of demand conditions.[2] Of course, these four features, and the model we propose, characterize other industries as well. In our model, semicollusion arises as an equilibrium outcome in markets with a positive level of uncertainty, *but only* when the increase in a cartel member's sales arising from an increase in its service quality comes primarily from non-cartel member firms (demand expansion) rather than from other cartel members (demand diversion). However, our model does not require the existence of a competitive fringe. It only requires that an increase in a cartel member's quality not only diverts sales to that member from the rest of the cartel, but also increases aggregate quantity demanded. In the absence of non-cartel firms demand expansion is likely to dominate demand diversion if the products of each firm are not very substitutable. Therefore, markets with a relatively strong demand diversion are markets with a small competitive fringe or markets in which the products of cartel members are very substitutable.

We recognize that there are many factors that affect the form of collusion. In this paper we focus our attention on the role of demand uncertainty and product substitutability and we identify conditions which determine the organizational form of the cartel. In particular, we consider a market that consists of n symmetric firms facing a stochastic demand. The firms are interested in forming a cartel and must decide on the form of collusion before demand uncertainty is resolved.[3] The cartel is assumed to be one of two types: (i) a cartel that fixes both price and quality, or (ii) a cartel that fixes price only. We choose these two types

2 Deltas, Serfes, and Sicotte (1999), and Deltas, Sicotte, and Tomczak (2001) document that the shipping agreements were typically not contingent on demand conditions, even though demand for shipping services was highly variable.

3 We motivate this assumption by the observation that in real world applications, the state of nature is prohibitively expensive to fully describe. Therefore, the cartel agreement, like most contracts, must be incomplete, in the sense that it is contingent on a coarser partition of the information set than is available to the firms themselves. For example, a cartel agreement may specify a collusive price as a function of only GNP and the price of a substitute product, even though these two factors do not fully specify the cartel's demand function.

of collusive agreements because they correspond to the two broad classes of shipping cartels. If the cartel allows the firms to choose the quality of their product non-cooperatively, the firms can delay committing to a quality level until after the realization of the demand uncertainty.[4] We assume that an increase in a firm's product quality leads to higher demand for its product. The increase in demand consists of two components: the first expands total industry output, the second results in a diversion of sales from the other firms. This second component demand diversion, is crucial to our results.

We show that for any given level of demand uncertainty, there exists a threshold level of demand substitutability below which collusion on price only would yield higher profits than collusion on both price and quality. One might be tempted to conclude that this result is a direct consequence of demand uncertainty, with higher demand uncertainty making semicollusion relatively more profitable. However, this is not the case. In particular, it is *not* true that for any given degree of demand substitutability, there exists a level of demand variability such that semicollusion leads to higher profits than full collusion. This statement is true only for sufficiently low levels of demand substitutability. Indeed, if the demand diversion effect is sufficiently strong, the ability to respond to the demand shock will *decrease* rather than increase the expected joint profits of the semicollusive cartel (which implies that the fully collusive cartel dominates for *all* levels of demand uncertainty). Therefore, there does *not* necessarily exist a trade-off between flexibility to respond to demand shocks and coordination on both price and quality. Such a trade-off exists only for sufficiently low level of demand diversion. On the other hand, the existence of at least some demand uncertainty is crucial. In the absence of any demand uncertainty, collusion on both price and quality yields higher cartel profits than semicollusion.[5]

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4 This assumption begs the question of why the firms do not jointly choose the quality level after the uncertainty is resolved. We motivate our assumption from the fact that it is easier for a single firm to change the quality of its own product than for all firms to agree on a given level of quality.

5 An entirely different strand of the cartel literature [e.g., Green and Porter (1984), Abreu, Pearce and Stacchetti (1986)] deals with the impact of demand variability that cannot be observed by cartel members on the ability to enforce the cartel agreement. In our paper, we follow Donsimoni et al. (1986), Cramton and Palfrey (1990) and others in assuming that the cartel agreement is enforceable.

Finally, the demand and cost framework that we adopt, though quite general, has a very important and special, but desirable, characteristic: For any given level of product quality, collusion on price is optimal regardless of the degree of demand uncertainty. That is, had product quality been exogenously fixed, collusion would always be optimal. Therefore, the departure from full collusion is due to the interplay of demand uncertainty with the ability of firms to affect market demand through changes in product quality.

In related work, Fox (1994) derives the comparative statics of shipping cartels in which member firms collude in the choice of price and choose the quality of service non-cooperatively. Her work adopts a deterministic framework in which the organizational form of the cartel is exogenously given. Our work complements hers by providing one possible mechanism for the endogenous determination of the cartel agreement. There also exists literature that examines the formation of semicollusive cartels when firms have two strategic variables. Fershtman and Gandal (1994), and Brod and Shivakumar (1999) analyze collusion in a market where firms first choose the level of cost reducing R&D (or capacity) and then compete in prices. They show that semicollusion may be less profitable than no collusion at all. Finally, the trade-off between commitment and flexibility under demand and cost uncertainty in the context of oligopoly models and strategic trade policy is examined by Spencer and Brander (1992), and Wong and Chow (1997).

Our paper is organized as follows. Section 2 outlines the general modeling framework. Section 3 develops the results and comparative statics of the model. The paper ends with a few concluding remarks. Appendix A shows an example of how our demand specification can be derived from micro-fundamentals, while Appendix B contains all the proofs.

2 Modeling Framework

2.1 Preliminaries

Consider an industry that consists of n symmetric firms. The demand for a firm's output depends on the price it charges, the prices that other firms charge, the quality of the firm's product, and the average quality of the competing firms' products. The quantity demanded is also influenced by a random disturbance. The total production cost of each firm is an increasing function of the firm's output and the quality of its product. We assume that the marginal cost is constant and independent of the level of product quality.[6]

We distinguish two different cartel organizational forms: Full collusion, in which firms collude with respect to both price and quality, and semicollusion, in which firms collude with respect to price only. We assume that under a fully collusive agreement, the cartel's choice of quality and price can neither be contingent on the realization of the demand disturbance nor can it be delayed until after the resolution of this demand uncertainty. We motivate this assumption by the observation that in real world applications the state of nature involves an exhaustive enumeration of different situations. The random disturbance is a real-valued function defined on the states of nature. The value of the random disturbance cannot be easily communicated amongst cartel members or verified by a third party. Therefore, a cartel agreement is contingent on these states of nature and not on the value that the random variable takes on for any given state of nature. If describing these states of nature is very costly, the cartel agreement must be incomplete in the sense that it is contingent on a coarser partition of the information set than is available to the firms themselves.[7] This parallels the motivation of incomplete contracts. Alternatively, one can base this assumption on models with asymmetric information. For instance, Serfes and Yannelis (1999), show that in a static Cournot duopoly with differential information it is not incentive compatible for the firms to base their collusive agreement on their private information. An incentive compatible cartel agreement can be conditioned only on the coarsest information partition.

Under full collusion, the cartel commits to a price and quality before any uncertainty is resolved. Under semicollusion, the cartel commits to a price before any uncertainty is resolved, but each individual firm can choose a quality level *after* it has observed the realization of the demand uncertainty. A firm's optimal choice of quality will be a function of the collusive price. The cartel chooses the price that maximizes the expected

6 This assumption is consistent with the interpretation of the costs of higher quality as organizational costs of producing better service, the costs of providing more frequent service in, say, the airline or other transportation industries, the cost of increasing the number of outlets, and R&D costs. In this respect, our analysis is similar to that in Dorfman and Steiner (1954).

7 In our model it is contingent on the coarsest possible partition. This greatly simplifies the exposition.

total firm profit, taking into consideration each firm's optimal response to this price. It is worth pointing out that even though we assume that collusion on price is always optimal, such collusion would arise in equilibrium if firm-level demand elasticity with respect to the price of competing firms is sufficiently high. This is highlighted in Appendix A in which the parametric model we use in Sect. 3.1 is derived from micro-foundations.

2.2 The Model

We assume that both types of cartel agreements are either binding or can be enforced by punishment strategies in a repeated game, i.e., we consider discount factors sufficiently high such that no firm would choose to deviate from either type of agreement. This approach has also been adopted by a large body of literature, including Donsimoni et al. (1986), Cramton and Palfrey (1990), and Fershtman and Gandal (1994). We also assume throughout the paper that the cartel members agree on a single price and not on a price-quality schedule. Therefore, we only need to specify the demand function of each firm as a function of the common cartel price and not in terms of the entire vector of all firms' prices. More formally, the demand function that a cartel member firm, i, faces is

$$q_i = \varepsilon f(p, S_i - \delta \bar{S}_{-i}) \ , \tag{1}$$

where q_i is the quantity demanded, p is the price, S_i represents the quality that firm i offers, $\bar{S}_{-i}$ is the average quality of the other firms, $\varepsilon \geq 0$ is a random variable, the mean of which is normalized to 1 and reflects the strength of the demand, and δ is the demand substitutability parameter. This parameter can be interpreted as follows. Suppose $f(\cdot, \cdot)$ is linear in quality. Then, if a firm increases its quality, the proportion of the increase in sales that is due to diversion from sales of the other $n-1$ cartel firms would be equal to δ. The proportion of the increase in sales that results from an overall expansion of the market (or diversion of sales from non-member firms) is $1-\delta$. Therefore, δ is low if the products of the cartel members are not very substitutable, aggregate demand is sensitive to quality, and/or there is a large competitive fringe. For demand functions that are not linear in quality the above interpretation is valid for small changes in quality around a symmetric equilibrium.[8] A demand function of this form could arise from a market where firms are both horizontally and vertically differentiated. In such a market, the higher quality firm does not get all consumers.[9]

The cost function for firm i is given by

$$C_i(S_i, q_i) = h_i(S_i) + cq_i \ . \tag{2}$$

Each member of the semicollusive cartel chooses the level of quality so as to maximize profits

$$\pi_i = (p - c)q_i - h_i(S_i) \tag{3}$$

after the uncertainty has been resolved. The solution to this problem, which is denoted by $S^*_{i,semi}$, is a function of the cartel price and the level of demand.

The semicollusive cartel chooses the price so as to maximize expected total firm profits

$$E\Pi_{semi} = E\left\{\sum_{i=1}^{n}\left[\pi_i(S^*_{i,semi})\right]\right\} \tag{4}$$

taking into account the reaction of the individual firms to the cartel price, as given by $S^*_{i,semi}$.

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8 If products are substitutes, δ cannot be negative since that would mean that an increase in the quality of a firm's product would increase the quantity demanded for each of the other firms. (We briefly discuss the possibility of product complementarities, i.e., the case of $\delta < 0$, later in the paper.) The value of δ cannot be greater than 1 either, since that would mean that a simultaneous increase in the quality of all firms' products would result in a reduction of demand for each one of them (which would imply a reduction of the overall demand).

9 In Appendix A, we derive a set of demand functions of this form from first principles. Further, we have obtained similar results (through numerical examples) when firm demand is derived using the standard logit model of discrete consumer choice. Consumer willingness to pay for quality is the same. However, consumer willingness to pay for a product also includes an idiosyncratic component, ensuring that firm demand curves are everywhere differentiable with respect to both price and quality (see McFadden, 1973). The results are the same as those obtained using the reduced form specification shown above. However, generalizing the preference structure to obtain general results would complicate the analysis without any additional insight.

The fully collusive cartel chooses price and quality simultaneously in order to maximize expected total firm profits

$$E\Pi_{full} = E\left\{ \sum_{i=1}^{n} [(p - c)q_i - h_i(S_i)] \right\} . \tag{5}$$

We note that the assumption of multiplicative demand shock and constant marginal cost has the following implication. If quality were exogenously fixed to any level, then expected cartel profits would have been independent of demand variability for any set of prices. As a consequence, collusion on price would have been always profitable. Therefore, any departure from full collusion in our model will be due to the interplay of the demand uncertainty with the ability to affect market demand by changing product quality.

In the next section, we determine the optimal organizational form of the cartel as a function of the degree of demand uncertainty and product substitutability.

3 Analysis

We begin the analysis by an illustrative example which highlights the effects arising from the interaction of demand uncertainty and product substitutability. We then derive some general results.

3.1 An Example

Consider an industry with n symmetric firms. Let the demand function that each firm faces be

$$q_i = \varepsilon \left[\frac{\alpha}{n} - \frac{\beta}{n} p + S_i - \delta \bar{S}_{-i} \right] ,$$

where q_i is the quantity demanded, p is the cartel price, S_i is the quality that firm i offers, $\bar{S}_{-i}$ is the average quality that the other firms offer, and ε is a shock that follows a distribution with mean 1 and variance σ^2. Let the cost function of each firm be

$$C_i = S_i^2 + cq_i .$$

Denote the second moment of ε by $E\varepsilon^2$. We make the following assumptions:

(i) $\alpha > \beta c$.
(ii) $4\beta - E\varepsilon^2 n(1-2\delta) > 0$.
(iii) $4\beta - n(1-\delta)^2 > 0$.

Assumptions (i) and (ii) state that the market demand is sufficiently strong so that the semicollusive cartel makes positive profits, and the market demand is sufficiently responsive to a price change so that these profits are finite. Similarly, Assumptions (i) and (iii) guarantee that the profits of the fully collusive cartel are positive and finite.

After observing the realization of the random variable ε, each firm in the semicollusive cartel chooses quality level S_i in order to maximize its profits,

$$\begin{aligned}\Pi_i &= (p-c)q_i - S_i^2 \\ &= (p-c)\varepsilon\left[\frac{\alpha}{n} - \frac{\beta}{n}p + S_i - \delta\bar{S}_{-i}\right] - S_i^2 \ .\end{aligned}$$

The optimal choice of quality, $S^*_{i,semi}$, will be a function of the price p, the parameter δ, and the shock ε. The semicollusive cartel chooses price in order to maximize the expected sum of profits

$$E\Pi_{semi} = E[(p-c)\varepsilon(\alpha - \beta p + (1-\delta)nS^*_{i,semi}(p,\varepsilon,\delta)) - nS^{*2}_{i,semi}(p,\varepsilon,\delta)] \ ,$$

taking into consideration each firm's optimal response to this price.

The fully collusive cartel chooses both price and quality in order to maximize expected total firm profits

$$E\Pi_{full} = E[(p-c)\varepsilon(\alpha - \beta p + (1-\delta)nS) - nS^2] \ ,$$

where S is the quality level of each firm. We first derive the optimal policy of the semicollusive cartel and the profits under this organizational form.

Lemma 1: The optimal price of the semicollusive cartel is given by

$$p^*_{semi} = \frac{2(\alpha + \beta c) - E\varepsilon^2 nc(1-2\delta)}{4\beta - E\varepsilon^2 n(1-2\delta)}$$

and the maximal profits by

$$E\Pi^*_{semi} = \frac{(\alpha - \beta c)^2}{4\beta - E\varepsilon^2 n(1-2\delta)} \quad . \tag{6}$$

The proof of this and all other results is contained in Appendix B. The effect of increased demand uncertainty on the cartel price is ambiguous and depends on the degree of demand diversion. For $\delta > 1/2$, p^*_{semi} and $E\Pi^*_{semi}$ are decreasing in the second moment $(E\varepsilon^2)$ of ε. For $\delta < 1/2$ they are both increasing in the second moment of ε.

We next turn to the derivation of the optimal policy of the fully collusive cartel and the associated profits.

Lemma 2: The optimal price and quality of the fully collusive cartel are given by

$$p^*_{full} = \frac{2(\alpha + \beta c) - cn(1-\delta)^2}{4\beta - n(1-\delta)^2}, \quad S^*_{full} = \frac{(\alpha - \beta c)(1-\delta)}{4\beta - n(1-\delta)^2}$$

and the maximal profits by

$$E\Pi^*_{full} = \frac{(\alpha - \beta c)^2}{4\beta - n(1-\delta)^2} \quad . \tag{7}$$

Notice that p^*_{full}, S^*_{full} and $E\Pi^*_{full}$ do not depend on $E\varepsilon^2$.

Figure 1 depicts the optimal semicollusive cartel's expected profits as a function of the variance for $\delta = .3, n = 2, \alpha = \beta = 1$ and $c = 0$ and Fig. 2 depicts the same relationship for $\delta = .7, n = 2, \alpha = \beta = 1$ and $c = 0$.

We are now ready to state the main proposition of this section.

Proposition 1: For any σ^2, there exists a level of demand diversion parameter $\hat{\delta}$ such that for any $\delta < \hat{\delta} = -\sigma^2 + \sqrt{\sigma^2(\sigma^2+1)}$ the semicollusive cartel yields higher profits than the fully collusive cartel.

Figure 3 plots a line depicting the locus of points where the profits of the two cartels are equal. Above and to the left of this line the fully collusive cartel is more profitable. Below and to the right of the line the semicollusive cartel is more profitable. Observe from the graph that for any level of variance, there exists a level of demand diversion such that the semicollusive cartel yields higher profits. Notice that it is *not* true that for any δ there exists some critical value of demand uncertainty $\hat{\sigma}^2$, such

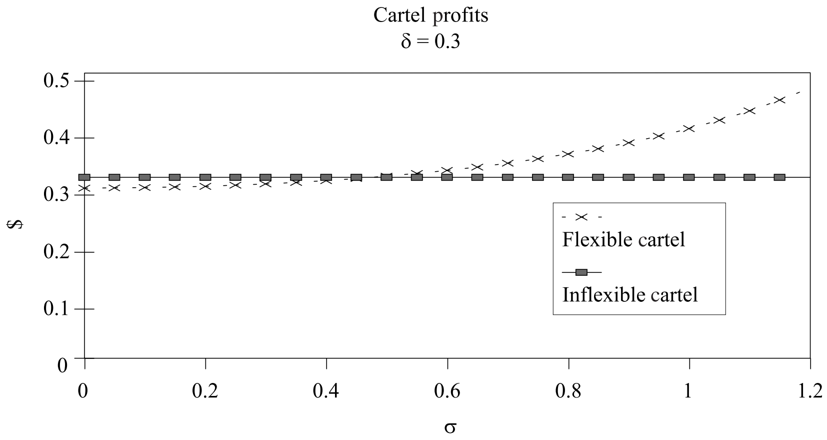

Fig. 1

that for $\sigma^2 > \hat{\sigma}^2$ the semicollusive cartel yields higher profits. For $\delta > 1/2$, collusion on price and quality is always better than collusion on price only. In fact, a stronger statement follows directly from Eq. (6):

Corollary 1: If $\delta < 1/2$, the maximum expected profits for the semicollusive cartel is an increasing function of σ^2. If $\delta > 1/2$, the reverse is true. If $\delta = 1/2$, the profits of semicollusive cartel are invariant to σ^2.

This Corollary states that for $\delta > 1/2$ the fully collusive cartel is not only more profitable than the semicollusive one, but the difference in the profits between the two organizational forms is *increasing* in the degree of demand uncertainty.[10]

To gain additional insight, we decompose the difference between the profits of the semicollusive cartel and the profits of the fully collusive cartel into a component that is due to the flexibility gained from the ability to adjust quality *ex post* and a component due to the loss of coordination from setting the quality cooperatively. Following the

10 Similar results can be obtained in a model in which product qualities are complements (i.e., $\delta < 0$). In this case, semicollusion is preferable if $0 > \delta > -\sigma^2 - \sqrt{\sigma^2(1+\sigma^2)}$, i.e., if the complementarity between the products is not too strong. Therefore, one can view the results more generally as arising from product differentiation rather than product substitutability. However, we focus on demand substituability as the theoretical model is motivated by markets in which products are likely to be substitutes rather than complements.

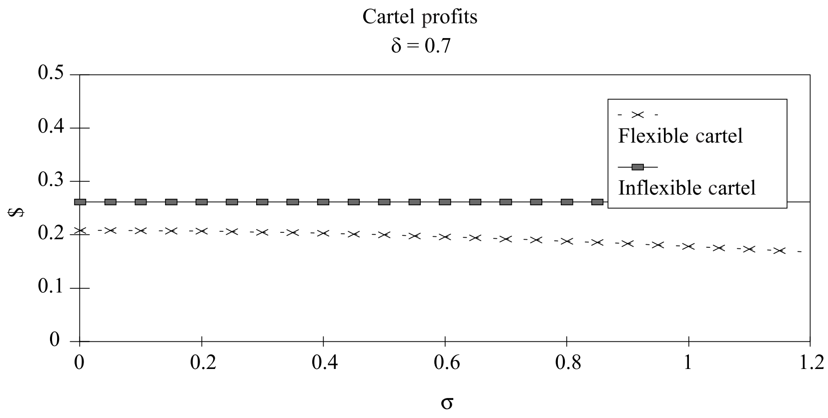

Fig. 2

terminology and rationale in Wong and Chow (1997), we refer to the first component of the difference as "option value" and to the second as "commitment value." To decompose the profit difference into these two components, we introduce the constrained "first best" cartel (from the point of view of the firms) in which the firms are able to collude on quality after the realization of the demand shock (but still have to collude on price before the realization of the demand shock). Note that our paper assumes that such a cartel is not feasible. Denote the expected profits of this first best cartel by $E\Pi_{FB}$. We can then write

$$E\Pi_{semi} - E\Pi_{full} = (E\Pi_{FB} - E\Pi_{full}) - (E\Pi_{FB} - E\Pi_{semi}) \ .$$

One can show, using similar steps to those in the above propositions, that the expected profits of the "first best" cartel are given by:

$$E\Pi_{FB} = \frac{(\alpha - \beta c)^2}{4\beta - n(1-\delta)^2 E\varepsilon^2} \ .$$

The option value, then, is given by

$$E\Pi_{FB} - E\Pi_{full} = \frac{n(1-\delta)^2\sigma^2(\alpha - \beta c)^2}{[4\beta - n(1-\delta)^2][4\beta - n(1-\delta)^2(1+\sigma^2)]} \ .$$

As expected, the option value vanishes when $\sigma^2 = 0$, and monotonically increases in σ^2. The commitment value, on the other hand, is given by

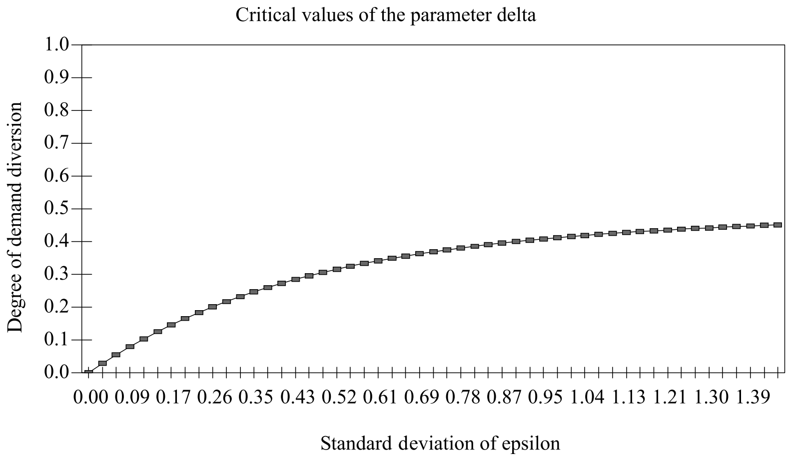

Fig. 3. Full collusion is more profitable to the left of the line, while semicollusion is more profitable to the right of the line

$$E\Pi_{FB} - E\Pi_{semi} = \frac{n\delta^2 E\varepsilon^2(\alpha - \beta c)^2}{[4\beta - n(1-\delta)^2 E\epsilon^2][4\beta - n(1-2\delta)E\epsilon^2]} \quad .$$

Note that the commitment value is zero when $\delta = 0$. This is intuitive in that the *ex ante* coordination in quality choice adds no value should the firms' products be completely independent.

It is also worth highlighting that the surprising result that, for high values of δ, increased uncertainty reduces (rather than increases) the profits of a semicollusive agreement, relies on the assumption that the demand shocks are common for all firms. If in addition to such a common shock, firm-level demand is subject to idiosyncratic shocks of sufficiently high variance, then an increase in the variance of the common shock increases the profitability of the semicollusive agreement for all values of δ. Therefore, under these conditions, there always exists a level of demand variability under which the semicollusive agreement dominates the fully collusive one. More concretely, suppose that the quantity sold by firm j is given by

$$q_j = \varepsilon_c \varepsilon_j \left[\frac{\alpha}{n} - \frac{\beta}{n} p + S_j - \delta \bar{S}_{-j}\right] \quad ,$$

where ε_c is the demand shock that is common to all firms with variance σ^2_{com}, and ε_j is an idiosyncratic i.i.d. shock to firm j with variance σ^2_{iid}. Following the solution approach utilized above for cartels with only common shocks, it can be shown that the expected firm profits under semicollusion are given by

$$E\Pi^*_{semi} = \frac{(\alpha - \beta c)^2}{4\beta - n(1 + \sigma^2_{com})(1 + \sigma^2_{iid} - 2\delta)} \ . \tag{8}$$

Since $\delta \leq 1$, the counter-intuitive result we obtain with respect to the non-monotonicity of profits with respect to the degree of common demand uncertainty still obtains for low levels of variance of idiosyncratic demand shocks. A somewhat mechanical intuition as to why the result fails to be obtained for high levels of σ^2_{iid} is as follows. Because the components of demand randomness enter multiplicatively, an increase in the common component increases the idiosyncratic randomness in a firm's demand. When $\sigma^2_{iid} > 1$, the increase in this type of randomness associated with an increase in σ^2_{com} is so large that the direct effect of an increase in σ^2_{com} never dominates. A less mechanical intuition is as follows. When the shock includes idiosyncratic components, the option value of flexibility is even higher, as each firm benefits from tailoring its quality to its own level of demand. In fact, with idiosyncratic shocks an ex post cartel that sets a uniform price and quality for all its members is not the first best cartel, as the return to quality differs across firms. Therefore, uniformity in quality induces an additional inefficiency (in terms of cartel profits) further tilting the trade-off towards the semicollusive cartel. In the remainder of the paper, we abstract from the possibility of any idiosyncratic demand shocks.

3.2 General Results

The above example demonstrates that there is no globally monotonic relationship between demand uncertainty and the relative profitability of the two cartel organizational forms. At best, one could hope to derive local results for relatively low values of demand substitutability. In this section we first derive such results and then demonstrate that the counter-intuitive inverse relationship between the profitability of semi-collusion and demand uncertainty is guaranteed to be obtained for high values of δ.[11] Finally, second order stochastic dominance is used as the measure of demand uncertainty because, when demand is nonlinear in quality, variance is no longer a sufficient statistic for the distribution of demand uncertainty.

We observe, at the outset, that under our demand and cost framework, the profits of the fully collusive cartel are independent of the degree of demand uncertainty. Therefore, the relative profitability of the two cartel forms depends on the effect of uncertainty on the profitability of semi-collusion. As Proposition 2 shows below, for sufficiently low levels of demand substitutability, an increase in demand uncertainty increases the profits of the semicollusive arrangement.

Proposition 2: There exists a level of demand substitutability $\Delta > 0$, such that for any $\delta \leq \Delta$, a mean-preserving spread in the distribution of ε leads to an increase in the profits of a semicollusive cartel.

The intuition for the proof is as follows. Consider, for the moment, the extreme case of $\delta = 0$. Then, conditional on the choice of price by the cartel, the firms can be thought of as stand-alone monopolies, i.e., there is no benefit to the firms of choosing quality cooperatively as there is no externality in each firm making this choice non-cooperatively. Then, for any given level of quality, firm profits would be linear in the level of demand, i.e., in ε. However, since firms can optimally adjust quality to the demand realization (at no cost to the other firms), profits will be convex in ε. Therefore, for $\delta = 0$, a mean-preserving spread in ε leads to a net gain in expected profits. The proposition shows that, in fact, there is a non-degenerate range of δ for which such a mean-preserving spread leads to higher profits. In our example above, this range is the [0,1/2) interval.

One, then, obtains the following corollary.

Corollary 2: Consider a set of demand conditions such that the two cartel organizational forms yield equal profits. Then, a mean-preserving spread in the distribution of ε results in semicollusion yielding higher profits than full collusion.

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11 Conditions under which the existence and uniqueness of equilibrium is guaranteed, and the associated proof of existence and uniqueness, are available from the authors upon request. These conditions form the generalization of Assumptions (i), (ii), and (iii) in Sect. 3.1.

The above result characterizes the geometry of the isoprofit locus. However, the result does not guarantee the existence of such a locus. It leaves open the possibility that semicollusion is never optimal (even for low δ). Proposition 3 below demonstrates the existence of such an isoprofit locus for low values of δ.

Proposition 3: For any level of demand uncertainty, i.e., for any distribution of ε, there exists a level of demand substitutability $\tilde{\delta} > 0$ such that for any $\delta < \tilde{\delta}$ semicollusion yields higher profits than full collusion.

The above results *guarantee* that for low levels of demand substitutability there exists a level of demand variability such that the semicollusive cartel dominates the fully collusive one. They do not preclude the possibility that for high values of δ a semicollusive cartel dominates the fully collusive one for *some* distributions and demand specifications. We next consider this possibility formally and show that for sufficiently high values of δ, the fully collusive cartel dominates the semicollusive one for *any* level of demand variability. The demonstration of this result proceeds in two steps. In Proposition 4 below, we show that the profits of a semicollusive cartel decline with the dispersion of the demand shock for sufficiently high values of δ. The result follows immediately from this proposition because the profits of the fully collusive cartel are independent of demand uncertainty and they exceed those of the semicollusive cartel when there is no demand uncertainty. This is stated in Corollary 3.

Proposition 4: There exists a level of demand substitutability $D < 1$, such that for any $\delta \geq D$, a mean-preserving spread in the distribution of ε leads to a decrease in the profits of a semicollusive cartel.

To obtain some informal insight, let us first consider the simplest (and most extreme) case of $\delta = 1$ and $\sigma^2 = 0$. Now, introduce some dispersion in the distribution of ε, but force firms not to adjust their quality levels to the realization of the value of ε. In that case, the firm profits will be linear in ε. If, in addition, firms could adjust their quality levels, *aggregate* revenues would remain unchanged, as with $\delta = 1$ the aggregate number of consumers does not increase with quality. Therefore, the change in the cartel profits will be determined by the change in the aggregate cost of investment in quality. Notice that if firms could adjust quality to the realization of ε, they would reduce it for draws of ε that are lower than 1 (leading to cost savings), and increase it for draws of ε that are higher than

1 (leading to cost increases). With a convex cost function in quality, the net sum of the cost savings and cost increases leads to higher costs (in expectation). Therefore, cartel profits are reduced.

More generally, consider the case of $\delta = 1$ and $\sigma^2 > 0$, and denote the equilibrium quality chosen by each firm for each realized value of ε by $S^*_{semi}(\varepsilon)$. Let the optimal price of the semicollusive cartel be p. Now, suppose there is a mean-preserving spread in the distribution of ε, and, for the moment, fix the cartel price at p. If cartel profits are concave in ε, then the mean-preserving spread will lead to a reduction in (expected) cartel profits. By continuity, this should hold for values of δ in the neighborhood of 1. We show this formally in the proof of Proposition 4 in the Appendix. Of course, the cartel will optimally adjust the price to the higher demand variance, counteracting to some degree the loss of profits described above. However, we show that because the price adjustment represents a second order effect, it does not dominate the profit decline induced from the mean-preserving spread in ε.

A direct implication of Proposition 4 is that the fully collusive cartel dominates the semicollusive one for any level of demand uncertainty when δ is sufficiently high. This is stated in Corollary 3 below.

Corollary 3: There exists a critical value of demand substitutability, $D < 1$, such that for any $\delta > D$ the profits of the fully collusive cartel exceed those of the semicollusive cartel for any level of demand variability.

This result follows from the observation that, for high levels of demand substitutability, the flexibility in choosing product quality leads to strong competition between the cartel members. This competition outweighs the benefits of tailoring the quality to the level of demand. In the terminology of Wong and Chow (1997), the commitment value always exceeds the option value for products that are sufficiently substitutable.

4 Conclusion

Forms of collusion between firms can be ranked on the basis of the number of the decision variables that are part of the collusive agreement. Semicollusive cartels allow the member firms to make many of the strategic decisions noncooperatively. In fully collusive cartels most or all of these decisions are made cooperatively. Cartel members are faced with the decision of how restrictive a cartel to form. Conventional wisdom suggests that a cartel that regulates all aspects of member firm behavior would lead to higher profits than a cartel that allows flexibility in member firm behavior. In this paper, we show that semicollusion can lead to higher profits than full collusion in the presence of demand uncertainty that cannot be incorporated into the cartel agreement.

The driving force behind this result is not uncertainty per se. If a strategic variable at the disposal of member firms results in large negative externalities to the other member firms, then allowing the cartel members to make this strategic choice non-cooperatively after the realization of the demand uncertainty would be detrimental to the profitability of the cartel. In this case, an *increase* in the demand uncertainty results in a *reduction* in the profits of the semicollusive cartel which implies that the fully collusive cartel dominates for *all* levels of demand uncertainty. If, on the other hand, the negative externalities are sufficiently small, then the higher the demand uncertainty, the higher the profits of the semicollusive cartel. Nevertheless, the presence of at least some demand uncertainty is crucial. If there is no demand uncertainty, then the fully collusive cartel yields profits that are higher than or equal to those of the semicollusive cartel regardless of the strength of the negative externality. These results suggest a reason why, even when allowed to do so, cartels choose not to collude on all aspects of their business. They also suggest that a change in industry structure that leads to greater substitutability between firm products increases the incentive for the firms to collude fully, rather than only partially.

Appendix A

A Microfoundations-based Derivation of the Demand Specification

In this appendix, (i) we derive our demand specification from a set of microfoundations using a parametric model, and (ii) we show in the context of that parametric model how collusion with respect to price can arise endogenously (rather than be assumed, as we do in the main body of the paper).

To motivate the parametric model, consider the following environment motivated by shipping cartels: A firm (one of many firms in a market) can ship its product using two shipping companies, A and B (which we will refer to as shippers). These shippers depart on different days, and therefore, to avoid holding stock in large quantities at either end of the route, the firm prefers to balance its shipments on both shippers. It is reasonable to assume that the firm has decreasing marginal revenue for its product in the destination market and increasing marginal cost. Therefore, profits are concave in the total amount shipped. Furthermore, because units shipped with either shipper are sold in the same market (and, thus, are to an extent substitutable), the marginal profit of a unit shipped with shipper A is a declining function of the number of units shipped with shipper B. Shippers can potentially differ not only in the prices they charge, P_A and P_B, but also in their quality levels, S_A and S_B. If the firm is using a higher quality shipper, its product will arrive in better shape and will be more desirable to consumers. Therefore, the firm's profits are increasing in the quality of a shipper in proportion to the amount that the firm is shipping with that shipper. In contrast, the profits of the firm are decreasing in the quality of the shipper that it is not utilizing, because a higher quality for that shipper implies that the products of the firm's competitors are arriving in better shape and are likely to be preferred by the consumers at the expense of the firm's own product.

A parametric specification of the firm's profit function consistent with the above environment is

$$\Pi^{firm}(q_A, q_B) = \alpha(q_A + q_B) - \frac{\beta}{2}(q_A^2 + q_B^2) - \gamma q_A q_B \\ + \eta q_A S_A - \zeta q_A S_B + \eta q_B S_B - \zeta q_B S_A - P_A q_A - P_B q_B \ .$$

The firm chooses q_A and q_B to maximize its profits, taking the quality of the products as given. The first order condition of profit maximization with respect to q_A yields

$$\frac{\partial \Pi^{firm}(\cdot)}{\partial q_A} = 0,$$
$$\alpha - \beta q_A - \gamma q_B + \eta S_A - \zeta S_B - P_A = 0 \ .$$

One can similarly obtain the first-order condition with respect to q_B. Solving each first-order condition for the respective price yields the demand system:

$$P_A = \alpha - \beta q_A - \gamma q_B + \eta S_A - \zeta S_B,$$
$$P_B = \alpha - \beta q_B - \gamma q_A + \eta S_B - \zeta S_A \ .$$

This system can be inverted to solve for the quantities shipped to yield

$$q_A = a - bP_A + cP_B + hS_A - eS_B,$$
$$q_B = a - bP_B + cP_A + hS_B - eS_A \ ,$$

where there is an one to one relationship between $\{a, b, c, h, e\}$ and $\{\alpha, \beta, \gamma, \eta, \zeta\}$. Our model is parameterized in the latter form (in which we constrain the prices to be the same, factor h out from the last two terms, and define $\delta = e/h$). [We omit the one-to-one mapping for brevity.]

Note that when b and c tend to infinity holding h and e constant, or equivalently as $\beta \approx \gamma$ with η and ζ adjusting so as to keep h and e constant, the model tends to the Bertrand competition model in terms of prices. In the limit, the firm will purchase from the lowest cost shipper. If both shippers charge the same price, then the quantity that he purchases from each shipper will be an increasing function of that shipper's quality and a decreasing function of the competing shipper's quality. In such a limiting case, if the marginal cost of shipping is constant, collusion in price is always optimal: In the absence of collusion in price, equilibrium profits are zero regardless of the realization of the demand shock (following standard arguments from the Bertrand model), while with collusion on price the profits will be positive. Therefore, one need only examine whether further collusion with respect to quality is also preferable. By continuity, when the parameter values are near enough to the Bertrand situation, collusion with respect to price will always be optimal.

Of course, the qualitative nature of our results is not likely to depend on a particular parametric form of preferences. As we note in a footnote, the same qualitative results are obtained using a standard logit discrete choice framework, even though such a framework can not be nested in our demand specification.[12]

Appendix B

Proofs of Lemmas and Propositions

Proof of Lemma 1: The F.O.C. of an individual firm's maximization problem is

$$\frac{\partial \pi_i}{\partial S_i} = 0 \Rightarrow (p - c)\varepsilon - 2S_i = 0 \ .$$

12 Note that in the model described above, as in the logit discrete choice framework, we can get symmetric equilibria, while in pure vertical differentiation models a strategy profile with the same quality choice is not an equilibrium.

This yields the firm's optimal choice of quality as a function of cartel price p

$$S^*_{semi} = \frac{(p-c)\varepsilon}{2} \quad .$$

Plugging S^*_{semi} into the profit function, summing up over all firms, and taking the expectation with respect to the distribution of ε yields,

$$\begin{aligned} E\Pi_{semi} &= E\left[(p-c)\varepsilon\left(\alpha - \beta p + \frac{(1-\delta)n(p-c)\varepsilon}{2}\right) - \frac{n(p-c)^2\varepsilon^2}{4}\right] \\ &= \alpha(p-c) - \beta p(p-c) \\ &\quad + \frac{(1-\delta)n(p-c)^2 E\varepsilon^2}{2} - \frac{n(p-c)^2 E\varepsilon^2}{4} \quad . \end{aligned}$$

The cartel chooses p to maximize $E\Pi_{semi}$. The F.O.C. with respect to p is

$$\begin{aligned} \frac{dE\Pi_{semi}}{dp} = 0 \Rightarrow \alpha - 2\beta p + \beta c + (1-\delta)n(p-c)E\varepsilon^2 \\ - \frac{n(p-c)E\varepsilon^2}{2} = 0 \\ \Rightarrow p^*_{semi} = \frac{2(\alpha+\beta c) - E\varepsilon^2 nc(1-2\delta)}{4\beta - E\varepsilon^2 n(1-2\delta)} \quad . \end{aligned}$$

Under Assumption (ii), the F.O.C. yields a maximum. Assumptions (i) and (ii) imply that price exceeds marginal cost. Finally, by plugging p^*_{semi} into $E\Pi_{semi}$ we get:

$$E\Pi^*_{semi} = \frac{(\beta c - \alpha)^2}{4\beta - E\varepsilon^2 n(1-2\delta)} \quad . \qquad \square$$

Proof of Lemma 2: The expected profits of the cartel when both price and quality are determined prior to the realization of the demand shock are

$$E\Pi_{full} = (p-c)(\alpha - \beta p + (1-\delta)nS) - nS^2 \quad .$$

The F.O.C.s with respect to p and S are

$$\frac{\partial E\Pi_{full}}{\partial S} = 0 \Rightarrow (1-\delta)(p-c)n - 2nS = 0,$$

$$\frac{\partial E\Pi_{full}}{\partial p} = 0 \Rightarrow \alpha - 2\beta p + \beta c + (1-\delta)nS = 0 \ .$$

The solution to the above system of equations yields

$$p^*_{full} = \frac{2(\alpha + \beta c) - cn(1-\delta)^2}{4\beta - n(1-\delta)^2}, \quad S^*_{full} = \frac{(\alpha - \beta c)(1-\delta)}{4\beta - n(1-\delta)^2} \ .$$

Under Assumptions (i) and (iii), this solution corresponds to a maximum and, moreover, p^*_{full} and S^*_{full} are positive.

Plugging p^*_{full} and S^*_{full} into $E\Pi_{full}$ yields

$$E\Pi^*_{full} = \frac{(\beta c - \alpha)^2}{4\beta - n(1-\delta)^2} \ . \qquad \square$$

Proof of Proposition 1: Collusion on both price and quality will be chosen over collusion on just price if

$$E\Pi^*_{full} > E\Pi^*_{semi} \Rightarrow \frac{(\beta c - \alpha)^2}{4\beta - n(1-\delta)^2} > \frac{(\beta c - \alpha)^2}{4\beta - E\varepsilon^2 n(1-2\delta)}$$

$$\Rightarrow 4\beta - E\varepsilon^2 n(1-2\delta) > 4\beta - n(1-\delta)^2 \ .$$

Using the fact that $E\varepsilon^2 = 1 + \sigma^2$, we write the above inequality as

$$(1+\sigma^2)(1-2\delta) < (1 - 2\delta + \delta^2) \Rightarrow \sigma^2(1-2\delta) < \delta^2 \ .$$

If $\delta > 1/2, (1-2\delta)$ is negative and the fully collusive cartel is always more profitable. If instead $\delta < 1/2$, then from above we get

$$\sigma^2 < \frac{\delta^2}{(1-2\delta)} \ .$$

Since $\delta^2/(1-2\delta)$ is an increasing function of δ, and positive for $\delta < 1/2$, it follows that for any level of demand uncertainty, σ^2, there exists a level of demand diversion $\hat{\delta}$ such that for any $\delta < \hat{\delta}$

$$\sigma^2 > \frac{\delta^2}{(1-2\delta)} \quad .$$

This, in turn, implies that $E\Pi^*_{full} < E\Pi^*_{semi} \; \forall \, \delta < \hat{\delta}$. Setting the two sides of the above expression equal to each other, solving for δ, and denoting the solution $\hat{\delta}$ yields $\hat{\delta} = -\sigma^2 + \sqrt{\sigma^2(\sigma^2+1)}$. □

Proof of Proposition 2: The proof proceeds in two parts. First, we show that the profits of semicollusion are convex in ε for low values of δ. We then use this fact to show that expected profits of a semicollusive cartel are increasing with a mean-preserving spread in the distribution of ε.

The F.O.C.s of Eq. (3) with respect to S_i are

$$(p-c)\varepsilon f_S(p, S_i - \delta \bar{S}_{-i}) - h'(S_i) = 0, \;\; i = 1, \ldots, n \quad .$$

At the symmetric equilibrium the F.O.C.s become

$$(p-c)\varepsilon f_S(p, (1-\delta)S) - h'(S) = 0 \quad .$$

Denote the solution by S^*_{semi} and the semicollusive cartel's maximal profits for each ε by

$$\pi(\varepsilon, \delta) = \varepsilon(p-c)f(p, (1-\delta)S^*_{semi}) - h(S^*_{semi}) \quad .$$

We first show that $\partial\pi^2(\varepsilon, 0)/\partial\varepsilon^2 > 0$, for all ε.

By differentiating $\pi(\varepsilon, \delta)$ with respect to ε and using the above F.O.C.s we get

$$\pi_\varepsilon(\varepsilon, \delta) = (p-c)f(p, (1-\delta)S^*_{semi}) - \delta f_S(p-c)(p, (1-\delta)S^*_{semi}) \frac{\partial S^*_{semi}}{\partial \varepsilon} \quad .$$

Differentiating π_ε with respect to ε and setting $\delta = 0$ we get

$$\pi_{\varepsilon\varepsilon}(\varepsilon, 0) = (p-c)f_S(p, S^*_{semi}) \frac{\partial S^*_{semi}}{\partial \varepsilon} > 0 \quad .$$

This is positive for every ε and $p > c$ since

$$\frac{\partial S^*_{semi}}{\partial \varepsilon} = \frac{-(p-c)f_S(p,(1-\delta)S^*_{semi})}{(1-\delta)\varepsilon(p-c)f_{SS}(p,(1-\delta)S^*_{semi}) - h''(S^*_{semi})} > 0, \text{ if } p > c \ .$$

We next show that there exists a neighborhood of strictly positive $\delta's$ such that the semicollusive cartel's profit function is convex in ε. Let $\varphi(\varepsilon, \delta) = \pi_{\varepsilon\varepsilon}(\varepsilon, \delta)$. Given our assumptions it can be easily seen that φ is a continuous function. Also, as we showed above, $\varphi(\varepsilon, 0) > 0$, for all ε. Now let $\Phi(\delta) = \min_\varepsilon \varphi(\varepsilon, \delta)$. The function Φ is continuous (by applying the Maximum Theorem) and has the property that $\Phi(0) > 0$. Thus, there exists a $\Delta > 0$ such that $\Phi(\delta) > 0$ for $\Delta \geqslant \delta > 0$. This completes the first part of the proof.

Before we proceed to the second part of the proof, we need to introduce some notation. Let $g(\cdot)$, the probability density function of ε, belong to a parametric family described by an L-dimensional parameter vector Θ. Let Ξ be a vector that results from a transformation of the parameter vector Θ, such that an increase in any of the first L_1 arguments of Ξ, keeping the remaining L_2 elements constant, results in a mean-preserving spread in ε. For illustration, consider a random variable X which is distributed uniformly on $[b_L, b_H]$. In this example, $\Theta = \{b_L, b_H\}$. We can re-parameterize this vector as $\Xi = \{\xi_1, \xi_2\}$ where $\xi_1 = b_H - b_L$ and $\xi_2 = (b_L + b_H)/2$. Notice that an increase in ξ_1, while keeping ξ_2 fixed, results in a mean-preserving spread in the distribution of X.

We now show that an increase in any of the first L_1 elements of Ξ has a positive effect on the expected profits of a semicollusive cartel. Consider such an element, ξ_j, of Ξ, where $1 \leq j \leq L_1$. The effect of an increase of ξ_j on $E\Pi^*_{semi}$ is

$$\frac{\partial E\Pi^*_{semi}(p^*_{semi}(\xi_j), \xi_j)}{\partial \xi_j} = \frac{\partial E\Pi_{semi}(p^*_{semi}(\xi_j), \xi_j)}{\partial p^*_{semi}} \frac{\partial p^*_{semi}}{\partial \xi_j} + \frac{\partial E\Pi_{semi}(p^*_{semi}(\xi_j), \xi_j)}{\partial \xi_j} \ . \tag{B.1}$$

The first term on the right hand side of (B.1) is zero from the F.O.C. Further, an increase in any of the first L_1 elements of Ξ yields a mean-preserving spread. Since profits for the semicollusive cartel are convex in ε, for any $\delta \leq \Delta$ and a given p, any mean-preserving spread of the distribution of ε yields higher expected profits for the semicollusive cartel. Thus, the second term of (B.1) is positive. □

Proof of Proposition 3: Suppose that the demand diversion parameter δ is 0. Then, there is no gain in choosing the level of quality S_i cooperatively. On the other hand, there is a private gain to each firm from being able to choose its level of quality after it has observed the resolution of the demand uncertainty. Therefore, $E\Pi^*_{semi} > E\Pi^*_{full}$.

Now suppose that the demand diversion parameter equals 1. Observe that the F.O.C.s of profit maximization for the fully collusive cartel with respect to p and S yield

$$f + (p - c)f_p = 0,$$

$$(p - c)f_S(1 - \delta) - h' = 0 \ .$$

Then, assuming f_S is finite and $h'(0) = 0$, we obtain $S^*_{full} \to 0$ as $\delta \to 1$. Also, $S^*_{semi} > S^*_{full}$ as $\delta \to 1$. This follows since, by using the F.O.C.s of the semicollusive cartel and taking the limit $\delta \to 1$, we get that

$$(p - c)\varepsilon f_S(p, 0) = h'(S_i), \ \forall i \ .$$

Assuming $f_S(p, 0) > 0$ and $h'(0) = 0$ for all i we obtain that $S^*_{semi} > 0$ (for $p > c$, and $\varepsilon > 0$). Since quality is costly, $E\Pi^*_{full} > E\Pi^*_{semi}$. Since $E\Pi^*_{semi}$ and $E\Pi^*_{full}$ are continuous in δ there exists a $\tilde{\delta} > 0$ such that for all $\delta < \tilde{\delta}$ the semicollusive cartel is more profitable than the fully collusive one.

We showed that for any level of the parameters of the distribution of ε, as long as it is not degenerate, we can find a sufficiently low δ such that the semicollusive cartel is more profitable. □

Proof of Proposition 4: The proof proceeds in two parts. First, we show that the profits of semicollusion are concave in ε for high values of δ. We then use this fact to show that expected profits of a semicollusive cartel are decreasing with a mean-preserving spread in the distribution of ε.

The first-order condition with respect to S_i of a firm's profit maximization problem is

$$(p-c)\varepsilon f_S(p, S_i - \delta\bar{S}_{-i}) - h'(S_i) = 0,\ i = 1, \ldots, n \ .$$

At the symmetric equilibrium the F.O.C.s become

$$(p-c)\varepsilon f_S(p, (1-\delta)S) - h'(S) = 0 \ .$$

Denote the solution by S^*_{semi} and the semicollusive cartel's maximal profits for each ε by

$$\pi(\varepsilon, \delta) = \varepsilon(p-c)f(p, (1-\delta)S^*_{semi}) - h(S^*_{semi}) \ .$$

We first show that $\partial\pi^2(\varepsilon, 0)/\partial\varepsilon^2 < 0$, for all ε. By differentiating $\pi(\varepsilon, \delta)$ with respect to ε and using the above F.O.C.s, we get

$$\pi_\varepsilon(\varepsilon,\delta) = (p-c)f(p,(1-\delta)S^*_{semi}) - \varepsilon(p-c)\delta f_S(p,(1-\delta)S^*_{semi})\frac{\partial S^*_{semi}}{\partial\varepsilon} \ .$$

Setting $\delta = 1$ the above equation becomes

$$\pi_\varepsilon(\varepsilon, 1) = (p-c)f(p, 0) - \varepsilon(p-c)f_S(p, 0)\frac{\partial S^*_{semi}}{\partial\varepsilon} \ .$$

Differentiating π_ε with respect to ε, we get

$$\pi_{\varepsilon\varepsilon}(\varepsilon, 1) = -(p-c)f_S(p, 0)\frac{\partial S^*_{semi}}{\partial\varepsilon} - \varepsilon(p-c)f_S(p, 0)\frac{\partial^2 S^*_{semi}}{\partial\varepsilon^2} \ .$$

The above expression is negative for every ε and $p > c$ since by totally differentiating the first-order condition of firm maximization with respect to S and ε we obtain

$$\frac{\partial S^*_{semi}}{\partial\varepsilon} = \frac{(p-c)f_S(p, 0)}{h''(S)} > 0$$

and

$$\frac{\partial^2 S^*_{semi}}{\partial\varepsilon^2} = 0 \ .$$

We next show that there exists a neighborhood of strictly positive $\delta' s$ such that the semicollusive cartel's profit function is concave in ε. Let

$\varphi(\varepsilon, \delta) = \pi_{\varepsilon\varepsilon}(\varepsilon, \delta)$. Given our assumptions it can be easily seen that φ is a continuous function. Also, as we showed above, $\varphi(\varepsilon, 1) < 0$, for all ε. Now let $\Phi(\delta) = \max_{\varepsilon} \varphi(\varepsilon, \delta)$. The function Φ is continuous (by applying the Maximum Theorem) and has the property that $\Phi(1) < 0$. Thus, there exists a $D < 1$ such that $\Phi(\delta) < 0$ for $D \leqslant \delta < 1$. This completes the first part of the proof.

The above shows that profits, conditional for any fixed p, are concave in ε. Therefore, any mean-preserving spread in the distribution of demand uncertainty, will lead, if p were held constant, to a reduction in cartel profits. Since p is chosen optimally and thus the adjustment of p is a second-order effect, one can use standard envelope theorem arguments to show that profits decline in the dispersion of ε even when the cartel optimally adjusts the price to the new distribution of ε. The formal derivation of these arguments is omitted because it follows the second half of the proof of Proposition 2 in the paper. □

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