# Price Discrimination in Two-Sided Markets[*]

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June 22, 2010

## Abstract

We examine the profitability and welfare implications of targeted price discrimination in two-sided markets. First, we show that equilibrium discriminatory prices exhibit novel features relative to discriminatory prices in one-sided models and uniform prices in two-sided models. Second, we compare the profitability of perfect price discrimination, relative to uniform prices in a two-sided market. The conventional wisdom from one-sided horizontally differentiated markets is that price discrimination hurts the firms and benefits consumers, *prisoners' dilemma*. We show that price discrimination, in a two-sided market, may actually *soften* the competition. Our results suggest that the conventional advice that price discrimination is good for competition based on one-sided markets may not carry over to two-sided markets.

**Keywords**: Price discrimination; Two-sided markets; Indirect network externalities.
**JEL Classification Codes:** D43, L13.

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[*] We would like to thank a coeditor and two referees for very helpful comments and suggestions. We would also like to thank Chris Adams, Christian Ahlin, Marcus Asplund, Luís Cabral, Jay Pil Choi, Bob Hunt, Alessandro Lizzeri, Steven Matusz, Leonard Nakamura and seminar participants at Drexel University, the Federal Reserve Bank of Philadelphia, Michigan State University, the 2008 International Industrial Organization Conference (IIOC), the 2008 NET Institute conference (NYU-Stern), the 2007 European Summer Meeting of the Econometric Society (ESEM), the 2007 Far Eastern Meeting of the Econometric Society (FEMES), the 2007 Society for the Advancement of Economic Theory (SAET) meeting, the 2007 Conference on Research in Economic Theory and Econometrics (CRETE), the 2007 Chinese Economists Society (CES) meeting and the 2007 Eastern Economic Association (EEA) meeting for helpful comments and suggestions. We thank the NET Institute (www.NETinst.org) for financial support.

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# 1 Introduction

The aim of this paper is to study the implications of price discrimination in two-sided markets.[1] For example, TV stations target advertisers, the one side of the market, with different advertising fees and viewers, the other side, with different subscription fees (Gil and Grichton (2010)), or newspapers offer low introductory rates to new subscribers (Asplund et al. (2008)) and different rates to advertisers.

There exists a relatively large literature on oligopolistic third-degree price discrimination in "one-sided" markets, but this paper is among the first ones that examine this problem in the context of a two-sided market.[2] The main message of the one-sided literature is that price discrimination intensifies price competition (prisoners' dilemma) and therefore it is beneficial for the consumers (at least on average).[3] The advice then given to policymakers and antitrust authorities is that they should not worry much about firms acquiring and using consumer information with the intention to customize prices, because after all firm competition for consumers dissipates profits and transfers most of the surplus to consumers.

Furthermore, it is well-known that the presence of indirect externalities in two-sided markets can intensify competition, e.g., Armstrong (2006a). Platforms have strong incentives to lower prices in order to sign-up more agents. Therefore, putting together the results from one-sided models with price discrimination and from two-sided models with no price discrimination, one would expect that price discrimination in a two-sided market will generate a very competitive environment with low prices and profits. This is true, but not always. We show that if the cross-group network externalities are strong and/or the marginal cost is low (e.g., digital products), then price discrimination increases platform profits and hurts consumer welfare.

The intuition for this result is as follows. In two-sided markets, price discrimination has two effects on competition. First, and similar to one-sided markets, when platforms can price discriminate, in equilibrium, any two prices charged by two platforms to an agent always differ by the difference in transportation costs. This flexibility in pricing reduces profits, a *negative effect.* How-

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[1] Two-sided (or multiple-sided) markets are markets that are organized around intermediaries or "platforms" with two (or multiple) sides who should join a platform in order for successful exchanges (trade) to take place, see Armstrong (2006a), Caillaud and Jullien (2003) and Rochet and Tirole (2003, 2006). For example, videogame platforms (e.g., Nintendo, Sony, Microsoft) need to attract both gamers and game developers. Newspapers need to attract advertisers and readers. Credit cards need merchants and users. Most media and advertising markets are two-sided markets, e.g., Anderson and Coate (2005) and Anderson and Gabszewicz (2006). Other examples of two-sided markets include, newspapers, scholarly journals, magazines, shopping malls, dating services and Business-to-Business (B2B) markets. More formally, a two-sided market is defined as one where the volume of transactions between end-users depends on the structure of the fees and not only on the overall level of fees charged by platforms [Rochet and Tirole (2006)].

[2] By "one-sided" markets we simply mean markets with no externalities. For a survey of the literature on oligopolistic price discrimination in one-sided markets we refer the reader to Armstrong (2006b) and Stole (2007). Jullien (2008) investigates how price discrimination helps a platform to coordinate the choices of consumers.

[3] See, for example, Thisse and Vives (1988), Shaffer and Zhang (1995), Bester and Petrakis (1996), Chen (1997), Corts (1998), Fudenberg and Tirole (2000) and Liu and Serfes (2004).

ever, price discrimination also has a second effect through the cross-group externality. In particular, it may render the cross-group externality irrelevant in equilibrium, and thus to improve profits relative to that under uniform pricing, a *positive effect*.[4] This is because, uniform equilibrium prices depend on the cross-group externality. A stronger externality increases each platform's incentives to cut prices and as a result equilibrium prices fall. Discriminatory prices, on the other hand, are, under certain conditions, *independent* of the cross-group externality.[5] The presence of the indirect externality intensifies competition and discriminatory prices fall. Under the reasonable assumption that prices cannot become negative, each platform, in the symmetric equilibrium, will charge zero price to the agents that are located closer to the rival platform and to its own agents will charge a premium which *only* depends on the transportation cost. Due to the "limit price" nature of the problem under perfect price discrimination and the assumption of non-negative prices, the feedback effect disappears in equilibrium. Hence, strong externalities imply that uniform prices will fall while discriminatory prices do not change, which further implies that price discrimination in such a case is more profitable. Price flexibility is a curse in one-sided markets, but it can be a blessing in a two-sided market. The result and intuition are similar even when we allow for imperfect price discrimination or when we allow agents on both sides to multi-home.

This is the first paper concerned with perfect price discrimination in a two-sided market. As we show, the features of the perfect price discrimination equilibrium are qualitatively very different from those in a one-sided market. For example, the prices a platform charges in its rival's turf are not constant (which is typically the case in one-sided models). This, in turn, has implications about the platform's prices in its own turf. Equilibrium prices are *not* distribution-free (as it is the case in one-sided perfect price discrimination models). Moreover, equilibrium prices under perfect price discrimination in a two-sided model may depend not only on the other-group externality (as it is the case under uniform pricing in a two-sided model), but also on the own-group externality. These new results have managerial implications that cannot be deduced from a one-sided perfect price discrimination model.

Our result has important theoretical and policy implications because it demonstrates that price discrimination is more likely to be anti-competitive in two-sided markets than it is in one-sided markets. More fundamentally, it suggests that two-sided markets can be very different from one-sided markets (see Economides and Tåg (2007), where a similar conclusion, regarding the difference between one-sided and two-sided markets, is reached).

Caillaud and Jullien (2003) and Armstrong (2006a) also allow for price discrimination. In Caillaud and Jullien agents in each group are homogeneous and therefore price discrimination means

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[4] In one-sided markets, cross-group externality is absent so only the first effect exists. Therefore, targeted price discrimination intensifies competition relative to uniform pricing (under the standard assumptions of uniform distribution and linear transportation cost).

[5] If, on the other hand, discriminatory prices depend on externality as well, then both effects are negative, and targeted price discrimination intensifies competition even more than in one-sided markets.

different prices charged to each group of agents, while within each group the price is constant. This is also the meaning of price discrimination in Armstrong (2006a), although he allows for heterogeneous populations of agents. In contrast, we allow the prices within each group to vary.

The result that targeted price discrimination may relax competition can potentially apply to settings other than two-sided markets, where there is a binding price floor under price discrimination but not under uniform pricing. For example, consider a two-period model with switching costs. In the second period, each firm has incentives to price discriminate between its own and the rival's customers. In particular, it may want to charge lower prices to its rival's customers to compensate for switching costs. Various studies (e.g., Chen (1997)) have shown that such price discrimination can lead to lower profits. However, price discrimination also makes the non-negative price constraint more likely to be binding, relative to the case of uniform pricing where the single price applies to its own customers as well. That is, price discrimination has a *positive* effect due to switching costs which is similar to the positive effect due to cross-group externality in our model.[6]

Our analysis can also apply to intermediate goods markets where price discrimination is more likely to raise antitrust concerns than in final goods markets. Indeed, in the Unites States price discrimination is illegal in intermediate goods markets under the Robinson-Patman act. Each platform in our model can be viewed as a Business-to-Business (B2B) website which matches input suppliers with producers [e.g., Caillaud and Jullien (2003)]. The Internet facilitates the collection and application of information about the users' preferences and characteristics, see FTC (2000). An interesting question which arises then is whether platforms should be restricted to charge uniform prices. We will return to this interpretation of our model in section 4.

The rest of the paper is organized as follows. In section 2, we present the benchmark model. In section 3, we perform the analysis. In section 4, we extend the benchmark model to allow agents to multi-home. We conclude in section 5.

## 2 The description of the benchmark model

There are two groups of agents $\ell = 1, 2$ and two horizontally differentiated platforms $k = A, B$.[7] We will denote the "other" group of agents by $m$. We capture platform differentiation as follows. There is a continuum of agents of group $\ell$ that is distributed on the $[0, 1]$ interval according to the distribution function $F_\ell(\cdot)$ with density $f_\ell$. The distributions are independent across the two groups of agents and symmetric about $\frac{1}{2}$, i.e., $F_\ell\left(\frac{1}{2}\right) = \frac{1}{2}$ and $f_\ell(x) = f_\ell(1-x)$. The two platforms are located at the two end points of each interval, with platform $A$ located at 0 and platform $B$

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[6] One can also introduce consumer heterogeneity in addition to that due to switching costs, in which case the non-negative price constraint is more likely to be binding under price discrimination. Of course, it remains to be seen whether firms' discounted profits over the two periods are higher under price discrimination or uniform pricing.

[7] Our benchmark model follows closely the model in Armstrong (2006a).

located at 1.

The common per-unit transportation cost of both groups is denoted by $t > 0$. We assume that each agent joins only one platform (single-homing).[8] Each member of a group who joins a given platform cares about the number of members from the other group who join the same platform. Denote by $n_{\ell k}$ the number of participants from group $\ell$ that platform $k$ attracts. The maximum willingness to pay for a member of group $\ell$ if he joins platform $k$ is given by $V + \alpha_\ell n_{mk}$, where $V$ is a stand-alone benefit each agent receives independent of the number of participants from the other group on platform $k$. The parameter $\alpha_\ell > 0$ measures the cross-group externality for group $\ell$ participants. The indirect utility of an agent from group $\ell$ who is located at point $x \in [0,1]$ is given by,

$$U_\ell = \begin{cases} V + \alpha_\ell n^e_{mA} - tx - p_{\ell A}(x), & \text{if he joins platform } A \\ V + \alpha_\ell n^e_{mB} - t(1-x) - p_{\ell B}(x), & \text{if he joins platform } B \end{cases} \tag{1}$$

where $p_{\ell k}(x)$ is platform $k$'s lump-sum charge to a group $\ell$ participant who is located at point $x$ and $n^e_{mk}$ denotes the expectations agents from group $\ell$ have about how many agents from group $m$ will join platform $k$. Under a uniform pricing rule prices are constant across all agents in the same group (prices are allowed to vary across groups), while under discriminatory pricing the price each agent pays depends on his preferences (location). We assume that $V$ is high enough which ensures that the market is covered. Platforms have constant marginal cost $c \geq 0$. We assume that prices cannot be negative.[9]

The timing of the game is as follows. In stage 1, the two platforms make, simultaneously, their pricing decisions. In stage 2, the agents decide which platform to join.

## 3 Analysis

We study two different price regimes. In the first regime each platform charges uniform prices to the agents of each group. In the second regime each platform can price discriminate perfectly the agents of each group. Then, we compare prices and profits between these two price regimes. We assume that each agent has rational expectations about how many agents from the other group will join each platform. Each agent observes all prices before he decides which platform to join (public prices), [e.g., Caillaud and Jullien (2003) and Armstrong (2006a)].[10]

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[8] We relax this assumption by allowing agents to multi-home in Section 4.

[9] In most cases negative prices are unrealistic and create perverse incentives [see also Armstrong (2006a) for a discussion on this issue]. If negative prices were allowed, viewers could, for example, subscribe to a TV channel, never watched it and got paid for that. More generally, agents will have incentives, when they get paid, to make multiple purchases and it will be difficult for the platforms to prevent this from happening.

[10] We consider private prices in section 3.5.

### 3.1 No price discrimination (uniform prices within each group of agents)

The next Proposition summarizes the main result when platforms cannot price discriminate within each group of agents with a general distribution of preferences.[11]

**Proposition 1** ***(Uniform prices)*** *If a symmetric equilibrium exists, then it is given by:*

$$p_{1A}^{*} = p_{1B}^{*} = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{f_1\left(\frac{1}{2}\right)} + c \text{ and } p_{2A}^{*} = p_{2B}^{*} = \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{f_2\left(\frac{1}{2}\right)} + c. \tag{2}$$

*The equilibrium profits are,*

$$\pi_A = \pi_B = \frac{t - \alpha_2 f_1\left(\frac{1}{2}\right)}{2 f_1\left(\frac{1}{2}\right)} + \frac{t - \alpha_1 f_2\left(\frac{1}{2}\right)}{2 f_2\left(\frac{1}{2}\right)}. \tag{3}$$

**Proof.** See Appendix. ■

Each platform serves one half of the members of each group. The equilibrium prices depend positively on the differentiation parameter $t$, negatively on the strength of the cross-group externality $\alpha_\ell$ and negatively on the number of marginal agents $f_\ell\left(\frac{1}{2}\right)$. When the externality for group $\ell$ is stronger platforms offer lower prices to the members of group $m$, all else equal. Potentially, prices can be negative, but we do not allow for this possibility.

#### 3.1.1 Uniform distribution

If we assume that the distribution is uniform ($f_1(x) = f_2(x) = 1$), then the equilibrium prices and profits are.[12]

$$p_{1A}^{*} = p_{1B}^{*} = t - \alpha_2 + c,\ p_{2A}^{*} = p_{2B}^{*} = t - \alpha_1 + c \tag{4}$$

and

$$\pi_A = \pi_B = t - \frac{(\alpha_1 + \alpha_2)}{2}. \tag{5}$$

### 3.2 Perfect price discrimination

Now we assume that platforms can price discriminate perfectly. Agent utility is given by (1) and platforms compete on an agent-by-agent basis. Each agent receives a targeted offer. Platform $A$'s

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[11] We were not able to come up with clean conditions on the distribution functions that would ensure the strict concavity (or quasi-concavity) of the objective functions. For instance, the monotone hazard rate property is not enough. When the distribution is uniform (see below), then the profit functions are strictly concave provided that $2t > (\alpha_1 + \alpha_2)$. When this condition holds, then a symmetric sharing equilibrium exists. Otherwise, one platform may corner the entire market.

[12] The existence of a symmetric equilibrium is guaranteed if $2t > \alpha_1 + \alpha_2$, see Armstrong (2006).

own territory is the $[0,1/2]$ interval and platform $B$'s own territory is the $[1/2,1]$ interval. The next Proposition summarizes the equilibrium. We focus on symmetric equilibria.

**Proposition 2** ***(Perfect price discrimination)*** *There are two distinct cases:*[13]

*(i) High marginal cost and/or low cross group externality,* $c \geq \max\{2\alpha_1+\alpha_2, \alpha_1+2\alpha_2\}$. *Suppose that*

$$t \geq 2(\alpha_1+\alpha_2)f_\ell(x), \ell = 1,2. \tag{6}$$

*The equilibrium prices are*

$$\begin{aligned} p^*_{\ell A}(x) &= t(1-2x) + p^*_{\ell B}(x), p^*_{\ell B}(x) = c - 2\alpha_m(1-F_m(x)) - \alpha_\ell(1-2F_m(x)), \textit{ for } x \leq \frac{1}{2} \textit{ and} \\ p^*_{\ell A}(x) &= c - 2\alpha_m F_m(x) - \alpha_\ell(2F_m(x)-1), p^*_{\ell B}(x) = p^*_{\ell A}(x) + t(2x-1), \textit{ for } x \geq \frac{1}{2}. \end{aligned} \tag{7}$$

*(ii) Low marginal cost and/or high cross-group externality,* $c \leq \min\{\alpha_1, \alpha_2\}$. *All prices in the rival platform's own territory are negative. Since negative prices are not allowed, they are replaced by zero. Suppose that*

$$t > c + \max\{\alpha_1, \alpha_2\} \textit{ and } t < (\alpha_1+\alpha_2)\min\{f_1(x), f_2(x)\}. \tag{8}$$

*The equilibrium prices are*

$$\begin{aligned} p^*_{\ell A}(x) &= t(1-2x), p^*_{\ell B}(x) = 0, \textit{ for } x \leq \frac{1}{2} \textit{ and} \\ p^*_{\ell A}(x) &= 0, p^*_{\ell B}(x) = t(2x-1), \textit{ for } x \geq \frac{1}{2}. \end{aligned} \tag{9}$$

**Proof.** See Appendix. ■

The idea behind the equilibrium prices in case $(i)$ is as follows. First, there is symmetry, that is, in equilibrium, $p^*_{\ell A}(x) = p^*_{\ell B}(1-x)$, for all $x \in [0,1]$, $\ell = 1,2$. Second, the two platforms split both groups evenly. Third, each agent is indifferent between buying from either platform (and buys from the one closer to his location). Prices are constructed as follows. If platform $A$, say, deviates to $x > 1/2$ in group $\ell$, its benefit from signing up an extra agent in group $\ell$ is $2\alpha_m F_m(x)$.[14] The cost to make an extra sale at price below marginal cost is $p^{dev}_{\ell A}(x) - c$. For the platform to be indifferent and hence not to want to deviate, we need

$$2\alpha_m F_m(x) = -(p^{dev}_{\ell A}(x) - c) \Rightarrow p^{dev}_{\ell A}(x) = c - 2\alpha_m F_m(x).$$

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[13] There is also a third case, case$(iii)$, that falls in between the two cases presented in this Proposition, i.e., $\min\{\alpha_1, \alpha_2\} < c < \max\{2\alpha_1+\alpha_2, \alpha_1+2\alpha_2\}$. Relative to case $(i)$ some prices in the rival platform's territory become negative. In equilibrium, these prices are replaced by zero and the prices charged by a platform in its own territory are equal to the transportation cost premium, as in (9). For the prices that are not negative the equilibrium is the same as in (7). We do not pursue this case further, as it does not add anything to our understanding of the problem.

[14] It is multiplied by 2 because an extra agent increases platform $A$'s share as well as decreases platform $B$'s market share in group $\ell$.

$p_{\ell A}^{dev}(x)$ is decided by the indifference condition for the agents, that is,

$$V - p_{\ell A}^{dev}(x) - tx + \alpha_\ell F_m(x) = V - p_{\ell B}^{*}(x) - t(1-x) + \alpha_\ell(1 - F_m(x)) \Rightarrow$$

$$\begin{aligned} p_{\ell B}^{*}(x) &= p_{\ell A}^{dev}(x) - \alpha_\ell(2F_m(x) - 1) + t(2x-1) \\ &= [c - 2\alpha_m F_m(x)] - \alpha_\ell(2F_m(x) - 1) + t(2x-1). \end{aligned}$$

These are the equilibrium prices for platform $B$ when $x \geq 1/2$, as stated in Proposition 2. Then, platform $A$'s equilibrium prices in $x \geq 1/2$ are

$$p_{\ell A}^{*}(x) = p_{\ell B}^{*}(x) + t(1-2x) = c - 2\alpha_m F_m(x) - \alpha_\ell(2F_m(x) - 1).$$

What remains is to confirm that indeed no platform has an incentive to deviate from the candidate equilibrium prices. Moreover, there is a continuum of (symmetric) price equilibria (when all prices are positive), that can be Pareto ranked (from the platforms' perspective) from the 'highest price' one to the 'lowest price' one. We assume that platforms can coordinate on the one that yields the highest prices and hence profits and this is the one we present in Proposition 2 (case (*i*)). More details can be found in the proof of Proposition 2 in the Appendix. As the externalities intensify, or the marginal cost decreases, some prices become negative. Negative prices are replaced by zero and we move to case (*ii*) in Proposition 2.

Case (*i*) of Proposition 2 presents novel results. There are two main differences between the perfect price discrimination equilibrium in a one-sided market and in a two-sided market. In a one-sided market a firm charges prices equal to marginal cost in the rival firm's territory, while in its own territory prices reflect the transportation cost difference. This is not the case in a two-sided market. First, in a two-sided market, the prices a platform charges in the rival's market (and some of the prices in its own market) are below marginal cost. Second, they are not constant and they are not distribution-free (that is, they depend on $F_\ell$). Notice, for example, from case (*i*) in Proposition 2, that when $x \geq 1/2$, $p_{\ell A}^{*}$ decreases in $x$ and depends on the distribution of agent preferences.

The intuition behind these two differences is as follows. A platform may be willing to charge a price below marginal cost to an agent, because an extra agent from one group is valuable to *all* the agents in the other group. This is a direct consequence of the network externalities and a platform's ability to customize prices. The second difference we mentioned above is more subtle. For an equilibrium to exist, a platform should be losing increasingly more money as it tries to poach the rival platform's agents. The reason for this is that the benefit from signing up one more agent increases with the market share of a platform, i.e., the more agents a platform has already signed up the higher the benefit of an additional agent. Thus, the price in the rival platform's territory should be decreasing in accordance to the mass of agents up to that point. Finally, equilibrium uniform prices depend on the externality in the other group (see (2)), while perfect price discrimination prices depend on the externalities in *both* groups. These new results should also serve as a guidance to managers in two-sided markets.

An interesting and possibly empirically testable implication is the following. Our results indicate that a platform's discriminatory prices in its own turf exhibit less dispersion in a two-sided market than in a one-sided market. This is because discriminatory prices decrease slower when moving from loyal to less loyal agents in a two-sided market. From (7), we can infer that $dp^{*}_{\ell A}\left(x\right)/dx = -2t + 2\alpha_m f_m\left(x\right) + 2\alpha_\ell f_m\left(x\right)$ when $x \leq \frac{1}{2}$, which under our assumptions is negative but greater than $-2t$, the corresponding price decline in a one-sided market.[15]

When the marginal cost is high, as in case (*i*) in Proposition 2, the equilibrium prices depend on the cross-group externalities. Actually, the presence of cross-group network externalities intensifies the competition further under perfect price discrimination relative to uniform pricing. Under uniform pricing, prices are reduced by $\alpha_m$, see (2), whereas under perfect price discrimination the price decline in group $\ell$, according to (7), ranges from $2\alpha_m + \alpha_\ell$ for the agents located at the extremes to $\alpha_m$ for the agents located at $x = 1/2$.

Nevertheless, the above comparison fails when the marginal cost is low and/or the network externalities are strong as in case (*ii*) in Proposition 2. Precisely because perfect price discrimination in a two-sided market generates a very competitive environment, prices fall so low that they reach the natural floor of zero. In this case, the network externality is priced-out of the equilibrium prices, see (9).[16]

Therefore, when the marginal cost is high and/or the externalities are low, perfect price discrimination unambiguously yields lower profits than uniform pricing. This result is in line with the comparison in one-sided markets, e.g., Thisse and Vives (1988). The difference arises when the marginal cost $c$ is low (e.g., digital products) and/or $\alpha$ is high, $c \leq \min\left\{\alpha_1, \alpha_2\right\}$, case (*ii*) in Proposition 2. (More on this comparison in the next section). After the next subsection, in the remaining of the paper we assume that we are in case (*ii*).

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[15] Our results also indicate that a platform's prices in its rival's turf exhibit more dispersion in a two-sided market than in a one-sided market. However, this is unlikely to be testable since should not expect to observe platform's prices in its weak market where it makes no sales.

[16] We acknowledge the possibility that nominal prices may be non-negative while actual prices are negative. This can happen when platforms give agents some kind of benefits (e.g., reimburse transportation costs when they are actually travel costs). This is similar to allow negative prices in our setting, and case (*i*) in Proposition 2 would apply.

### 3.2.1 Uniform distribution

Suppose that the distribution is uniform, i.e., $f_1(x) = f_2(x) = 1$. The equilibrium profits when all prices are positive, as in case ($i$) in Proposition 2, are

$$\begin{aligned}\pi_A &= \int_0^{1/2} \left[t(1-2x) - 2\alpha_2(1-x) - \alpha_1(1-2x)\right] dx + \int_0^{1/2} \left[t(1-2x) - 2\alpha_1(1-x) - \alpha_2(1-2x)\right] dx \\ &= \frac{t}{2} - \alpha_1 - \alpha_2 = \pi_B. \qquad (10)\end{aligned}$$

Condition (6) guarantees that the above profits are non-negative.

On the other hand, when all unconstrained prices in the rival platform's territory are negative (in which case they are replaced by zero), as in case ($ii$) in Proposition 2, the equilibrium profits are

$$\pi_A = \int_0^{1/2} \left[t(1-2x) - c\right] dx + \int_0^{1/2} \left[t(1-2x) - c\right] dx = \frac{t}{2} - c = \pi_B. \qquad (11)$$

Condition (8) together with $c \leq \min\{\alpha_1, \alpha_2\}$ guarantee that the above profits are positive. Notice that the network externalities affect the equilibrium profits given by (10), while they do not affect the equilibrium profits given by (11).

## 3.3 Price and profit comparison

We compare the equilibrium uniform prices given by (4) with the discriminatory prices given by (9). So, we assume that $c \leq \min\{\alpha_1, \alpha_2\}$.[17] Discriminatory prices, as it is the case in one-sided markets that are characterized by horizontal differentiation, are decreasing in the degree of agent loyalty to a platform. Agents located very close to one or the other platform pay higher prices than those located in the middle. The highest price is $t$ and the lowest is 0. If we compare these prices with the no discriminatory prices, $\frac{t-\alpha_\ell f_m\left(\frac{1}{2}\right)}{2f_m\left(\frac{1}{2}\right)} + c$, we will see that it is possible that nearly all agents pay higher prices under price discrimination if $c$ is small and $\alpha_\ell$ is high enough. It then becomes obvious that there exists a threshold for the cross-group externality parameters above which perfect price discrimination benefits the platforms (relative to uniform pricing). The next Proposition summarizes the profit comparison.

**Proposition 3** ***(Profit comparison)*** *If the marginal cost is low enough and/or the cross-group externalities are strong enough, then perfect price discrimination is more profitable than uniform pricing.*

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[17] For this comparison, we focus on case ($ii$) of Proposition 2, because, as we mentioned above, under case ($i$) the standard ranking of profits between uniform and discriminatory prices from one-sided markets carries over to two-sided markets.

When the distribution is uniform, equilibrium profits increase with price discrimination if and only if $t/2 - c > t - (\alpha_1 + \alpha_2)/2$, (we compare (11) with (5)). This is the case if and only if,

$$t < (\alpha_1 + \alpha_2 - 2c) . \tag{12}$$

Furthermore, the necessary and sufficient condition for a market sharing equilibrium under uniform pricing to exist is $2t > (\alpha_1 + \alpha_2)$. Under perfect discrimination the condition we need, from Proposition 2 condition (8), is

$$t > c + \max\{\alpha_1, \alpha_2\} \text{ and } t < (\alpha_1 + \alpha_2).$$

Therefore, if $c$ is low and/or the externalities are strong, i.e., $c \leq \min\{\alpha_1, \alpha_2\}$, there exists a range of parameters such that uniform equilibrium prices are given by (4), discriminatory prices are given by (9) and price discrimination leads to higher profits. For example, if $\alpha_1 = \alpha_2 = \alpha$, for the above assertion to be true we need $t$ to be in the nonempty interval $[c + \alpha,\ 2(\alpha - c)]$.

The main idea behind the price and profit comparison is that the externalities are priced in the equilibrium uniform prices, but not in the discriminatory prices. When platforms lower their prices below marginal cost due to the externality, the natural price floor of zero is reached before the price goes down all the way to $c - 2\alpha_m(1 - F_m(x)) - \alpha_\ell(1 - 2F(x))$, for $x \leq 1/2$, (which is negative). So, strong enough externalities combined with low marginal cost (i.e., $c \leq \min\{\alpha_1, \alpha_2\}$) make perfect price discrimination more profitable (relative to uniform prices). This intuition does not rely on specific modeling assumptions and it is likely to hold in more general models.

Armstrong (2006a) compares price discrimination with uniform prices. In his model price discrimination is defined as the uniform pricing rule in our model, i.e., when platforms charge each group a different price. A uniform pricing rule in Armstrong's model is when a platform charges both groups the same price. Armstrong shows that price discrimination is profitable if and only if,

$$(t_1 - t_2)^2 > (\alpha_1 - \alpha_2)^2 . \tag{13}$$

Our condition (12) for a profitable price discrimination is qualitatively different from Armstrong's condition (13). In our case the levels matter, whereas in Armstrong's case the differences matter more. If the transportation parameters are the same across groups ($t_1 = t_2$), as it is the case in our model, then price discrimination is never profitable in Armstrong's model, while it may be in our model.

Finally, as it is well-known [e.g., Thisse and Vives (1988)], price discrimination, in one-sided markets when preferences are uniformly distributed and platforms are symmetric, *always* leads to a prisoners' dilemma. The profits under perfect price discrimination are $t/4$, while under a uniform pricing rule they are $t/2$ (with marginal cost $c$ equal to zero). In contrast, in two-sided markets, when (12) is satisfied perfect price discrimination yields higher profits than uniform prices.

### 3.4 Intermediate goods markets

When the market is an intermediate goods markets, e.g., B2B market, then our result implies that price discrimination will lead to higher input prices if and only if platforms have detailed information about the preferences of the participants and the marginal cost is low (and/or externalities are strong). To arrive at this result, we can assume that each firm is seeking to buy only one unit of the input and each input seller sells only one unit. The platforms facilitate the matching process between the two sides [as in Caillaud and Jullien (2003)]. Let's assume that platforms have very good (perfect) information about the agents. If platforms are allowed to customize their prices then firms end up paying higher prices for the right to trade a unit of the input. Now if we assume that the prices the participants pay to join a platform do not affect the bargaining process between an input supplier and a firm that will ensue once a matching takes place, then a higher price charged by a platform will lead to a higher overall price a firm will have to pay in order to acquire its input. If firms can pass part of this extra cost on to consumers, then price discrimination is anti-competitive. However, the reverse is true if platforms do not possess very detailed information about the participants (as in the uniform price case). In this case the cost of acquiring the input is reduced due to price discrimination.

### 3.5 Prices are private

So far we have assumed that each agent observes all prices before he chooses which platform to join. Here we assume that prices are private, in the sense that each agent only observes his own price. Given the cross-group externalities, beliefs are important in this case. What is an agent's belief about the offers made to other agents if he receives an out-of equilibrium offer? If beliefs are passive [e.g., McAfee and Schwartz (1994)] and price offers are secret, then equilibrium prices do not depend on the cross-group externalities. In particular, the equilibrium prices are,

$$\begin{aligned} p_{\ell A} &= t\,(1-2x)+c \text{ and } p_{\ell B}=c, \text{ for } x\leq\frac{1}{2} \text{ and} \\ p_{\ell A} &= c \text{ and } p_{\ell B}=t\,(2x-1)+c, \text{ for } x\geq\frac{1}{2}. \end{aligned}$$

To see this, suppose that a platform raises unilaterally its prices to a group of agents in its territory. Each agent, however, continues to believe that market shares will not change and *given* that agents are indifferent, in equilibrium, between the two platforms they will *all* switch to the rival platform. Hence, such a deviation is unprofitable. Price cuts would also be unprofitable because a reduction in price to an agent (or a group of agents) will not lead to higher market share. Thus, when prices are private equilibrium discriminatory prices do not depend on the externality parameter, as in the case of public prices and low marginal cost, i.e., case (*ii*) of Proposition 2. Given that no new insights are derived under the assumption of private prices, in the rest of the paper we go back to assuming that prices are public.

# 4 Agents are allowed to multi-home

We would like to investigate the robustness of the comparison between uniform and discriminatory pricing to an extension to the benchmark model. We allow agents to multi-home. Prices are public. Our result does not change qualitatively. One difference is that equilibrium discriminatory prices, when agents multi-home, depend positively (on average) on the cross-group externality. In order to cut down on the number of different cases that we will have to examine, we assume that the marginal cost $c$ is zero, which is the analogue of case (*ii*) in Proposition 2. We maintain the assumption that the distribution is uniform and we set $\alpha_1 = \alpha_2 = \alpha$. The indirect utility of an agent from group $\ell$ who is located at point $x \in [0, 1]$ is given by,[18]

$$U_\ell = \begin{cases} V + \alpha n^e_{mA} - tx - p_{\ell A}(x), & \text{if he joins platform } A \\ V + \alpha n^e_{mB} - t(1-x) - p_{\ell B}(x), & \text{if he joins platform } B \\ V + \theta + \alpha - t - p_{\ell A}(x) - p_{\ell B}(x) & \text{if he joins both platforms.} \end{cases} \tag{14}$$

The incremental maximum willingness to pay of an agent from group $\ell$ who chooses to multi-home by joining platform $k$ is given by $\theta + \alpha(1 - n^e_{mk})$. The first effect (*product variety* effect) is captured by the parameter $\theta$, where $\theta \in [0, V]$, and the second effect (*indirect externality* effect) is given by the term $\alpha(1 - n^e_{mk})$. For example, the utility of an agent who chooses to read a second newspaper increases because he gets to see more classified advertisements (indirect externality effect), but also because the second newspaper covers different issues than the first one (product variety effect). Or, a second credit card allows the holder to have transactions with more merchants (indirect externality effect), but also increases his credit limit (product variety effect). More generally, agent utility can increase, when he joins a second platform, independent of the indirect externality effect, because platforms are differentiated and agents value "variety."

The disutility of the agent who chooses to multi-home also increases and this is captured by the parameter $t$ ($t = tx + t(1-x)$). We assume that the total transportation cost is additive. Agents choose the option that gives them the highest indirect utility. We maintain the assumption that $t > \alpha$.

In general, there are three possible type of equilibria: i) single-homing, ii) partial multi-homing and iii) complete multi-homing. In the first equilibrium, no agent multi-homes, in the second one a fraction of the agents multi-homes and in the third one all agents multi-home. Due to symmetry the outcomes are the same across the two groups of agents. We will focus on the second type of equilibrium. Figure 1 is consistent with the partial multi-homing equilibrium and depicts the indirect utilities when prices within each group are uniform.

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[18] This utility specification has also been used in Kim and Serfes (2006) in a one-sided framework.

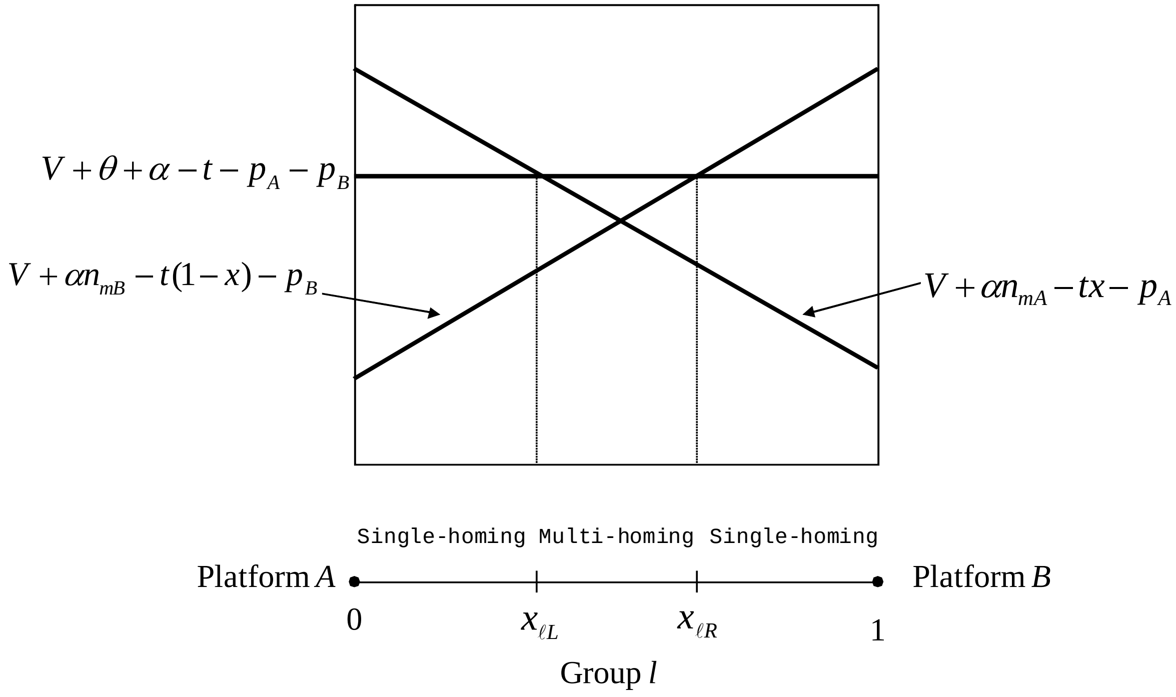

Figure 1: Indirect utilities: Partial multi-homing

There are two marginal agents in group $\ell$, where $\ell = 1, 2$, located at $x_{\ell L}$ and $x_{\ell R}$ respectively (the subscript $L$ stands for left and the subscript $R$ stands for right). $x_{\ell L}$ is indifferent between joining platform $A$ only and joining both platforms, whereas $x_{\ell R}$ is indifferent between joining both platforms and joining $B$ only. Agents in $[0, x_{\ell L}]$ and in $[x_{\ell R}, 1]$ single-home and in $[x_{\ell L}, x_{\ell R}]$ they multi-home.

## 4.1 Uniform prices (UP)

We assume that prices are constant within each group of agents. Let $p_{\ell A}$ and $p_{\ell B}$ denote platform $A$ and $B$'s price in group $\ell = 1, 2$ respectively. The marginal agents in group $\ell = 1, 2$ are defined by,

$$x_{\ell L}^{UP}: \quad V + \alpha n_{mA}^{e} - tx - p_{\ell A} = V + \theta + \alpha - t - p_{\ell A} - p_{\ell B},$$

$$x_{\ell R}^{UP}: \quad V + \alpha n_{mB}^{e} - t\,(1-x) - p_{\ell B} = V + \theta + \alpha - t - p_{\ell A} - p_{\ell B},$$

with $n_{mA}^{e} = x_{mR}^{UP}$, and $n_{mB}^{e} = 1 - x_{mL}^{UP}$, $m = 1, 2$. From these equations, we can obtain the marginal agents as follows,

$$x_{1L}^{UP} = \frac{(-t^2 + \alpha p_{2A} - \alpha\theta + t\alpha - tp_{1B} + t\theta)}{(\alpha - t)(t + \alpha)}$$

$$x_{1R}^{UP} = \frac{(\alpha\theta - t\alpha - t\theta + tp_{1A} + \alpha^2 - \alpha p_{2B})}{(\alpha - t)(t + \alpha)}$$

$$x_{2L}^{UP} = \frac{(t\theta + t\alpha - \alpha\theta + \alpha p_{1A} - tp_{2B} - t^2)}{(\alpha - t)(t + \alpha)}$$

$$x_{2R}^{UP} = \frac{(-t\theta - t\alpha + \alpha\theta + tp_{2A} - \alpha p_{1B} + \alpha^2)}{(\alpha - t)(t + \alpha)}.$$

Then, the platforms' problems are,

$$\max_{p_{1A},\, p_{2A}} \quad \pi_A = p_{1A}x_{1R}^{UP} + p_{2A}x_{2R}^{UP},$$

$$\max_{p_{1B},\, p_{2B}} \quad \pi_B = p_{1B}(1 - x_{1L}^{UP}) + p_{2B}(1 - x_{2L}^{UP}).$$

Solving the first order conditions, we can obtain the candidate equilibrium prices and profits as,

$$p_{1A}^* = p_{1B}^* = p_{2A}^* = p_{2B}^* = \frac{(t-\alpha)(\alpha+\theta)}{2t-\alpha},$$

$$\pi_A = \pi_B = \frac{2t(t-\alpha)(\alpha+\theta)^2}{(2t-\alpha)^2(\alpha+t)}. \tag{15}$$

In this candidate equilibrium,

$$0 < x_{\ell L}^{UP} \equiv \frac{2t^2 - \alpha^2 - t\theta}{(t+\alpha)(2t-\alpha)} < x_{\ell R}^{UP} \equiv \frac{t(\alpha+\theta)}{(t+\alpha)(2t-\alpha)} < 1,$$

if $\theta \in \left[\theta_1 \equiv \frac{6t(t-\alpha)}{(5t-\alpha)}, \theta_2 \equiv \frac{2t^2-\alpha^2}{t}\right]$.[19] That is, only the agents strictly in the middle multi-home (see figure 1), and we have partial multi-homing.

What is worth observing is that the equilibrium profits (15) are decreasing in $\alpha$ when $\alpha$ exceeds a given threshold (its specific value is omitted) and approach zero as $\alpha$ tends to $t$. For low values of $\alpha$ equilibrium profits can be increasing in the externality parameter. There are two opposing effects present as the indirect externality $\alpha$ increases. First, as in the single-homing case, incentives for unilateral price cuts increase. Second, agents are willing to pay more to join a second platform, which gives platforms incentives to raise their prices. This second effect arises because of the multi-homing assumption.[20] When $\alpha$ is high, the first effect is more dominant, while for low $\alpha$ the second effect may be more dominant.

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[19] The restrictions on the parameter $\theta$ can be understood as follows: i) $\theta < \theta_2$ guarantees that $x_{\ell L}^{UP} > 0$ and $x_{\ell R}^{UP} < 1$ and ii) $\theta > \frac{2t^2-\alpha^2-\alpha t}{2t}$ guarantees that $x_{\ell L}^{UP} < x_{\ell R}^{UP}$. Moreover, $\theta \geq \theta_1 \geq \frac{2t^2-\alpha^2-\alpha t}{2t}$, ensures that no global unilateral deviation is profitable. Profit functions are strictly concave when partial multi-homing is assumed, but a platform can deviate in a way that partial multi-homing vanishes. We want our solutions to the first order conditions to be immune from such a deviation. So, if $\theta$ falls in the above interval, then the candidate equilibrium becomes an equilibrium.

[20] Multi-homing changes the platforms' incentives to unilaterally change prices and therefore it generates new insights. Choi (2006), for example, shows that, when multi-homing is allowed in two-sided markets, tying can be welfare-enhancing because it induces more consumers to multi-home.

## 4.2 Perfect price discrimination (PD)

Now, we assume that platforms can identify the exact location of each agent. We first need to identify the locations of the marginal agents. The agent who is located at $x_{\ell L}^{PD}$ is indifferent between joining from platform $A$ only and joining from both platforms. This implies that this agent obtains zero utility from joining $B$, when platform $B$ is charging zero price. Then, $x_{1L}^{PD}$ is defined by,

$$x_{1L}^{PD}: \quad V + \alpha n_{2A}^{e} - tx - p_{1A} = V + \theta + \alpha - t - (p_{1A} + 0) \Rightarrow \theta + \alpha(1 - n_{2A}^{e}) = t(1 - x).$$

Similarly the other marginal agents are defined by,

$$x_{1R}^{PD}: \quad V + \alpha n_{2B}^{e} - t(1-x) - p_{1B} = V + \theta + \alpha - t - (0 + p_{1B}) \Rightarrow \theta + \alpha(1 - n_{2B}^{e}) = tx.$$

$$x_{2L}^{PD}: \quad V + \alpha n_{1A}^{e} - tx - p_{2A} = V + \theta + \alpha - t - (p_{2A} + 0) \Rightarrow \theta + \alpha(1 - n_{1A}^{e}) = t(1 - x).$$

$$x_{2R}^{PD}: \quad V + \alpha n_{1B}^{e} - t(1-x) - p_{2B} = V + \theta + \alpha - t - (0 + p_{2B}) \Rightarrow \theta + \alpha(1 - n_{1B}^{e}) = tx.$$

Also, $n_{mA}^{e} = x_{mR}^{PD}$, and $n_{mB}^{e} = 1 - x_{mL}^{PD}$, $m = 1, 2$. From these equations, we can obtain the locations of the marginal agents, $\ell = 1, 2$,

$$x_{\ell L}^{PD} = \frac{t - \theta}{t + \alpha} \text{ and } x_{\ell R}^{PD} = \frac{\theta + \alpha}{t + \alpha}.$$

Then, $n_{mA} = n_{mB} = \frac{\theta+\alpha}{t+\alpha}$. When $x \in (x_{\ell L}^{PD}, x_{\ell R}^{PD})$, both platforms set prices so that all agents are indifferent between joining both platforms or that platform alone,

$$V + \alpha n_{mA}^{e} - tx - p_{\ell A}(x) = V + \alpha n_{mB}^{e} - t(1-x) - p_{\ell B}(x) = V + \theta + \alpha - t - p_{\ell A}(x) - p_{\ell B}(x).$$

From this equation, we can solve for $p_{\ell A}(x)$ and $p_{\ell B}(x)$. When $\theta \in \left[\frac{t-\alpha}{2}, t\right]$ we have $0 \leq x_{\ell L}^{PD} \leq x_{\ell R}^{PD} \leq 1$, and partial multi-homing is an equilibrium. The equilibrium prices and profits are given by,

$$p_{\ell A}^{*}(x) = \begin{cases} \frac{t(\theta+\alpha-x(t+\alpha))}{(t+\alpha)}, & \text{for } x \in \left[x_{\ell L}^{PD} \equiv \frac{t-\theta}{t+\alpha}, x_{\ell R}^{PD} \equiv \frac{\theta+\alpha}{t+\alpha}\right] \\ t(1-2x), & \text{for } x \leq x_{\ell L}^{PD} \equiv \frac{t-\theta}{t+\alpha} \\ 0, & \text{for } x \geq x_{\ell R}^{PD} \equiv \frac{\theta+\alpha}{t+\alpha} \end{cases} \tag{16}$$

and

$$p_{\ell B}^{*}(x) = \begin{cases} \frac{t(\theta-t+x(t+\alpha))}{(t+\alpha)}, & \text{for } x \in \left[x_{\ell L}^{PD} \equiv \frac{t-\theta}{t+\alpha}, x_{\ell R}^{PD} \equiv \frac{\theta+\alpha}{t+\alpha}\right] \\ t(2x-1), & \text{for } x \geq x_{\ell R}^{PD} \equiv \frac{\theta+\alpha}{t+\alpha} \\ 0, & \text{for } x \leq x_{\ell L}^{PD} \equiv \frac{t-\theta}{t+\alpha}. \end{cases}$$

$$\pi_A = \pi_B = \frac{t\left(t^2 - 2t\theta + 2\theta^2 + 2\alpha\theta + \alpha^2\right)}{(t+\alpha)^2}. \tag{17}$$

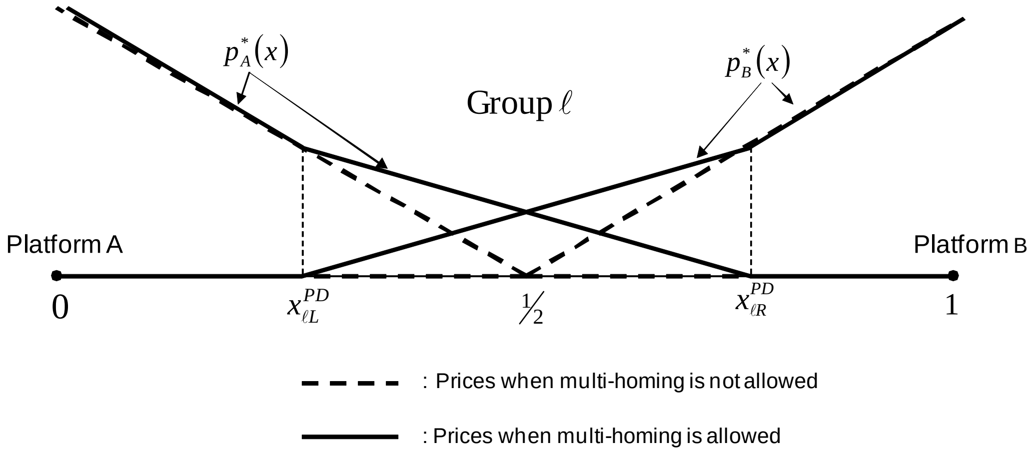

Figure 2: Price comparison under perfect discrimination between when multi-homing is allowed and when it is not

The solid lines in figure 2 depict the equilibrium prices as given by (16). (The dashed lines on the same figure depict the equilibrium prices under perfect discrimination when multi-homing is not allowed, as given by (9)). The price functions under multi-homing exhibit two kinks, one at $x_{\ell L}^{PD}$ and the other at $x_{\ell R}^{PD}$. The agents that multi-home are located in the interval $\left[x_{\ell L}^{PD}, x_{\ell R}^{PD}\right]$. The agents in $\left[0, x_{\ell L}^{PD}\right]$ join platform $A$ exclusively and the agents in $\left[x_{\ell R}^{PD}, 1\right]$ join platform $B$ exclusively. The differences between when multi-homing is not allowed and when it is are the following: i) when multi-homing is allowed platforms make more sales (i.e., $x_{\ell L}^{PD} < \frac{1}{2}$ and $x_{\ell R}^{PD} > \frac{1}{2}$) and ii) equilibrium prices are (weakly) higher when agents are allowed to multi-home. In particular, the prices are the same between the two regimes for the agents who join one platform exclusively, but higher when multi-homing is allowed for the agents who join both platforms. This is because under multi-homing each agent is indifferent between joining one platform exclusively and joining both platforms, which softens price competition.[21]

It can be readily verified that equilibrium prices (for the agents who multi-home) and profits increase with $\alpha$. When some agents multi-home equilibrium prices are affected by the cross-group externalities. The reason is that equilibrium prices keep the agents who multi-home indifferent between joining one and two platforms. In other words, platforms in equilibrium extract all the incremental surplus from the agents who choose to multi-home. Hence, the externalities are priced in the discriminatory equilibrium prices. (Recall that when multi-homing is not allowed, perfect discriminatory prices are free of the externality parameters, when $c < \min\{\alpha_1, \alpha_2\}$ (case (*ii*) of Proposition 2), which holds here because we have assumed that $c = 0$). As the indirect externality

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[21] This can be better seen by observing that in the multi-homing region the prices are falling slower (slope is equal to $-t$), as we move in the middle of the intervals, than when multi-homing is not allowed (slope is equal to $-2t$). In the latter case a unilateral price cut induces agents to switch platforms (*business stealing*), whereas in the former case a similar price cut results in more agents joining both platforms (*demand creation*).

increases the incremental benefit from joining a second platform also increases. This allows platforms to sustain higher equilibrium discriminatory prices as a function of $\alpha$. On the other hand, as expected, the prices for the agents who single-home are not affected by $\alpha$.

## 4.3 Comparing uniform pricing and perfect PD

### 4.3.1 Price and profit comparison

In this comparison, for brevity, we focus implicitly on the parameter range that is common between the two parameter ranges for which (15) and (17) constitute an equilibrium. By comparing (15) with (17), it can be shown that price discrimination is always more profitable.

This sharp prediction is very likely to be model specific. However, we believe that the effects we have identified at the end of each of the previous two subsections are likely to hold in more general models. These effects yield the following predictions as the indirect externality $\alpha$ increases: i) uniform prices decrease (after the externality exceeds a given threshold) and ii) discriminatory prices increase. Hence, price discrimination should yield higher profits than uniform pricing at least when these externalities are strong enough. This result echoes the prediction from our benchmark model where multi-homing is not allowed.

### 4.3.2 Social welfare

Due to multi-homing, aggregate demand is elastic, so social welfare comparisons are meaningful. The equilibrium under perfect price discrimination is efficient. This can be seen as follows. Given symmetry and the fact that each agent who single-homes joins the closest platform, what matters for efficiency is total output, i.e., the number of agents who multi-home. First, note that, in the partial multi-homing case, each platform is a local monopoly, because platforms do not compete head-on for agents. Second, each platform extracts each agent's entire incremental surplus from multi-homing (i.e., from joining a second platform). Therefore, the private benefit is aligned with the social benefit, which implies that social surplus is maximized under perfect price discrimination. If the equilibrium under uniform prices differs, then we can conclude that the uniform price equilibrium is inefficient. This is indeed the case. Comparing $x_{\ell R}^{UP}$ and $x_{\ell R}^{PD}$, we can easily find that,

$$x_{\ell R}^{UP} < x_{\ell R}^{PD},$$

since $t > \alpha$.

By symmetry, we can show that,

$$x_{\ell L}^{UP} < x_{\ell L}^{PD}.$$

This implies that more agents multi-home under perfect price discrimination than under uniform pricing. We can conclude by stating that perfect price discrimination is efficient, while uniform prices result in an inefficient equilibrium (less output than the first-best).

# 5 Conclusion

We examine the issue of price discrimination in two-sided markets. We assume that there are two symmetric horizontally differentiated platforms and two groups of agents. Agents from both groups must join a platform for successful trades to take place. Platforms possess information about the agents' brand preferences which can be used to customize prices. We derive new results regarding the equilibrium discriminatory prices. When indirect externality is weak (relative to marginal cost), contrary to predictions from one-sided models, equilibrium prices are not distribution-free. Moreover, they do depend on both group externalities, as opposed to uniform prices in two-sided models which only depend on the other-group externality.

Then, we compare the profitability of price discrimination with uniform pricing in a two-sided market. Our main result indicates that when the indirect externality is strong perfect price discrimination yields higher profits relative to those under uniform prices. This result is in sharp contrast with the prisoners' dilemma prediction in oligopolistic one-sided price discrimination models.

Our results have new and clear managerial implications, regarding pricing strategies in two-sided markets. Moreover, in a two-sided market, firms may have stronger incentives to collect consumer information which allows them to price discriminate. This should happen when the cross-group externality is strong relative to marginal cost.

# Appendix A: Proof of Proposition 1

The location of the marginal agent from group $\ell$, who is indifferent between $A$ and $B$, is given by,

$$\begin{aligned} V+\alpha_\ell n_{mA}^e - tx - p_{\ell A} &= V+\alpha_\ell n_{mB}^e - t(1-x) - p_{\ell B} \Rightarrow \\ x_\ell &= \frac{\alpha_\ell\left(n_{mA}^e - n_{mB}^e\right) - p_{\ell A} + p_{\ell B} + t}{2t}. \end{aligned} \tag{18}$$

where $n_{mA}^e = F_m(x_\ell^e)$ and $n_{mB}^e = 1 - F_m(x_\ell^e)$. Therefore, the implicit functions for the market shares are given by,

$$x_1 = \frac{\alpha_1\left[2F_2(x_2^e)-1\right]-(p_{1A}-p_{1B})+t}{2t} \text{ and } x_2 = \frac{\alpha_2\left[2F_1(x_1^e)-1\right]-(p_{2A}-p_{2B})+t}{2t}.$$

Since expectations are rational we must have $x_\ell = x_\ell^e$, or $n_{mk}^e = n_{mk}$. By invoking the implicit function theorem we can derive the effect of prices on the market shares,

$$\begin{aligned} \frac{\partial x_1}{\partial p_{1A}} &= \frac{\partial x_2}{\partial p_{2A}} = -\frac{t}{2\left[t^2-\alpha_1\alpha_2 f_1(x_1) f_2(x_2)\right]}, \frac{\partial x_1}{\partial p_{2A}} = -\frac{\alpha_1 f_2(x_2)}{2\left[t^2-\alpha_1\alpha_2 f_1(x_1) f_2(x_2)\right]} \\ \text{and } \frac{\partial x_2}{\partial p_{1A}} &= -\frac{\alpha_2 f_1(x_1)}{2\left[t^2-\alpha_1\alpha_2 f_1(x_1) f_2(x_2)\right]}. \end{aligned} \tag{19}$$

For the Jacobian of the system of the implicit functions to have a non-zero determinant it must be that $t^2-\alpha_1\alpha_2 f_1(x_1) f_2(x_2) \neq 0$, for all $x_1$ and $x_2$. We further assume that $t^2-\alpha_1\alpha_2 f_1(x_1) f_2(x_2) > 0$, for all $x_1$ and $x_2$.

The platforms' profit functions are given by,

$$\begin{aligned}\pi_A &= (p_{1A}-c)n_{1A}+(p_{2A}-c)n_{2A}=(p_{1A}-c)F_1(x_1)+(p_{2A}-c)F_2(x_2) \text{ and}\\ \pi_B &= (p_{1B}-c)n_{1B}+(p_{2B}-c)n_{2B}=(p_{1B}-c)\left[1-F_1(x_1)\right]+(p_{2B}-c)\left[1-F_2(x_2)\right].\end{aligned}$$

The first order conditions of platform $A$ are given by,

$$\begin{aligned}\frac{\partial \pi_A}{\partial p_{1A}} &= F_1(x_1)+(p_{1A}-c)f_1(x_1)\frac{\partial x_1}{\partial p_{1A}}+(p_{2A}-c)f_2(x_2)\frac{\partial x_2}{\partial p_{1A}}=0,\\ \frac{\partial \pi_A}{\partial p_{2A}} &= F_2(x_2)+(p_{2A}-c)f_2(x_2)\frac{\partial x_2}{\partial p_{2A}}+(p_{1A}-c)f_1(x_1)\frac{\partial x_1}{\partial p_{2A}}=0.\end{aligned}$$

Each first order condition has three terms. Suppose platform $A$ lowers its price to group $\ell$ agents. The first two terms in each first order condition capture the reduction in inframarginal rents and the increase in marginal agents respectively. Since more agents from group $\ell$ join platform $k$, platform $k$ becomes more attractive to the members of group $m$. The third term represents the additional revenue from the increase in the number of agents from group $m$ that join platform $k$.

We look for a symmetric equilibrium where platforms charge the same prices to each group. We assume that regularity conditions hold so that a symmetric sharing equilibrium exists.[22] Using (19), the symmetric solution to the system of first order conditions is given by,

$$p^*_{1A}=p^*_{1B}=\frac{t-\alpha_2 f_1\left(\frac{1}{2}\right)}{f_1\left(\frac{1}{2}\right)}+c \text{ and } p^*_{2A}=p^*_{2B}=\frac{t-\alpha_1 f_2\left(\frac{1}{2}\right)}{f_2\left(\frac{1}{2}\right)}+c.$$

The equilibrium profits are,

$$\pi_A=\pi_B=\frac{t-\alpha_2 f_1\left(\frac{1}{2}\right)}{2f_1\left(\frac{1}{2}\right)}+\frac{t-\alpha_1 f_2\left(\frac{1}{2}\right)}{2f_2\left(\frac{1}{2}\right)}.$$

## Appendix B: Proof of Proposition 2

Given the length of this proof, we first provide an outline. The proof is divided into 4 subsections: **B1-B3** deal with case $(i)$ of the Proposition and case $(ii)$ is considered in **B4**.

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[22] We were not able to come up with clean conditions on the distribution functions that would ensure the strict concavity (or quasi-concavity) of the objective functions. For instance, the monotone hazard rate property is not enough. When the distribution is uniform, then the profit functions are strictly concave provided that $2t > (\alpha_1+\alpha_2)$. When this condition holds, then a symmetric sharing equilibrium exists. Otherwise, one platform may corner the entire market.

- In **B1**, we provide two specific equilibria when $c$ is high/$\alpha_1$ and $\alpha_2$ are low: the 'highest prices' one and the 'lowest prices' one.

  The 'highest price' equilibrium is presented as case (*i*) in the Proposition. We also explain how the equilibrium prices are constructed in both equilibria.

- In **B2**, we show that the 'highest price' equilibrium is indeed an equilibrium.

  We first showed that when firms choose prices according to case (*i*), agents served by either platform are connected on or off equilibrium. We then show that neither platform has incentive to deviate.

- In **B3**, we explain the idea of the 'lowest prices' equilibrium but skip the details, and a continuum of equilibria with prices in between the 'highest prices' and the 'lowest prices.'

- In **B4**, we prove case (*ii*) in the Proposition, first by showing that agents served by either platform are connected, then by showing that neither platform has incentive to deviate.

## B1. Two specific equilibria when $c$ is high/$\alpha_1$ and $\alpha_2$ are low

We will show that under perfect price discrimination there is a continuum of (symmetric) price equilibria, that can be Pareto ranked (from the platforms' perspective) from the 'highest price' one to the 'lowest price' one. Case *A)* below refers to the 'highest prices' equilibrium and case *B)* to the 'lowest prices'. Then, we assume that platforms can coordinate on the one that yields the highest prices and hence profits and this is what we show in Proposition 2, that is, case (*i*).

We assume that marginal cost and/or low cross group externality, $c \geq \max\{2\alpha_1+\alpha_2, \alpha_1+2\alpha_2\}$ so the prices, in the 'highest prices' equilibrium, are non-negative. We also need $t \geq 2(\alpha_1+\alpha_2)f_\ell(x)$, $\ell = 1, 2$ to prevent deviation. A weaker (thus redundant) condition $t \geq (\alpha_1+\alpha_2)f_\ell(x)$ is needed for 'connectedness.'

*A)* **'Highest prices'** equilibrium

$$\begin{aligned} p^*_{\ell A}(x) &= t\,(1-2x) + p^*_{\ell B}(x) \text{ and } p^*_{\ell B} = c - 2\alpha_m(1-F_m(x)) - \alpha_\ell(1-2F_m(x)), \text{ for } x \leq \frac{1}{2} \\ p^*_{\ell A}(x) &= c - 2\alpha_m F_m(x) - \alpha_\ell(2F_m(x)-1) \text{ and } p^*_{\ell B}(x) = p^*_{\ell A} + t\,(2x-1)\,, \text{ for } x \geq \frac{1}{2}. \end{aligned}$$

*B)* **'Lowest prices'** equilibrium

$$\begin{aligned} p^{**}_{\ell A}(x) &= c - 2\alpha_m F_m(x) - \alpha_\ell(2F_m(x)-1) \text{ and } p^{**}_{\ell B}(x) = p^{**}_{\ell A}(x) - t(1-2x), \text{ for } x \leq \frac{1}{2} \\ p^{**}_{\ell A}(x) &= p^{**}_{\ell B}(x) - t(2x-1) \text{ and } p^{**}_{\ell B}(x) = c - \alpha_\ell(1-2F_m(x)) - 2\alpha_m(1-F_m(x)), \text{ for } x \geq \frac{1}{2}. \end{aligned}$$

Before we prove the Proposition, we first want to: (1) explain how the prices in *(A)* and *(B)* are constructed; and (2) point out a unique feature of these discriminatory equilibrium prices, which does not exist in the uniform price equilibrium.

How do we construct the equilibrium prices?

They are designed so the following conditions hold:

- On equilibrium path
  - There is symmetry: $p^*_{\ell A}(x) = p^*_{\ell B}(1-x)$, $p^{**}_{\ell A}(x) = p^{**}_{\ell B}(1-x)$, $\forall x \in [0,1]$, $\ell = 1,2$.
  - The two platforms split both groups evenly.
  - Each agent is indifferent between buying from either platform.

- Off equilibrium path
  - Assume that platform $A$ deviates. Let $x_\ell$ denote the marginal agent in group $\ell = 1,2$, and let $\pi^{dev}_A(x_1,x_2)$ denote platform $A$'s deviation profit.
  - We want platform $A$ to choose $x_1 = x_2 = x$. With $x_1 = x_2$, let $\pi_A(x)$ denote its corresponding deviation profit.
  - In the 'highest prices' equilibrium provided in $A$),

    $$\pi_A(x = 1/2) = \pi_A(x > 1/2) > \pi_A(x < 1/2).$$

    That is, platform $A$ is indifferent between whether or not to sign more agents in the other platform's turf, but strictly prefers not to lose agents in its own turf.
  - $\pi_A(x = 1/2) = \pi_A(x > 1/2)$ also governs how the equilibrium prices are constructed. When platform $A$ deviates, let $x_\ell$ denote the marginal agents in group $\ell = 1,2$. We want the optimal deviation to be $x_1 = x_2 = x$. The benefit of signing an extra agent in group $\ell$ is $2\alpha_m F_m(x)$. The cost is to make an extra sale at price below marginal cost, $p^{dev}_{1A}(x) - c$. Equilibrium prices are constructed so that when $\min\{x_1,x_2\} \geq 1/2$, we have

    $$\begin{cases} p^{dev}_{\ell A}(x_\ell) - c + 2\alpha_m F_m(x_m) = 0 \Rightarrow \frac{\partial \pi^{dev}_A}{\partial x_\ell} = 0, & \text{if } x_1 = x_2 \geq 1/2, \\ p^{dev}_{\ell A}(x_\ell) - c + 2\alpha_m F_m(x_m) > 0 \Rightarrow \frac{\partial \pi^{dev}_A}{\partial x_\ell} > 0, & \text{if } x_m > x_\ell \geq 1/2. \end{cases}$$

    The first equation implies that $\pi_A(x = 1/2) = \pi_A(x > 1/2)$. What remains is to verify that $\pi_A(x = 1/2) = \pi_A(x < 1/2)$.
  - In the second equilibrium given in $B$),

    $$\pi_A(x = 1/2) = \pi_A(x < 1/2) > \pi_A(x > 1/2).$$

    That is, platform $A$ is indifferent between whether or not to sign fewer agents in its own turf, but strictly prefers not to sign more agents in the other platform's turf. Again the equality $\pi_A(x = 1/2) = \pi_A(x < 1/2)$ governs how the equilibrium prices are constructed and what remains is to verify that $\pi_A(x = 1/2) > \pi_A(x > 1/2)$.

A unique feature of the discriminatory prices

In the uniform pricing case, price in each group (say $\ell$) only depends on the externality parameter of the other group ($\alpha_m$),

$$p^*_{\ell A} = p^*_{\ell B} = \frac{t - \alpha_m f_\ell(\frac{1}{2})}{f_\ell(\frac{1}{2})}.$$

However, under perfect price discrimination, price in each group depends on the externalities of both groups. For example, when $x \leq \frac{1}{2}$,

$$p^*_{\ell A}(x) = t(1-2x) + c - 2\alpha_m(1 - F_m(x)) - \alpha_\ell(1 - 2F_m(x)), \quad \ell \neq m = 1, 2.$$

Let's see why. We have explained that these equilibrium prices are designed so that when a platform (say $A$) deviates, it always wants to choose $x_1 = x_2$. When $x_1 = x_2 \geq \frac{1}{2}$, we want platform $A$ to be indifferent between (i) staying put; (ii) signing an extra agent in either group; (iii) sign an extra agent in both groups. Let's consider signing an extra agent in group $\ell = 1, 2$ for example. Let $p^*_{\ell A}(x)$ denote the deviation price for an agent located at $x \in [0, x_\ell]$ in group $\ell = 1, 2$. The cost of signing an extra agent (at $x_1$) is an extra sale at price $p^{dev}_{\ell A}(x_1) < c$. Clearly this deviation price will depend on platform $A$'s market share in group $m \neq \ell$. This is where $\alpha_\ell(1 - 2F_m(x))$ comes from. The benefit is that the increase of market share in group $\ell$ will be enjoyed by agents in group $m$ who buy from platform $A$, in the form of $2\alpha_m F_m(x_m)$. Combined, both $\alpha_1$ and $\alpha_2$ enter into the equilibrium prices.[23]

Why is this feature absent in the uniform pricing case? Recall that equilibrium is symmetric and $x_1 = x_2 = \frac{1}{2}$. No firm has incentive to change its price in either group. Note that, when $x_1 = x_2 = \frac{1}{2}$, $F_m(x) = \frac{1}{2}$. Therefore, in the expression for $p^*_{\ell A}$,

$$-2\alpha_m(1 - F_m(x)) - \alpha_\ell(1 - 2F_m(x)) = -\alpha_m,$$

and $\alpha_\ell$ disappears.

### B2. Show that A) is an equilibrium

Recall that the prices are

$$\begin{aligned} p^*_{\ell A} &= t\,(1-2x) + p^*_{\ell B} \text{ and } p^*_{\ell B} = c - 2\alpha_m(1 - F_m(x)) - \alpha_\ell(1 - 2F_m(x)), \text{ for } x \leq \frac{1}{2} \\ p^*_{\ell A} &= c - 2\alpha_m F_m(x) - \alpha_\ell(2F_m(x) - 1) \text{ and } p^*_{\ell B} = p^*_{\ell A} + t\,(2x - 1)\,, \text{ for } x \geq \frac{1}{2}. \end{aligned}$$

We prove **A)** is an equilibrium in two steps. In step 1, we show that in any deviation, platform $A$ must be serving agents who are 'connected' in each group (a concept that will be defined below).

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[23] This implies that, when marginal cost is sufficiently higher relative to externality parameters, perfect price discrimination intensifies competition not only in the $t$ term, but also in the terms involving externality parameters. That is, the two effects of price discrimination on firms' profits are both negative.

Then, in step 2, we show that under this deviation structure, platform $A$ has no incentive to deviate. Due to symmetry, we only look at the case of platform $A$ deviating.

**Step 1: Agents served by platform $A$ are connected.**

By connected, we mean that if a platform, say platform $A$, has signed up agent $x'$ in group $\ell = 1, 2$, then it will also sign up all agents with $x < x'$ in group $\ell$.[24] In the equilibrium which is symmetric, each platform serves its own half of the market, thus the agents served by each platform are 'connected'. Note that the equilibrium price $p^*_{\ell A}(x)$ decreases with $x$,

$$\frac{dp^*_{\ell A}(x)}{dx} = \begin{cases} -2[t - (\alpha_1 + \alpha_2)]f_m(x) < 0, & \text{if } x \leq 1/2, \\ -2(\alpha_1 + \alpha_2)f_m(x) < 0, & \text{if } x > 1/2. \end{cases}$$

Next, we prove that the same property (connectedness) holds for deviations as well. Suppose that platform $A$ deviates. Consider a setup where platform $A$'s market share in group 2, after a deviation, is $s_2$. Suppose now platform $A$ is contemplating to sign up one more agent located at $x$ in group 1 with $x > 1/2$. Let $p^{dev}_{1A}(x)$ denote the maximum deviation price if it wants to attract this agent. The benefit of signing any agent $x$ in group 1 (enjoyed by all agents in group 2 who buy from platform $A$) is the same, so the choice only depends on the cost of signing them, $p^{dev}_{1A}(x) - c$. The deviation price $p^{dev}_{1A}(x)$ is decided by the following equation

$$V - p^{dev}_{1A}(x) - tx + \alpha_1 s_2 = V - p^*_{1B}(x) - t(1-x) + \alpha_1(1 - s_2).$$

This implies that,

$$\begin{aligned} p^{dev}_{1A}(x) &= t(1-2x) + \alpha_1(2s_2 - 1) + p^*_{1B}(x) \\ &= [t(1-2x) + \alpha_1(2s_2 - 1)] + [c - 2\alpha_2 F_2(x) - \alpha_1(2F_2(x) - 1) + t(2x - 1)] \\ &= c + \alpha_1(2s_2 - 1) - 2\alpha_2 F_2(x) - \alpha_1(2F_2(x) - 1). \end{aligned}$$

Then,

$$\frac{dp^{dev}_{1A}(x)}{dx} = -2(\alpha_1 + \alpha_2)f_2(x) < 0.$$

Since deviation prices decrease with $x$ when $x \geq 1/2$, it is always more profitable for platform $A$ to sign up agents closer to 1/2 first. Therefore, agents served by platform $A$ in the right segment ($[1/2, 1]$), after a deviation, are connected.

Next, we prove that agents in the left segment $[0, 1/2]$ are also connected after a deviation on the part of platform $A$.

Following similar logic as above, we can derive the deviation price as the following,

$$\begin{aligned} p^{dev}_{1A}(x) &= t(1-2x) + \alpha_1(2s_2 - 1) + p^*_{1B}(x) \\ &= [t(1-2x) + \alpha_1(2s_2 - 1)] + [c - 2\alpha_2(1 - F_2(x)) - \alpha_1(1 - 2F_2(x))]. \end{aligned}$$

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[24] This rules out the case where a platform finds it profitable to sign up agents who belong to disjoint intervals.

Then,

$$\frac{dp_{1A}^{dev}(x)}{dx} = -2[t-(\alpha_1+\alpha_2)f_2(x)] < 0.$$

The above derivative is negative because we have assumed strong horizontal differentiation, i.e., $t > 2(\alpha_1+\alpha_2)f_\ell(x), \ell = 1, 2$. Since deviation prices decrease with $x$ when $x \leq 1/2$, it's always more profitable for platform $A$ to sign up agents closer to 0 first. Therefore, agents in group 1 served by platform $A$ in the left segment are connected as well. Similarly one can show that agents in group 2 served by platform $A$ are also connected.

**Step 2: Show that platform $A$ has no incentive to deviate**

We have shown that agents served by either platform in each group are connected. Let $x_\ell$ denote the marginal agent in group $\ell = 1, 2$. This implies that, in group $\ell = 1, 2$, agents in $[0, x_\ell]$ are served by platform $A$, while agents in $(x_\ell, 1]$ are served by platform $B$. Platform $A$ has three types of deviations:

- Type 1 deviation: signing up fewer agents in both groups, or $\max\{x_1, x_2\} \leq 1/2$.
- Type 2 deviation: signing up more agents in both groups, or $\min\{x_1, x_2\} \geq 1/2$.
- Type 3 deviation: signing up more agents in one group, but fewer agents in the other group, or $\max\{x_1, x_2\} \geq 1/2 \geq \min\{x_1, x_2\}$.

Next we show that neither type of deviation can be profitable for platform $A$.

Type 1 deviation: $\max\{x_1, x_2\} \leq 1/2$.

In this case, platform $A$ signs up fewer agents in both groups (relative to the candidate equilibrium share which is 1/2). We will show that this is dominated by $x_1 = x_2 = 1/2$, and thus platform $A$ has no incentive for such a deviation.

First, we derive platform $A$'s deviation price in each group, starting with group 1. Let $p_{1A}^{dev}(x)$ denote platform $A$'s deviating price targeting the agent located at $x \in [0, x_1]$. Since platform $A$ is the deviating platform, it will choose $p_{1A}^{dev}(x)$ such that the agent is indifferent between the two platforms, that is,[25]

$$V - p_{1A}^{dev}(x) - tx + \alpha_1 F_2(x_2) = V - p_{1B}^*(x) - t(1-x) + \alpha_1(1 - F_2(x_2)).$$

Solving for $p_{1A}^{dev}(x)$, we can obtain,

$$\begin{aligned} p_{1A}^{dev}(x) &= [t(1-2x) - \alpha_1(1-2F_2(x_2))] + p_{1B}^*(x) \\ &= [t(1-2x) - \alpha_1(1-2F_2(x_2))] + [c - 2\alpha_2(1-F_2(x)) - \alpha_1(1-2F_2(x))]. \end{aligned}$$

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[25] Since the agents are connected, platform $A$ will sign all agents $x \leq x_1$ and nobody else in group 1. For any $x > x_1$, since platform $A$ will not make sales, we do not need derive the deviation price since all we need is for the deviation price to be sufficiently high, e.g., infinite. Another candidate is the equilibrium price $p_{1A}^*(x)$ for $x > x_1$.

Similarly, we can obtain

$$\begin{aligned} p_{2A}^{dev}(x) &= [t(1-2x) - \alpha_2(1-2F_1(x_1))] + p_{2B}^*(x) \\ &= [t(1-2x) - \alpha_2(1-2F_1(x_1))] + [c - 2\alpha_1(1-F_1(x)) - \alpha_2(1-2F_1(x))]. \end{aligned}$$

Note that

$$\frac{\partial p_{\ell A}^{dev}(x)}{\partial x_\ell} = 0, \quad \frac{\partial p_{mA}^{dev}(x)}{\partial x_\ell} = 2\alpha_m f_\ell(x_\ell), \quad \ell \neq m = 1, 2.$$

Platform $A$'s deviation profit is

$$\pi_A^{dev}(x_1, x_2) = \int_0^{x_1} \left(p_{1A}^{dev}(x) - c\right) f_1(x)dx + \int_0^{x_2} \left(p_{2A}^{dev}(x) - c\right) f_2(x)dx$$

Therefore,

$$\begin{aligned} \frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} &= \left(p_{1A}^{dev}(x_1) - c\right) f_1(x_1) + \int_0^{x_2} \frac{\partial p_{2A}^{dev}(x)}{\partial x_1} f_2(x)dx \\ &= \left(p_{1A}^{dev}(x_1) - c\right) f_1(x_1) + \int_0^{x_2} 2\alpha_2 f_1(x_1) f_2(x)dx \\ &= \left(p_{1A}^{dev}(x_1) - c\right) f_1(x_1) + 2\alpha_2 f_1(x_1) F_2(x_2) \\ &= \left(p_{1A}^{dev}(x_1) - c + 2\alpha_2 F_2(x_2)\right) f_1(x_1) \\ &= [t(1-2x_1) - \alpha_1(1-2F_2(x_2)) - 2\alpha_2(1-F_2(x_1)) - \alpha_1(1-2F_2(x_1)) + 2\alpha_2 F_2(x_2)]\, f_1(x_1) \\ &= [t(1-2x_1) - 2(\alpha_1+\alpha_2)(1-F_2(x_1)-F_2(x_2))]\, f_1(x_1) \end{aligned}$$

Assume that $x_1 < x_2$, then $F_2(x_1) < F_2(x_2)$, and

$$\frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} \geq [t(1-2x_1) - 2(\alpha_1+\alpha_2)(1-2F_2(x_1))]\, f_1(x_1).$$

Let $g(x_1)$ denote the term inside the square bracket, i.e.,

$$g(x_1) \equiv t(1-2x_1) - 2(\alpha_1+\alpha_2)(1-2F_2(x_1)).$$

$g(x_1)$ decreases with $x_1$ since

$$\frac{dg(x_1)}{dx_1} = -2[t - 2(\alpha_1+\alpha_2)f_2(x_1)] < 0.$$

Combining this with $g(x_1 = 1/2) = 0$, it must be that

$$g(x_1 < 1/2) > 0.$$

Therefore,

$$\frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} > 0.$$

Whenever $\max\{x_1, x_2\} < 1/2$, if $x_1 < x_2$, then platform $A$ has incentive to increase $x_1$. Similarly, it can be shown that if $x_1 > x_2$, then platform $A$ has incentive to increase $x_2$. That is, platform $A$ has incentive to equalize $x_1$ and $x_2$.

It can be further seen that at $x_1 = x_2 = x < 1/2$,

$$\frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} > 0, \quad \frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_2} > 0,$$

and platform $A$ has incentive to increase both $x_1$ and $x_2$. To summarize, platform $A$ has no incentive for Type 1 deviation.

<u>Type 2 deviation:</u> $\min\{x_1, x_2\} \geq 1/2$.

In this case, platform $A$ signs up more agents in both groups. We will show that

$$\pi_A(x_1 = x_2 = 1/2) = \pi_A(x_1 = x_2 > 1/2) > \pi_1(x_1 \neq x_2).$$

Thus platform $A$ has no incentive for such a deviation.

First, we derive platform $A$'s deviation price in each group, starting with group 1. Let $p_{1A}^{dev}(x)$ denote platform $A$'s deviating price targeting the agent located at $x \in [0, x_1]$. The agent located at $x$ is indifferent between the two platforms if and only if[26]

$$V - p_{1A}^{dev}(x) - tx + \alpha_1 F_2(x_2) = V - p_{1B}^*(x) - t(1-x) + \alpha_1(1 - F_2(x_2)),$$

$$\Rightarrow p_{1A}^{dev}(x) = [t(1-2x) - \alpha_1(1 - 2F_2(x_2))] + p_{1B}^*(x).$$

The formula for $p_{1B}^*(x)$ depends on whether $x \leq 1/2$ or $x > 1/2$ as the following,

$$p_{1B}^*(x) = \begin{cases} c - 2\alpha_2(1 - F_2(x)) - \alpha_1(1 - 2F_2(x)), & \text{if } x \leq 1/2, \\ c - 2\alpha_2 F_2(x) - \alpha_1(2F_2(x) - 1) + t(2x-1), & \text{if } x > 1/2. \end{cases}$$

This implies that

$$p_{1A}^{dev}(x) = \begin{cases} [t(1-2x) - \alpha_1(1 - 2F_2(x_2))] + [c - 2\alpha_2(1 - F_2(x)) - \alpha_1(1 - 2F_2(x))], & \text{if } x \leq 1/2, \\ c - 2\alpha_2 F_2(x) - 2\alpha_1(F_2(x) - F_2(x_2)), & \text{if } x > 1/2. \end{cases}$$

Note that the formula for $p_{1A}^{dev}(x)$ here for $x \leq 1/2$ is the same as that in Type 1 deviations, but the formula when $x > 1/2$ differs because the $p_{1B}^*(x)$ takes a different form when $x > 1/2$. Moreover, platform $A$'s deviation prices are higher than the equilibrium prices, i.e. $p_{1A}^{dev}(x) > p_{1A}^*(x)$. The intuition is the following. $p_{1A}^*(x)$ and $p_{1B}^*(x)$ are designed so that all agents are indifferent between the two platforms when they split the market equally. However, in a Type 2 deviation, we have

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[26] Again this deviation price applies to $x \leq x_1$. Platform $A$ does not make sales to agents located at $x > x_1$, and all we need is that $p_{1A}^{dev}(x)$ is sufficient high when $x > x_1$, e.g., infinite.

$x_2 \geq 1/2$. Since the agents (with $x \leq x_1$) are indifferent buying from platform $A$ at $p_{1A}^{dev}(x)$ and buying from platform $B$ at $p_{1B}^*(x)$, it must be that $p_{1A}^{dev}(x) > p_{1A}^*(x)$.[27]

Similarly, it can be shown that

$$p_{2A}^{dev}(x) = \begin{cases} [t(1-2x) - \alpha_2(1-2F_1(x_1))] + [c - 2\alpha_1(1-F_1(x)) - \alpha_2(1-2F_1(x))], & \text{if } x \leq 1/2, \\ c - 2\alpha_1 F_1(x) - 2\alpha_2(F_1(x) - F_1(x_1)), & \text{if } x > 1/2. \end{cases}$$

Note that

$$\frac{\partial p_{1A}^{dev}(x)}{\partial x_1} = 0, \quad \frac{\partial p_{2A}^{dev}(x)}{\partial x_1} = 2\alpha_2 f_1(x_1) \text{ regardless of whether } x \leq 1/2,$$

which is exactly the same as the derivatives in Type 1 deviations.

Platform $A$'s deviation profit is

$$\pi_A^{dev}(x_1, x_2) = \int_0^{x_1} \left(p_{1A}^{dev}(x) - c\right) f_1(x)dx + \int_0^{x_2} \left(p_{2A}^{dev}(x) - c\right) f_2(x)dx.$$

Therefore,

$$\begin{aligned} \frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} &= \left(p_{1A}^{dev}(x_1) - c\right) f_1(x_1) + \int_0^{x_2} \frac{\partial p_{2A}^{dev}(x)}{\partial x_1} f_2(x)dx \\ &= \left(p_{1A}^{dev}(x_1) - c\right) f_1(x_1) + \int_0^{x_2} 2\alpha_2 f_1(x_1) f_2(x)dx \\ &= \left(p_{1A}^{dev}(x_1) - c\right) f_1(x_1) + 2\alpha_2 f_1(x_1) F_2(x_2) \\ &= \left(p_{1A}^{dev}(x_1) - c + 2\alpha_2 F_2(x_2)\right) f_1(x_1) \\ &= [-2\alpha_2 F_2(x_1) - 2\alpha_1(F_2(x_1) - F_2(x_2)) + 2\alpha_2 F_2(x_2)] f_1(x_1) \\ &= 2(\alpha_1 + \alpha_2)(F_2(x_2) - F_2(x_1)). \end{aligned}$$

Similarly it can be shown that

$$\frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_2} = 2(\alpha_1 + \alpha_2)(F_1(x_1) - F_1(x_2)).$$

It's easy to see that at any $x_1 = x_2 = x \geq 1/2$,

$$\frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} = \frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_2} = 0.$$

Next, if $x_\ell > x_m$, then $\frac{\partial \pi_A^{dev}(x_1,x_2)}{\partial x_\ell} < 0$, and $\frac{\partial \pi_A^{dev}(x_1,x_2)}{\partial x_m} > 0$. That is, platform $A$ has incentive to equalize $x_1 = x_2$. Therefore,

$$\pi_A(x_1 = x_2 = 1/2) = \pi_A(x_1 = x_2 > 1/2) > \pi_1(x_1 \neq x_2),$$

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[27] This is due to the following equilibrium feature. In the equilibrium, all agents are indifferent between buying from either platform. If platform $A$ lowers price for a single agent $x > 1/2$ without changing any other price, then all agents (possibly except $x$) will strictly prefer platform $A$. This is how a platform can raise prices and sign more agents – by lowering price for a single agent to get it started. It can be shown that the deviation market structure is unique, and without coordination of agents.

and platform $A$ has no incentive for Type 2 deviations either.

Type 3 deviation: $\max\{x_1, x_2\} \geq 1/2 \geq \min\{x_1, x_2\}$.

Without loss of generality, assume that $x_1 \leq 1/2 \leq x_2$. Note that whether $x_2 \geq 1/2$ does not change the formula of $p_{1A}^{dev}(x)$ or $\frac{\partial p_{2A}^{dev}(x)}{\partial x_1}$ at all, relative to those we derived in Type 1 deviations. With $x_2 > x_1$, platform $A$ would have incentive to increase $x_1$, up until $x_1 = 1/2$.[28] Then it becomes a Type 2 deviation, and we have already shown that platform $A$ has no incentive for Type 2 deviations.

## B3. The 'lowest prices' equilibrium and a continuum of equilibria

### The 'Lowest prices' equilibrium

Recall that the prices are,

$$\begin{aligned} p_{\ell A}^{**}(x) &= c - 2\alpha_m F_m(x) - \alpha_\ell(2F_m(x) - 1) \text{ and } p_{\ell B}^{**}(x) = p_{\ell A}^{**}(x) - t(1-2x), \text{ for } x \leq \frac{1}{2} \\ p_{\ell A}^{**}(x) &= p_{\ell B}^{**}(x) - t(2x-1) \text{ and } p_{\ell B}^{**}(x) = c - \alpha_\ell(1 - 2F_m(x)) - 2\alpha_m(1 - F_m(x)), \text{ for } x \geq \frac{1}{2}. \end{aligned}$$

These prices are constructed similarly as those in the 'highest prices' equilibrium. In both cases, when a platform deviates, it is optimal to have $x_1 = x_2 = x$. In the 'highest prices' equilibrium, we want

$$\pi_A(x = 1/2) = \pi_A(x > 1/2) > \pi_A(x < 1/2).$$

That is, platform $A$ is indifferent between signing more agents in the other platform's turf, but strictly prefers not to low agents in its own turf.

On the contrary, in the 'lowest prices' equilibrium, we want

$$\pi_A(x = 1/2) = \pi_A(x < 1/2) > \pi_A(x > 1/2).$$

That is, platform $A$ is indifferent between whether or not lose agents in its own turf, but strictly prefers not to sign more agents in the other platform's turf.

The equality $\pi_A(x = 1/2) = \pi_A(x > 1/2)$ governs how the equilibrium prices are constructed, and what remains is to verify that $\pi_A(x = 1/2) = \pi_A(x < 1/2) > \pi_A(x > 1/2)$.[29]

### A continuum of equilibria

It can be shown that there is a continuum of equilibria as the following. Note that $p_{\ell j}^{*}(x) > p_{\ell j}^{**}(x)$, $\ell = 1, 2$, $j = A, B$. That is, the prices in the 'highest prices' equilibrium are higher than

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[28] This is quite intuitive. Platform $A$ would have more incentive to sign up an extra agent in group 1, if this leads to larger benefit to be reaped from group 2 agents. Since there are more agents in group 2 in Type 3 deviations ($x_2 \geq 1/2$ here relative to $x_2 \leq 1/2$ in Type 1 deviations) thus more benefit from an extra agent in group 1, platform $A$ would have even more incentive to raise $x_1$.

[29] The whole proof is very similar to that of the 'highest prices' equilibrium. The detailed proof is available upon request.

those in the 'lowest prices' equilibrium.[30] Next, we claim that any set of $(p_{\ell A}(x), p_{\ell B}(x))$ constitutes an equilibrium if the following (sufficient but not necessary) conditions are satisfied:

- $p_{\ell j}(x) \in [p^{**}_{\ell j}(x), p^{*}_{\ell j}(x)]$ and $p_{\ell A}(x) = p_{\ell B}(x) + t(1-2x)$, $\ell = 1, 2$, $j = A, B$.
- Suppose that platform $A$ deviates. Its deviation prices $p^{dev}_{\ell A}(x)$ depends on $p^{*}_{\ell B}(x)$, and we want $p^{dev}_{\ell A}(x)$ to decrease with $x$. This guarantees that agents served by either platform are connected on or off equilibrium path.

## B4. Case (ii) in Proposition 2: $c \leq \min\{\alpha_1, \alpha_2\}$

When $c \leq \min\{\alpha_1, \alpha_2\}$, the disadvantaged platform's prices given in Case $(i)$ are either negative or zero, and the negative prices are replaced by zero. This applies to $p^{*}_{\ell B}(x)$ when $x \leq \frac{1}{2}$ and to $p^{*}_{\ell A}(x)$ when $x \geq \frac{1}{2}$.

Similar to Case $(i)$, we divide the proof of Case $(ii)$ in two steps. In step 1, we show that in any deviation, platform $A$ must be serving agents who are 'connected' in each group. Then, in step 2, we show that under this deviation structure, platform $A$ has no incentive to deviate. Due to symmetry, we only look at the case where platform $A$ deviates. Note that, in equilibrium, it already charges zero price in the other platform's turf $(1/2, 1]$. Therefore, it will not able to lower price there further (otherwise the price will be negative) to sign more agents.[31] Thus we only consider deviations where platform $A$ would sign weakly fewer agents in each group.

**Step 1: Agents served by platform $A$ are connected.**

In the equilibrium which is symmetric, each platform serves its own half of the market, thus the agents served by each platform are 'connected'. Next, we prove that the same property (connectedness) holds for deviations as well. Since platform cannot deviate and sign more agents, we focus on deviations where platform $A$ sign fewer agents only.

Consider a setup where platform $A$'s market share in group 2, after a deviation, is $s_2$. Suppose now platform $A$ is contemplating whether to sign up an agent located at $x$ in group 1 with $x < 1/2$. Let $p^{dev}_{1A}(x)$ denote the maximum deviation price if it wants to attract this agent. The benefit of signing any agent $x$ in group 1 (enjoyed by all agents in group 2 who buy from platform $A$) is the

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[30] This can be seen from the following comparison: (1) when $x > 1/2$, platform $A$ is indifferent between signing an extra agent in the 'highest prices' equilibrium, but it loses profit in the 'lowest prices' equilibrium; (2) when $x < 1/2$, platform $A$ loses profit if it loses an agent in the 'highest prices' equilibrium, while it's indifferent in the 'lowest prices' equilibrium.

[31] As in Case (i), when platform $A$ deviates, it can actually raise prices and sign more agents. However, to get started, it has to lower prices for some $x > 1/2$ first before it can raise price for other agents. While this is possible in Case (i), it can't be done here.

same, so the choice only depends on the cost of signing them, $p_{1A}^{dev}(x) - c$. The deviation price $p_{1A}^{dev}(x)$ is decided by the following equation

$$V - p_{1A}^{dev}(x) - tx + \alpha_1 s_2 = V - p_{1B}^{*}(x) - t(1-x) + \alpha_1(1-s_2).$$

This implies that,

$$\begin{aligned} p_{1A}^{dev}(x<1/2) &= t(1-2x) + \alpha_1(2s_2-1) + p_{1B}^{*}(x) \\ &= t(1-2x) + \alpha_1(2s_2-1). \end{aligned}$$

Then,

$$\frac{dp_{1A}^{dev}(x<1/2)}{dx} = -2t < 0.$$

Since deviation prices decrease with $x$ when $x \leq 1/2$, it's always more profitable for platform $A$ to sign up agents closer to 0 first. Therefore, agents in group 1 served by platform $A$ in the left segment are connected.

Even when platform $A$ signs fewer agents, it may still choose to sign some agents in $(1/2, 1]$. Next, we rule out this probability. Following similar logic as above, we can derive the deviation price as the following,

$$\begin{aligned} p_{1A}^{dev}(x>1/2) &= t(1-2x) + \alpha_1(2s_2-1) + p_{1B}^{*}(x) \\ &= t(1-2x) + \alpha_1(2s_2-1) + t(2x-1) \\ &= \alpha_1(2s_2-1). \end{aligned}$$

Compare the following deviation prices, we have

$$\begin{aligned} p_{1A}^{dev}(x<1/2) &= t(1-2x) + \alpha_1(2s_2-1) \\ &> \alpha_1(2s_2-1) \\ &= p_{1A}^{dev}(x>1/2) \end{aligned}$$

Since $p_{1A}^{dev}(x<1/2) > p_{1A}^{dev}(x=1/2)$ and platform $A$ signs fewer agents in deviation, it will not sign any agent $x > \frac{1}{2}$. Therefore, the agents platform $A$ serves in group 1 are connected. Similarly one can show that agents in group 2 served by platform $A$ are also connected.

**Step 2: Show that platform $A$ has no incentive to deviate**

We have shown that in each group, agents served by either platform are connected. Let $x_\ell$ denote the marginal agent in group $\ell = 1, 2$. This implies that, in group $\ell = 1, 2$, agents in $[0, x_\ell]$ are served by platform $A$, while agents in $(x_\ell, 1]$ are served by platform $B$. Platform $A$ has only one type of deviations possible, i.e., to sign up fewer agents in both groups, or $\max\{x_1, x_2\} \leq 1/2$.

Next we show platform $A$ has no incentive to deviate. This proof is similar to the proof for Type 1 deviation in Case (i), with the difference being $p^*_{\ell B}(x) = 0$ now.

First, we derive platform $A$'s deviation price in each group, starting with group 1. Let $p^{dev}_{1A}(x)$ denote platform $A$'s deviating price targeting the agent located at $x \in [0, x_1]$. Since platform $A$ is the deviating platform, it will choose $p^{dev}_{1A}(x)$ such that the agent is indifferent between the two platforms, that is,[32]

$$V - p^{dev}_{1A}(x) - tx + \alpha_1 F_2(x_2) = V - p^*_{1B}(x) - t(1-x) + \alpha_1(1 - F_2(x_2)).$$

Solving for $p^{dev}_{1A}(x)$, we can obtain,

$$\begin{aligned} p^{dev}_{1A}(x) &= [t(1-2x) - \alpha_1(1 - 2F_2(x_2))] + p^*_{1B}(x) \\ &= t(1-2x) - \alpha_1(1 - 2F_2(x_2)). \end{aligned}$$

Similarly, we can obtain

$$\begin{aligned} p^{dev}_{2A}(x) &= [t(1-2x) - \alpha_2(1 - 2F_1(x_1))] + p^*_{2B}(x) \\ &= t(1-2x) - \alpha_2(1 - 2F_1(x_1)). \end{aligned}$$

Note that

$$\frac{\partial p^{dev}_{\ell A}(x)}{\partial x_\ell} = 0, \quad \frac{\partial p^{dev}_{mA}(x)}{\partial x_\ell} = 2\alpha_m f_\ell(x_\ell), \quad \ell \neq m = 1, 2.$$

Platform $A$'s deviation profit is

$$\pi^{dev}_A(x_1, x_2) = \int_0^{x_1} \left(p^{dev}_{1A}(x) - c\right) f_1(x)dx + \int_0^{x_2} \left(p^{dev}_{2A}(x) - c\right) f_2(x)dx$$

Therefore,

$$\begin{aligned} \frac{\partial \pi^{dev}_A(x_1, x_2)}{\partial x_1} &= \left(p^{dev}_{1A}(x_1) - c\right) f_1(x_1) + \int_0^{x_2} \frac{\partial p^{dev}_{2A}(x)}{\partial x_1} f_2(x)dx \\ &= \left(p^{dev}_{1A}(x_1) - c\right) f_1(x_1) + \int_0^{x_2} 2\alpha_2 f_1(x_1) f_2(x)dx \\ &= \left(p^{dev}_{1A}(x_1) - c\right) f_1(x_1) + 2\alpha_2 f_1(x_1) F_2(x_2) \\ &= \left(p^{dev}_{1A}(x_1) - c + 2\alpha_2 F_2(x_2)\right) f_1(x_1) \\ &= [t(1-2x_1) - \alpha_1(1 - 2F_2(x_2)) - c + 2\alpha_2 F_2(x_2)]\, f_1(x_1) \\ &= [t(1-2x_1) - \alpha_1 - c + 2(\alpha_1 + \alpha_2) F_2(x_2)]\, f_1(x_1) \end{aligned}$$

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[32] Since the agents are connected, platform $A$ will sign all agents $x \leq x_1$ and nobody else in group 1. For any $x > x_1$, since platform $A$ will not make sales, we do not need derive the deviation price since all we need is for the deviation price to be sufficiently high, e.g., infinite. Another candidate is the equilibrium price $p^*_{1A}(x)$ for $x > x_1$.

Assume that $x_1 < x_2$, then $F_2(x_1) < F_2(x_2)$, and

$$\frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} \geq [t(1-2x_1) - \alpha_1 - c + 2(\alpha_1+\alpha_2)F_2(x_1)]\, f_1(x_1).$$

Let $g(x_1)$ denote the term inside the square bracket, i.e.,

$$g(x_1) \equiv t(1-2x_1) - \alpha_1 - c + 2(\alpha_1+\alpha_2)F_2(x_1).$$

$$\frac{dg(x_1)}{dx_1} = -2[t - (\alpha_1+\alpha_2)f_2(x_1)] \geq 0 \quad (\text{ since } t \leq (\alpha_1+\alpha_2)f_\ell(x)).$$

Moreover,

$$g(x_1 = 0) = t - \alpha_1 - c > 0 \quad (\text{ since } t > c + \max\{\alpha_1, \alpha_2\}).$$

Combining $g(x_1 = 0) > 0$ and $\frac{dg(x_1)}{dx_1} \geq 0$, it must be that

$$g(x_1 < 1/2) > 0.$$

Therefore,

$$\frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} > 0,$$

and platform $A$ has incentive to increase $x_1$.

Similarly, it can be shown that if $x_1 > x_2$, then platform $A$ has incentive to increase $x_2$. That is, platform $A$ has incentive to equalize $x_1$ and $x_2$.

It can be further seen that at $x_1 = x_2 < 1/2$,

$$\frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_1} > 0, \quad \frac{\partial \pi_A^{dev}(x_1, x_2)}{\partial x_2} > 0,$$

and platform $A$ has incentive to increase both $x_1$ and $x_2$ (until $x_1 = x_2 = 1/2$). That is, platform $A$ has no incentive to deviate from $x_1 = x_2 = \frac{1}{2}$.

# References

[1] Anderson, S.P. and S. Coate, 2005, "Market Provision of Broadcasting: A Welfare Analysis," *Review of Economic Studies*, 72, 947-972.

[2] Anderson, S.P. and J.J. Gabszewicz, 2006, "The Media and Advertising: A Tale of Two-Sided Markets," in the *Handbook of the Economics of Art and Culture*, vol. 1, edited by V.A. Ginsburgh and D. Throsby.

[3] Armstrong, M., 2006a, "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37, 668-691.

[4] Armstrong, M., 2006b, "Recent Developments in the Economics of Price Discrimination," in *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress*, eds. Blundell, Newey and Persson, Cambridge University Press.

[5] Asplund, M., R. Eriksson and N. Strand, 2008, "Price Discrimination in Oligopoly: Evidence from Regional Newspapers," *Journal of Industrial Economics*, LVI, 333-346.

[6] Bester, H. and E. Petrakis, 1996, "Coupons and Oligopolistic Price Discrimination," *International Journal of Industrial Organization,* 14, 227-242.

[7] Caillaud, B. and B. Jullien, 2003, "Chicken & Egg: Competition Among Intermediation Service Providers," *RAND Journal of Economics*, 34, 309-328.

[8] Chen, Y., 1997, "Paying Customers to Switch," *Journal of Economics & Management Strategy,* 6, 877-897.

[9] Choi, J-P., 2006, "Tying in Two-Sided Markets with Multi-Homing," NET Institute working paper #06-04.

[10] Corts, K., 1998, "Third-Degree Price Discrimination in Oligopoly," *RAND Journal of Economics,* 29, 306-323.

[11] Economides, N. and J. Tåg, 2007, "Net Neutrality on the Internet: A Two-Sided Market Analysis," NET Institute working paper #07-14.

[12] Federal Trade Commission (FTC), 2000,"Entering the $21^{\text{st}}$ Century: Competition Policy in the World of B2B Electronic Market Places."

[13] Fudenberg, D. and J. Tirole, 2000, "Customer Poaching and Brand Switching," *RAND Journal of Economics,* 31, 634-657.

[14] Gil, R. and D. Riera-Crichton, 2010, "Price Discrimination and Competition in Two-Sided Markets: Evidence from the Spanish Local TV Industry," working paper.

[15] Jullien, B., 2008, "Price Skewness and Competition in Multi-Sided Markets," Working Paper.

[16] Kim, H. and K. Serfes, 2006, "A Location Model with Preference for Variety," *Journal of Industrial Economics*, 4, 569-595.

[17] Liu, Q. and K. Serfes, 2004, "Quality of Information and Oligopolistic Price Discrimination," *Journal of Economics & Management Strategy*, 13, 671-702 .

[18] Liu, Q. and K. Serfes, 2005, "Imperfect Price Discrimination in a Vertical Differentiation Model," *International Journal of Industrial Organization*, 23, 341-354.

[19] McAfee, P. and M. Schwartz, 1994, "Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity and Uniformity," *American Economic Review*, 84, 210-230.

[20] Rochet, J-C. and J. Tirole, 2003, "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 4, 990-1029.

[21] Rochet, J-C. and J. Tirole, 2006, "Two-Sided Markets: A Progress Report," *RAND Journal of Economics*, 37, 645-667.

[22] Shaffer, G. and Z.J. Zhang, 1995, "Competitive Coupon Targeting," *Marketing Science*, 14, 395-415.

[23] Shaffer, G. and Z.J. Zhang, 2002, "Competitive One-to-One Promotions," *Management Science*, 48, 1143-1160.

[24] Stole, L.A., 2007, "Price Discrimination and Imperfect Competition," in *The Handbook of Industrial Organization*, III.

[25] Thisse, J.F. and X. Vives, 1988, "On the Strategic Choice of Spatial Price Policy," *American Economic Review*, 78, 122-137.