

Matching and Contracting in the Venture Capital Market*

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Abstract

We propose an equilibrium model of the venture capital (VC) market with two-sided matching between heterogeneous VCs and entrepreneurs. Each VC matches endogenously with an entrepreneur, and offers a contract that specifies a capital investment and equity allocation. We show that competition among VCs for the most promising entrepreneurial ideas leads to more efficient capital endowments of new ventures throughout the *entire* VC market, which in turn improves the prospects of success across all ventures (*endogenous matching effect*). We also show that endogenous matching amplifies the impact of economic cycles on VC investments and survival rates of new ventures.

Keywords: Entrepreneurship, venture capital, matching, contracts, moral hazard, incentives

JEL classifications: C78, D86, G24, L26, M13

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1 Introduction

The venture capital (VC) market has long been recognized as a fundamental source for entrepreneurs to finance their ventures, and thus to bring their innovative ideas to market (Kaplan, Sensory, and Strömberg (2009)). A hallmark of the VC market is the process by which entrepreneurs are matched to VCs, and the corresponding impact of VC financing on entrepreneurial success. The literature has identified two forces at work: VCs with higher expertise select superior ventures to invest in (the "selection" effect); and VCs with higher expertise provide greater value-added to their ventures (the "treatment" effect). Sørensen (2007) provides empirical evidence that the selection effect is almost twice as important as the treatment effect in explaining observed differences in IPO rates across VC investors. Clearly, then, the matching process matters greatly in the VC market.¹ To better understand the matching process and its implications for the financing of new ventures, we propose an equilibrium model of the VC market consisting of heterogeneous entrepreneurs and VCs. We demonstrate that—in addition to the selection and treatment effect—there is an *endogenous matching effect* that has a considerable impact on the financing of start-ups and their likelihood of survival, and amplifies business cycles in the VC market.²

The *endogenous matching effect* originates from the competition between VCs for entrepreneurial ideas with exceptional market potential. Endogenous matching in the VC market leads to a number of novel theoretical, empirical, and policy implications. First, we show that endogenous matching enhances the efficiency of the entire VC market through higher capital endowments of start-ups, which in turn improves the prospects of success across all ventures. Moreover, as a result of endogenous matching, entry of VCs during economic booms does not only benefit entrepreneurs who then get capital to start their own ventures, but may also lead

¹Matching is not only relevant in VC markets, but also matters in general principal-agent relationships (see, e.g., Akerberg and Botticini (2002), Besley and Ghatak (2005), and Serfes (2005, 2008)).

²This *endogenous matching effect* does not appear in Sørensen's framework as he treats the contracts between entrepreneurs and VCs as exogenous. In contrast, we endogenize the contract choice of VCs, which, in our model, is also determined by alternative contract offers for their entrepreneurs.

to more favorable contract terms for all other entrepreneurs in this market (the *ripple effect*). Because of this positive externality on the entire market, VC entry should be promoted in order to spur indispensable innovation. In economic downturns, on the other hand, exit of some VCs can trigger a wave of failures as all remaining VCs respond by substantially reducing their investment levels, thereby jeopardizing the survival of their ventures (the *unraveling effect*). From a policy perspective, this also implies that failure of small VCs should be averted as this can impair the efficiency of the entire VC market, and thus hamper the creation of promising innovation. In this sense, a VC can be "too small to fail". It is therefore important that empirical researchers—using our model predictions and following along the lines of Sørensen (2007)—quantify and evaluate the relative magnitude of the identified *endogenous matching effect* in the VC market.

We develop an equilibrium model of the VC market with two-sided endogenous matching between a collection of VCs, that are heterogeneous in terms of their expertise, and a collection of entrepreneurs, that are heterogeneous with respect to the quality (or market potential) of their business ideas. We show that, under reasonable conditions, the matching equilibrium is positive assortative (PAM): VCs with high expertise match with entrepreneurs that have business ideas with high market potentials.³ Once matched with an entrepreneur, each VC offers a contract which stipulates an allocation of equity and the VC's capital investment. Each entrepreneur then needs to exert (unobservable) effort which affects the success or failure of his venture (moral hazard).⁴ Due to endogenous matching in our framework, each contract must not only satisfy the usual incentive compatibility constraint associated with the moral hazard problem, but also a condition that guarantees that an entrepreneur cannot be better off by contracting with an alternative VC. In this sense, VCs are competing for entrepreneurs with promising business

³Sørensen (2007) provides empirical evidence that the matching equilibrium in the VC market is indeed positive assortative.

⁴Thus, our framework exhibits a typical one-sided moral hazard problem with respect to the entrepreneur's effort, while the investment of the VC is contractible. For models with double-sided moral hazard in the context of the financing of new ventures, see e.g. Casamatta (2003), Repullo and Suarez (2004), and de Bettignies (2008).

ideas which—as we will show—plays a key role in the design of optimal VC contracts and the overall efficiency of the VC market.⁵

While the analysis of isolated entrepreneur-VC relationships in a contractual context undoubtedly provides important insights, a main implication of our study is that endogenous matching in the VC market significantly affects the structure and properties of VC contracts. Specifically, we show that each VC is forced to transfer more surplus to its matched entrepreneur through more attractive contracts than would occur in the absence of endogenous matching (i.e., as in an isolated principal-agent relationship). This in turn results in better capital endowments of new ventures, and hence, improves their prospects of success. Despite this positive effect of endogenous matching on the overall efficiency of the VC market, we show that equilibrium investment levels and survival rates of new ventures are always below their first-best levels. We therefore argue that any managerial or policy implications should not be based on an isolated analysis of VC contracts; rather, it is indispensable to consider the two-sided VC market as a whole in order to account for the effects of endogenous matching.

We also use our framework with endogenous matching to identify the effects of economic cycles on equilibrium VC contracts. We show that VC entry during economic booms improves the outside options of all entrepreneurs, forcing VCs to offer more capital in order to attract entrepreneurs with highly promising business ideas. Our prediction that the more intense the competition, the higher the amount of funding each portfolio company receives is consistent with evidence in Gompers and Lerner (2000). This has an important and interesting implication: VC-backed ventures that are founded during economic booms enjoy higher survival rates than ventures started during recessions, even after accounting for the characteristics of their respective product markets. We refer to this mechanism—which arises because of endogenous matching—as the *ripple effect* of market entry since it benefits all entrepreneurs in the market, and not just those who would have remained unfunded in the absence of VC entry.

⁵de Bettignies and Chemla (2008) also consider the effects of competition for business ideas, though in a different context. In their model, a firm wants to attract managers with high quality ideas for new ventures.

The *ripple effect* throughout the VC market—triggered by VC entry—leads to an important policy implication: Regulatory hurdles, which hamper the entry of potential financiers, should be eliminated to spur innovation.⁶ In this sense, facilitating entry of VCs is integral not only to fund *more* innovation, but also to *improve* the funding of promising innovative ideas, which in turn makes new ventures more likely to succeed.

The *ripple effect* in the context of an economic expansion can turn into an *unraveling effect* in the context of an economic contraction. We show that exit of VCs from the market can adversely affect the survival rates of new VC-backed ventures, potentially triggering a wave of failures in the VC market. The reason is as follows. Market exits inevitably curb the degree of competition among the remaining VCs for entrepreneurs with high quality business ideas. Due to fewer alternatives, entrepreneurs are then compelled to accept VC contracts that provide them with less capital to turn their innovative ideas into marketable products. Lower capital endowments of new ventures, however, deteriorate their prospects of success.

Our paper is closest in spirit to the equilibrium models of the VC market as devised by Inderst and Müller (2004), Silviera and Wright (2010), and Sørensen (2007).⁷ Our paper differs in two important aspects from Inderst and Müller (2004) and Silviera and Wright (2010). First, both papers consider a *search* equilibrium, whereas we study *matching*. The search equilibria in their models are characterized by the ratio of the number of VCs to the number of entrepreneurs, commonly referred to as "market thickness" in the search theory literature. Using a matching framework, however, we show that it is not just the market thickness that matters; the qualities of entrepreneurial ideas and the expertise of VCs are also critical for the optimal design of VC

⁶This implication is not only restricted to the VC market, but does also apply to angel investments. For the latter, our paper provides clear and intuitive arguments against the reform bill recently proposed by Senator Chris Dodd from the District of Connecticut. This reform bill would raise regulatory hurdles for angel investors by requiring them to register with the U.S. Securities and Exchange Commission (SEC), and by doubling their income and wealth baselines; see Farrell (2010).

⁷Dam (2007) also studies matching between venture capitalists and firms. However, his setting and predictions are substantially different from ours. He considers a market where venture capitalists are heterogeneous with respect to their monitoring ability, and firms are heterogeneous with respect to their levels of initial wealth. The matching equilibrium in his framework is negative assortative (NAM): venture capitalists with higher monitoring ability invest in firms with lower initial wealth.

contracts, especially since they determine the (endogenous) outside options of entrepreneurs. Second, Inderst and Müller (2004) consider homogenous VCs and entrepreneurs, while Silveira and Wright (2010) introduce ex-post heterogeneity, i.e., the heterogeneity arises after a VC is matched with an entrepreneur. In contrast, we explicitly account for different match qualities by allowing entrepreneurs to be ex-ante heterogenous with respect to the market potential of their ideas, and VCs with respect to their expertise.

Sørensen (2007) devises a structural model based on a two-sided matching model of VCs and entrepreneurs. The matching model in Sørensen (2007) differs from ours in the following respects. First, in Sørensen (2007), contracts between VCs and entrepreneurs are exogenous. By contrast, we derive the optimal contract featuring an equity stake and investment by the VC. Second, Sørensen does not model the link between all entrepreneur-VC pairs that occurs via their outside options. We show that this link gives rise to the *endogenous matching effect*, which is crucial towards understanding the inter-relationship between the VC market as a whole and the properties of individual VC contracts. Third, we examine the effect of economic cycles on equilibrium VC contracts. We find that such considerations reveal the *ripple effect* that permeates the VC market as ventures enter and exit, which in turn generates important insights into the functioning and possible failure of the VC market.

The remainder of the paper is structured as follows. Section 2 introduces the features of our model. Section 3 derives the optimal contract of an entrepreneur-VC pair in the absence of endogenous matching. Section 4 incorporates two-sided matching in the VC market and characterizes the equilibrium contracts. Section 5 investigates the roles of economic expansions and contractions in the VC market, and how they affect the contractual relationships between entrepreneurs and VCs. Section 6 discusses the implications when allowing each VC to finance multiple (heterogeneous) entrepreneurs. Section 7 then addresses empirical ramifications of the model for future research. Section 8 concludes. All proofs for our lemmas and propositions can be found in the Appendix.

2 Preliminaries

2.1 The Structure and Timing of the Model

Consider a market of $n_E \geq 2$ risk-neutral entrepreneurs and $n_{VC} \geq 2$ risk-neutral VCs. There are more entrepreneurs than venture capitalists, i.e., $n_E \geq n_{VC}$. All entrepreneurs are wealth constrained and their reservation utilities are normalized to zero.⁸ There are four dates:

1. Entrepreneurs are endowed with ideas for new ventures.
2. Each VC matches with an entrepreneur and offers him a contract that consists of an equity share of the venture and an investment.
3. Entrepreneurs exert unobservable effort.
4. Profits of ventures are realized and payments are made.

At date 1, each entrepreneur $i, i \in E = \{1, \dots, n_E\}$, conceives an innovative business idea, denoted μ_i , where a higher value of μ_i reflects an idea of superior quality (or market potential). Entrepreneurial ideas can be ranked according to their respective quality, i.e., $\mu_{n_E} \geq \dots \geq \mu_2 \geq \mu_1$. To commercially exploit his idea, each entrepreneur relies on a capital investment K_i as well as on managerial and marketing expertise, both provided by a VC.⁹ Let x_j denote the expertise of VC $j, j \in V = \{1, \dots, n_{VC}\}$, where a superior expertise is reflected by a higher x_j , i.e., $x_{n_{VC}} \geq \dots \geq x_2 \geq x_1$. Entrepreneurial ideas and VC expertise are common knowledge.¹⁰ At

⁸While we initially assume a zero outside option for every entrepreneur, a strictly positive reservation utility for entrepreneurs with high-quality projects arises endogenously in our matching framework.

⁹The dependency of entrepreneurs on VC expertise excludes debt as a viable form of financing their start-ups. However, if VC expertise was not crucial, debt financing could be preferred in some cases; see Ueda (2004) and Thiele and Tombak (2010). To account for debt financing as an alternative to VC financing, one could extend our framework by assuming a strictly positive outside option for entrepreneurs which reflects their expected utility when using debt to start their ventures. As will become clear from our analysis, however, this would not change our results.

¹⁰While a potential information asymmetry problem with respect to the qualities of entrepreneurial ideas is not the focus of this paper, one could extend our framework to account for such private information. As is well known from the standard adverse selection literature, entrepreneurs with high quality ideas would then extract an information rent.

date 2, each VC matches with exactly one entrepreneur.¹¹ A VC j then makes its entrepreneur i a *take-it-or-leave-it* offer with a contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ that specifies an equity share λ_{ij} of the venture for the entrepreneur as well as the VC's capital investment K_{ij} . The cost of capital faced by each VC is exogenous and denoted $r > 0$.

At date 3, after signing the contract Γ_{ij} , entrepreneur i exerts effort e_{ij} to turn his idea into a marketable product. Implementing effort e_{ij} imposes the cost $c(e_{ij}) = e_{ij}^2/2$. The entrepreneur's effort e_{ij} —which cannot be observed by the VC—determines the likelihood of whether the venture succeeds ($Y_{ij} = 1$) or fails ($Y_{ij} = 0$). Specifically, we assume that $\text{Prob}[Y_{ij} = 1|e_{ij}] = e_{ij}$.¹² Finally, at date 4, the profit of each venture is realized, and payments according to the respective contracts are made.

A key property of our framework is the reliance of entrepreneurs on the expertise of VCs to turn promising ideas into marketable products. Intuitively, the value of this expertise is closely related to the quality of the entrepreneur's specific idea. To capture this notion, let $\Omega_{ij} \equiv \Omega(\mu_i, x_j) > 0$ denote the *match quality* between an entrepreneur with idea μ_i and a VC with expertise x_j . The match quality Ω_{ij} is strictly increasing in both the entrepreneur's idea quality μ_i and the VC's expertise x_j , where ideas and expertise are complements (i.e., $\partial^2\Omega(\mu_i, x_j)/\partial x_i\partial\mu_j > 0$). This in turn allows us to rank ventures according to their respective match quality, where we henceforth refer to the resulting collection of ventures—consisting of entrepreneurs each matched with one VC—as a *ladder*.

Let $\Pi_{ij} \in \{0, \pi(K_{ij}, \Omega_{ij})\}$ denote the gross profit of the venture started by entrepreneur i and financed by VC j , where $\pi(K_{ij}, \Omega_{ij}) \geq 0$ denotes the gross profit of a *successful* venture

¹¹To keep our framework as simple as possible, we focus on one-to-one matching. However, our analysis can easily be extended to a many-to-one matching, where a venture capital firm can contract with multiple entrepreneurs. We discuss the implications in greater detail in Section 6.

¹²To ensure interior solutions, we will assume that the venture's potential profit, denoted by π , is always smaller than one, so that even the first-best effort level e_{ij}^{fb} guarantees that $\text{Prob}[Y_{ij} = 1|e_{ij}^{fb}] < 1$.

(i.e., $Y_{ij} = 1$). Given the entrepreneur's effort e_{ij} , the VC's capital investment K_{ij} , and the match quality Ω_{ij} , the expected profit of entrepreneur i 's venture financed by VC j is

$$E[\Pi_{ij}|e_{ij}, K_{ij}, \Omega_{ij}] = \pi(K_{ij}, \Omega_{ij})e_{ij}. \quad (1)$$

The realization of Π_{ij} is uncertain from an *ex ante* perspective, implying that the VC cannot perfectly infer from the realized profit Π_{ij} the entrepreneur's effort level e_{ij} , leading to a typical moral hazard problem.

We now elaborate on the underlying properties of the gross profit $\pi(K_{ij}, \Omega_{ij})$ of a *successful* venture. A venture's profit $\pi(K_{ij}, \Omega_{ij})$ can only be strictly positive if (i) the entrepreneur implements effort ($e_{ij} > 0$), (ii) the VC provides capital ($K_{ij} > 0$), (iii) the entrepreneur has a marketable idea ($\mu_i > 0$); and (iv), the VC is equipped with expertise ($x_i > 0$). Technically, we assume

$$\pi(0, \Omega(\mu_i, x_j)) = \pi(K_{ij}, \Omega(0, x_j)) = \pi(K_{ij}, \Omega(\mu_i, 0)) = 0.$$

Furthermore, we assume that $\pi(K_{ij}, \Omega_{ij})$ is concave increasing in the VC's investment K_{ij} and in the match quality Ω_{ij} . Finally, we make the following assumptions to ensure interior solutions:

$$\left. \frac{\partial \pi(K_{ij}, \Omega_{ij})}{\partial K_{ij}} \right|_{K_{ij}=0} = \infty, \quad \left. \frac{\partial \pi(K_{ij}, \Omega_{ij})}{\partial \Omega_{ij}} \right|_{K_{ij}=0} = \left. \frac{\partial \pi(K_{ij}, \Omega_{ij})}{\partial K_{ij}} \right|_{\Omega_{ij}=0} = 0.$$

These assumptions imply that the first unit of capital K_{ij} is very productive, but only as long as the match quality in question is strictly positive. We further assume that a higher match quality Ω_{ij} makes every unit of capital K_{ij} more productive, $\partial^2 \pi(K_{ij}, \Omega_{ij}) / (\partial K_{ij} \partial \Omega_{ij}) \geq 0$.

2.2 The Efficient (First-Best) Contract

To evaluate how matching affects efficiency in the VC market and by extension provide policy implications, we first derive the contract between entrepreneur i and VC j which maximizes the total surplus from the venture. To do so, suppose a social planner can dictate entrepreneur i 's effort level e_{ij} as well as the investment K_{ij} by VC j . The social planner would choose the (first-best) effort level e_{ij}^{fb} and the (first-best) investment K_{ij}^{fb} that maximize each venture's expected total surplus S_{ij} . The problem of the social planner can thus be stated as follows for a venture with match quality Ω_{ij} :

$$\max_{\{e_{ij}, K_{ij}\}} S_{ij}(e_{ij}, K_{ij}) = \pi(K_{ij}, \Omega_{ij})e_{ij} - e_{ij}^2/2 - rK_{ij}. \quad (2)$$

Notice that $S_{ij}(e_{ij}, K_{ij})$ is not affected by the entrepreneur's share λ_{ij} of the venture's profit $\pi(K_{ij}, \Omega_{ij})$ (if successful) since λ_{ij} determines only the allocation of surplus, and not the size of the total surplus. The following lemma characterizes the solution to this problem.

Lemma 1 (First-Best Contract) *The socially efficient investment K_{ij}^{fb} by VC j is implicitly characterized by*

$$\frac{\partial \pi(K_{ij}^{fb}, \Omega_{ij})}{\partial K_{ij}} = \frac{r}{\pi(K_{ij}^{fb}, \Omega_{ij})}; \quad (3)$$

and the socially efficient effort level e_{ij}^{fb} of entrepreneur i is

$$e_{ij}^{fb} = \pi(K_{ij}^{fb}, \Omega_{ij}). \quad (4)$$

To ensure that $e_{ij}^{fb} < 1$, and hence $\text{Prob}[Y_{ij} = 1 | e_{ij}^{fb}] < 1$, we assume that $\pi(K_{ij}^{fb}, \Omega_{ij}) < 1$ for the socially optimal capital investment K_{ij}^{fb} and any possible match quality Ω_{ij} .¹³ We will draw on the socially efficient investment K_{ij}^{fb} and effort level e_{ij}^{fb} as emphasized by Lemma 1 in

¹³Alternatively, one could incorporate a parameter $\gamma > 0$ in the entrepreneur's effort cost function so that $c(e_{ij}) = \gamma e_{ij}^2/2$. Assuming a sufficiently high cost parameter γ would then also ensure that $\text{Prob}[Y_{ij} = 1 | e_{ij}^{fb}] < 1$.

subsequent sections to characterize the effect of endogenous matching on the efficiency of VC contracts in a second-best environment.

3 The Venture Capital Market in the Absence of Matching

To establish a benchmark that enables one to identify the impact of endogenous matching, we first derive the optimal VC contract in the absence of endogenous matching. To do so, we proceed in two steps. First, in Section 3.1, we characterize the entrepreneur's effort choice for a *given* VC contract. Second, we derive the optimal VC contract in Section 3.2 by accounting for the entrepreneur's effort choice.

3.1 An Entrepreneur's Effort

We begin our analysis by investigating entrepreneur i 's effort choice for a given contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ offered by VC j , with $i \in \{1, \dots, n_{VC}\}$ and $j \in \{1, \dots, n_E\}$. For a given match quality Ω_{ij} , capital investment K_{ij} , and share of the venture's profit λ_{ij} , the entrepreneur chooses his effort e_{ij} to maximize his expected utility

$$\max_{\{e_{ij}\}} U_i(e_{ij}, \lambda_{ij}, K_{ij}, \Omega_{ij}) = \lambda_{ij}\pi(K_{ij}, \Omega_{ij})e_{ij} - e_{ij}^2/2. \quad (5)$$

The entrepreneur's effort choice is thus

$$e_{ij}^* = \lambda_{ij}\pi(K_{ij}, \Omega_{ij}). \quad (6)$$

Clearly, the entrepreneur's effort e_{ij}^* is increasing in his profit share λ_{ij} , the investment K_{ij} , his idea quality μ_i , and finally, in the specific expertise of the VC x_i . This is very intuitive: a higher capital investment or a better match quality makes every unit of the entrepreneur's effort

more productive. In response, it is optimal for the entrepreneur to put in more effort in order to maximize his expected utility.

Finally, notice that the entrepreneur's participation constraint is always satisfied (assuming an initial outside option of zero) because potential losses (up to K_{ij}) are only incurred by the VC firm—which is rooted in the entrepreneur's limited wealth. His limited liability further implies that the entrepreneur extracts an economic rent, which can be shown (using the Envelope Theorem) to be strictly increasing in his profit share λ_{ij} , the VC's capital investment K_{ij} , and in the specific match quality Ω_{ij} of his venture.

3.2 A Venture Capitalist's Contract Choice

In this section, we derive the optimal contract offered by a VC in the absence of endogenous matching. This contract is not only aimed at securing the VC an adequate return on its investment, but also at motivating the entrepreneur to implement sufficient effort in order to turn his idea into a profitable product. Clearly, to indirectly control the entrepreneur's effort choice, the VC can simply adjust his share λ_{ij} of the venture's profit. Less obvious, however, is the fact that the VC can also utilize its investment K_{ij} to influence the entrepreneur's effort. As discussed in Section 3.1, a higher capital investment makes every unit of the entrepreneur's effort more productive, which in turn induces him to implement more effort.

Consider the contract $\Gamma_{ij} = \{\lambda_{ij}, K_{ij}\}$ offered by VC j to entrepreneur i . The VC adjusts this contract to maximize its expected profit, taking the entrepreneur's effort choice e_i^* into account. The VC's expected profit is thus given by

$$\Pi_{ij}^{VC}(\lambda_{ij}, K_{ij}, e_{ij}^*, \Omega_{ij}) = (1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij})e_{ij}^* - rK_{ij}. \quad (7)$$

By accounting for e_{ij}^* as defined by (6), the expected profit of VC j becomes

$$\Pi_{ij}^{VC}(\lambda_{ij}, K_{ij}, \Omega_{ij}) = \lambda_{ij}(1 - \lambda_{ij})\pi^2(K_{ij}, \Omega_{ij}) - rK_{ij}. \quad (8)$$

Before deriving the optimal contract Γ_{ij}^* , it is insightful to unravel the relationship between the allocation of equity and the VC's expected profit. We start by briefly discussing the extreme cases: all shares are either allocated to the VC ($\lambda_{ij} = 0$) or the entrepreneur ($\lambda_{ij} = 1$). Due to the lack of any prospective compensation, it is obvious that the entrepreneur refuses to implement effort whenever the VC holds all shares (i.e., $\lambda_{ij} = 0$), implying a zero profit for the VC as $\text{Prob}[Y_{ij} = 1 | e_{ij} = 0] = 0$, i.e., $\Pi_{ij}^{VC}(\lambda_{ij} = 0, K_{ij}, \Omega_{ij}) = 0$. On the other hand, if the entire profit of the venture accrues to the entrepreneur (i.e., $\lambda_{ij} = 1$), the VC clearly does not benefit from the new enterprise. We thus conclude that financing the new venture can only be profitable for the VC as long as *both* parties hold some shares (i.e., $\lambda_{ij} \in (0, 1)$).

The optimal VC contract Γ_{ij}^* —comprising the entrepreneur's share λ_{ij}^* and the VC's investment K_{ij}^* —is dictated by two objectives: (i) to provide the entrepreneur with sufficient effort incentive to turn his idea into a profitable product; and (ii), to equip the new venture with sufficient capital, not only aimed at ensuring its survival, but also at generating an adequate return on investment for the VC.

The following lemma characterizes the VC contract that maximizes (8).

Lemma 2 (VC Contract without Endogenous Matching) *Suppose there is no endogenous matching in the VC market. Then, the optimal contract between entrepreneur i and VC j comprises the profit share $\lambda_{ij}^* = 1/2$ for the entrepreneur, and the capital investment K_{ij}^* which is implicitly defined by*

$$\pi(K_{ij}^*, \Omega_{ij}) \frac{\partial \pi(K_{ij}^*, \Omega_{ij})}{\partial K_{ij}} = 2r. \quad (9)$$

In the absence of endogenous matching, it is optimal for the financiers to equally split the shares of new ventures.¹⁴ We will demonstrate in the next section that the split of shares is *not* even when accounting for endogenous matching in the VC market. Finally, by comparing (9) with (3), it becomes clear that the optimal contract $\{\lambda_{ij}^*, K_{ij}^*\}$ entails under-provision of capital from a social perspective, i.e., $K_{ij}^* < K_{ij}^{fb}$. A comparison of (6) with (4) further reveals that entrepreneur i 's effort level under this contract is also inefficiently low, i.e., $e_{ij}^* < e_{ij}^{fb}$.

Using the optimal contract $\Gamma_{ij}^* = \{\lambda_{ij}^*, K_{ij}^*\}$ stated in Lemma 2, one can derive the expected utility of entrepreneur i matched with VC j :

$$U_{ij}^V(K_{ij}^*, \Omega_{ij}) = \pi^2(K_{ij}^*, \Omega_{ij})/8. \quad (10)$$

The superscript V indicates that the entire bargaining power in this relationship rests with the VC. Entrepreneur i 's expected utility $U_{ij}^V(K_{ij}^*, \Omega_{ij})$ constitutes his lowest possible expected utility level in our matching framework, which will play a fundamental role in our subsequent analysis.

The next lemma elaborates on how the optimal contract $\Gamma_{ij}^* = \{\lambda_{ij}^*, K_{ij}^*\}$ responds to different match qualities, as measured by Ω_{ij} .

Lemma 3 (Properties of the VC Contract without Endogenous Matching) *Suppose there is no matching in the VC market. Then, entrepreneur i 's share λ_{ij}^* is independent of the match quality Ω_{ij} , while the capital investment K_{ij}^* is increasing in Ω_{ij} .*

A better match quality—stemming either from a more promising entrepreneurial idea (μ_i) or superior VC expertise (x_j)—enhances the payoff for the VC when supplying the entrepreneur with more capital K_{ij}^* . On the other hand, providing the entrepreneur with the share $\lambda_{ij}^* = 1/2$

¹⁴Note that this result is rooted in our specific assumptions about the entrepreneur's effort cost function $c(e_{ij})$, and how his effort affects the performance of the venture. However, the specific value of λ_{ij}^* is of minor importance for our subsequent analysis as we are mainly interested in how matching in the VC market alters the *relative* allocation of equity between entrepreneurs and their financiers.

already maximizes the VC's return on investment, and is thus independent of the venture's specific match quality.

Applying the Envelope Theorem to (8) and (10) yields the following lemma, which emphasizes the effect of the match quality Ω_{ij} on the expected utility of entrepreneur i and the expected profit of VC j .

Lemma 4 (Impact of Match Quality without Endogenous Matching) *Suppose there is no endogenous matching in the VC market. Entrepreneur i , $i = 1, \dots, n$, and VC j , $j = 1, \dots, n$, strictly benefit from a superior match quality Ω_{ij} , i.e., $dU_{ij}^V/d\Omega_{ij} > 0$ and $d\Pi_{ij}^{VC}/d\Omega_{ij} > 0$.*

Both parties strictly benefit from a superior match quality, rooted either in a more promising entrepreneurial idea or in greater VC expertise. This observation will have important implications for the properties of the matching process in the VC market, which we now turn to.

4 The Matching of Entrepreneurs with Venture Capitalists

4.1 Properties of Matching in the Venture Capital Market

After having derived the optimal VC contract in the absence of endogenous matching, we are now well equipped to investigate the optimal contracts offered by VCs when forming partnerships with entrepreneurs in a two-sided heterogeneous market. To do so, we proceed in two steps. In this section, we first identify the general properties of the matching process in the VC market. Then, in the subsequent section, we elaborate on the optimal adjustment of VC contracts when taking endogenous matching into account.

In a traditional principal-agent setting, an entrepreneur is forced to accept the offer of a specific VC. However, if entrepreneurs are free to choose the VC with the most attractive offer, it is intuitively clear that optimal VC contracts must take into consideration the best alternative available to an entrepreneur. VC j thus designs the contract $\{\lambda_{ij}, K_{ij}\}$ to maximize its

expected profit subject to the entrepreneur receiving at least his outside value u_{ij} , which we will characterize later. The constrained optimization problem of VC j can thus be expressed as follows:

$$\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij}) \equiv \max_{\{\lambda_{ij}, K_{ij}\}} (1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij})e_{ij}^* - rK_{ij} \quad (11)$$

s.t.

$$\lambda_{ij}\pi(K_{ij}, \Omega_{ij})e_{ij}^* - (e_{ij}^*)^2/2 \geq u_{ij}, \quad (12)$$

where $e_{ij}^* = \lambda_{ij}\pi(K_{ij}, \Omega_{ij})$. The maximized objective function $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ defines the *bargaining frontier* between VC j and entrepreneur i . Whether the entrepreneur's individual rationality (IR) constraint (12) is binding clearly hinges on his specific reservation utility u_{ij} . We can infer from our previous analysis that U_{ij}^V , as defined by (10), constitutes the lower bound of u_{ij} . This is the utility level that entrepreneur i receives when contracting with VC j in the absence of endogenous matching, assuming that the entire bargaining power rests with the VC. The maximum value of u_{ij} , on the other hand, is denoted by U_{ij}^E , which constitutes entrepreneur i 's expected utility in case the entire bargaining power rests with him, and not with VC j as considered above.¹⁵ The next lemma emphasizes an important property of the bargaining frontier defined by $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$.

Lemma 5 (Bargaining Frontier) *The bargaining frontier $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ is decreasing in the entrepreneur's reservation utility u_{ij} for $u_{ij} \in [U_{ij}^V, U_{ij}^E]$.*

Figure 1 visualizes the result highlighted by lemma 5. The bold curve depicts the bargaining frontier $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$, which consists of the Pareto-optimal utilities for the (i, j) pair.

We can now define the equilibrium in the VC market when each VC matches with one entrepreneur (*one-to-one matching*).

¹⁵As will become clear, our subsequent analysis does not rely on the specific functional form of U_{ij}^E . For completeness purposes, we formally derive U_{ij}^E in the Appendix.

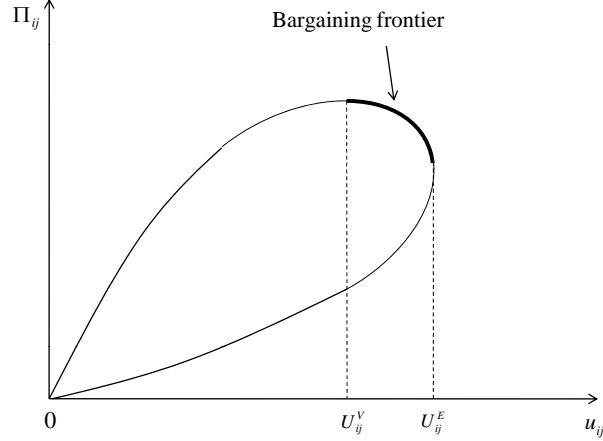


Figure 1: The Bargaining Frontier

Definition 1 (VC Market Equilibrium) *An equilibrium in the VC market consists of a one-to-one matching function $m : E \rightarrow V$ and payoff allocations $\Pi^* : V \rightarrow \mathbb{R}$ and $u^* : E \rightarrow \mathbb{R}_+$ that satisfy the following two conditions:*

- (i) *Feasibility of (Π^*, u^*) with respect to m : For all $i \in E$, $\{\Pi^*(m(i)), u^*(i)\}$ is on the bargaining frontier $\bar{\Pi}(u_{i,m(i)}, \Omega_{i,m(i)})$.*
- (ii) *Stability of m with respect to $\{\Pi^*, u^*\}$: There do not exist a pair $(i, j) \in E \times V$, where $m(i) \neq j$, and outside value $u > u^*(i)$ such that $\bar{\Pi}_{ij}(u, \Omega_{ij}) > \Pi^*(j)$.*

The two conditions guarantee the existence of a stable matching equilibrium in the VC market.¹⁶ Specifically, the *feasibility* condition requires that the payoffs for VCs and entrepreneurs must be attainable, which is guaranteed whenever the payoffs of any $(i, m(i))$ pair are on the bargaining frontier $\bar{\Pi}_{i,m(i)}(u_{i,m(i)}, \Omega_{i,m(i)})$. Moreover, the *stability* condition ensures that all matched VCs and entrepreneurs cannot become strictly better off by breaking their current partnership (and matching with a new VC or entrepreneur).

¹⁶Note that we consider a Nash equilibrium in a simultaneous move game, i.e., all VCs make their contract offers simultaneously, and then, all entrepreneurs accept their contracts at the same time. However, since the idea qualities μ_i , $i = 1, \dots, n_E$, of entrepreneurs as well as the expertise x_j , $j = 1, \dots, n_{VC}$, of VCs are common knowledge, we would obtain the same matching equilibrium in a sequential move game.

We would obtain *positive assortative matching* (PAM) whenever entrepreneurs with high quality ideas are matched with VCs enjoying high expertise. The opposite occurs with *negative assortative matching* (NAM). The next definition formalizes the characteristics of positive versus negative assortative matching.

Definition 2 (Assortative Matching in the VC Market) *Consider two entrepreneurs i and i' with idea qualities $\mu_i > \mu_{i'}$, and suppose that entrepreneur i is matched with VC $j = m(i)$ and entrepreneur i' is matched with VC $j' = m(i')$. The matching equilibrium is positive assortative (PAM) if the VCs' expertise satisfy $x_j > x_{j'}$; and negative assortative (NAM) if $x_{j'} > x_j$.*

Applying the criteria identified by Legros and Newman (2007) to our framework, we can infer that the matching equilibrium is positive assortative if: (i) the cross-partial derivative of the bargaining frontier $\bar{\Pi}_{ij}(u_{ij}, \Omega_{ij})$ with respect to the entrepreneur's idea quality μ_i and the VC's expertise x_j is positive, i.e., $\partial^2 \bar{\Pi}_{ij} / (\partial \mu_i \partial x_j) \geq 0$; and (ii), it is relatively easier for a high (versus low) expertise VC to transfer surplus to an entrepreneur, i.e., $\partial^2 \bar{\Pi}_{ij} / \partial u_{ij} \partial x_j \geq 0$. The first condition is the standard complementarity condition that guarantees positive assortative matching in models with transferable utility (see Shapley and Shubik (1972) and Becker (1973)). However, as shown by Legros and Newman, this is *not* a sufficient condition to guarantee PAM whenever utility is non-transferable, as in our framework.¹⁷

The next lemma derives sufficient conditions for PAM to arise in the VC market.

Lemma 6 (PAM in the VC Market) *The matching between VCs and entrepreneurs is positive assortative (PAM) if the following two conditions are satisfied:*

- (i) *The profit function $\pi(K_{ij}, \Omega_{ij})$ is increasing and concave in the capital investment K_{ij} and match quality Ω_{ij} , with complementarity between the two, i.e. $\partial^2 \pi(K_{ij}, \Omega_{ij}) / (\partial K_{ij} \partial \Omega_{ij}) \geq 0$.*

¹⁷In our framework, utility can be transferred via the investment K and profit share λ . These two instruments, however, transfer surplus imperfectly as they also affect the size of the surplus. Due to entrepreneurs' limited liability, side payments from entrepreneurs to VCs are not feasible, which is an important characteristic of VC markets; see Sørensen (2007) for a discussion.

(ii) The match quality $\Omega_{ij} = \Omega(\mu_i, x_j)$ is increasing in the entrepreneur's idea quality μ_i and the VC's expertise x_j , with complementarity between the two, i.e. $\partial^2 \Omega(\mu_i, x_j) / (\partial \mu_i \partial x_j) \geq 0$.

Our assumption of the venture's gross profit $\pi(K_{ij}, \Omega_{ij})$ and the match quality function $\Omega(\mu_i, x_j)$ exhibiting diminishing marginal returns is not only very intuitive, but also implies (in addition to the two emphasized complementarity conditions) that matching in the VC market is positive assortative. Moreover, PAM is also efficient from a social perspective. Our conditions guarantee that a social planner would also prefer matching to be positive assortative. Interestingly, the conditions that guarantee that PAM is socially efficient are weaker than the ones stated in Lemma 6 because it is ultimately irrelevant for the social planner how easily the matched parties can transfer surplus.

Given there is PAM, the top n_{VC} entrepreneurs will match with the n_{VC} venture capitalists. The remaining $n_E - n_{VC}$ entrepreneurs will remain unmatched. Therefore, the market consists of n_{VC} matched VC-entrepreneur pairs. The highest quality entrepreneur is n_E and the lowest quality is 1. Hence, the last matched entrepreneur is $n_E - n_{VC} + 1$.

4.2 Matching and the Design of Venture Capital Contracts

Before we can draw any inferences about matching and the optimal design of VC contracts, it is essential to first derive conditions for a given entrepreneur-VC match being affected by an improved alternative for the entrepreneur as captured by a higher reservation utility u_{ij} . This is important because some matched entrepreneurs might be "out of reach" of other VCs. For these partnerships, alternative matches are irrelevant, and the optimal VC contract is thus identical to the one derived in Section 3 in the absence of endogenous matching.

Consider two adjacent entrepreneur-VC matches $(i, m(i))$ and $(i - 1, m(i - 1))$. Lemma 6 provides a useful characterization of the matching problem. Accordingly, entrepreneur i is the most desirable entrepreneur for all VCs that are ranked below VC $m(i)$, but among those, it is

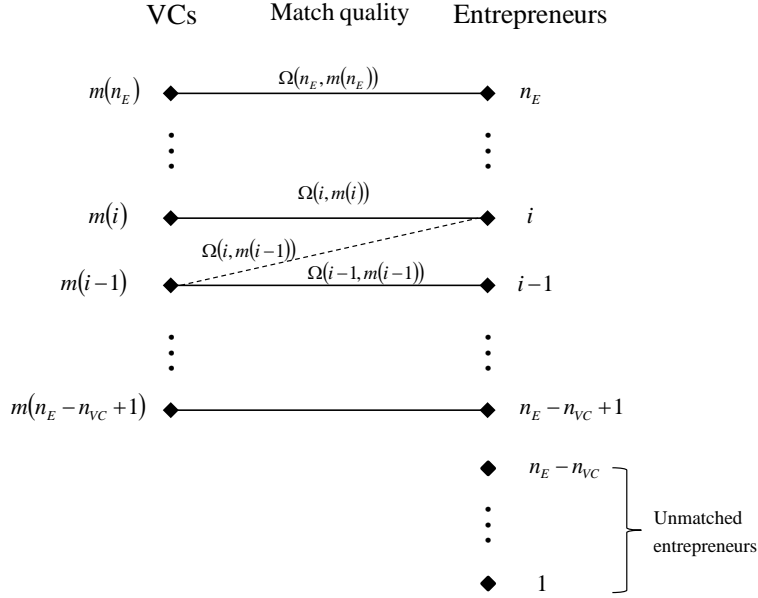


Figure 2: The VC Market Equilibrium with Positive Assortative Matching (PAM)

VC $m(i - 1)$ which has the highest willingness to pay for entrepreneur i .¹⁸ Therefore, VC $m(i)$ directly competes for entrepreneur i only with VC $m(i - 1)$ (i.e., the one ranked directly below $m(i)$), which in turn allows us to define the outside option of entrepreneur i when he is matched with VC $m(i)$.¹⁹

Figure 2 is a graphical depiction of the VC market equilibrium with its corresponding ladder of matched pairs exhibiting PAM, and unmatched entrepreneurs that are endowed with sufficiently poor quality ideas so that they do not receive VC financing. Solid lines represent (stable) matches, while the dashed line represents the outside option of entrepreneur i , which is to contract with VC $m(i - 1)$.

¹⁸The positive cross-partial derivative of the bargaining frontier with respect to the entrepreneur's idea quality and VC's expertise, and the fact that it is easier to transfer surplus to an entrepreneur as the match quality of the specific venture increases, ensure that the highest bid for entrepreneur i , among all venture capitalists below $m(i)$, originates from venture capitalist $m(i - 1)$.

¹⁹In contrast, it is not optimal for VC $m(i)$ to make a contract offer to the lower ranked entrepreneur $i - 1$ because such a match would result in a lower expected profit for VC $m(i)$; see Lemma 4.

Let $u_{i,m(i-1)}$ denote the maximum utility that entrepreneur i can obtain from VC $m(i-1)$, provided that VC $m(i-1)$ does not become worse off relative to its current match with entrepreneur $i-1$. Formally, $u_{i,m(i-1)}$ is defined by

$$u_{i,m(i-1)} \equiv \max_{\{\lambda_{i,m(i-1)}, K_{i,m(i-1)}\}} \lambda_{i,m(i-1)}^2 \pi^2(K_{i,m(i-1)}, \Omega_{i,m(i-1)})/2 \quad (13)$$

s.t.

$$\lambda_{i,m(i-1)}(1 - \lambda_{i,m(i-1)})\pi^2(K_{i,m(i-1)}, \Omega_{i,m(i-1)}) - rK_{i,m(i-1)} = \Pi^*(m(i-1)),$$

where $\Omega_{i,m(i-1)}$ denotes the match quality when entrepreneur i matches with the lower ranked VC $m(i-1)$. Consequently, the equilibrium utility of entrepreneur i is $u^*(i) = \max \{u_{i,m(i-1)}, U_{i,m(i)}^V\}$.²⁰ According to the properties of the bargaining frontier as stated in Lemma 5, $u_{i,m(i-1)}$ is increasing in $\Omega_{i,m(i-1)}$, and decreasing in $\Pi^*(m(i-1))$. Hence, $u_{i,m(i-1)}$ depends negatively on the equilibrium profit of the VC that is right below entrepreneur i in the ladder (i.e., $m(i-1)$), and positively on the match quality $\Omega_{i,m(i-1)}$ with VC $m(i-1)$. A higher equilibrium profit $\Pi^*(m(i-1))$ reduces the incremental profit of VC $m(i-1)$ if it contracts with entrepreneur i instead of $i-1$. The maximum bid which VC $m(i-1)$ is willing to make, therefore decreases, all else equal. On the other hand, a higher match quality, $\Omega_{i,m(i-1)}$, increases VC $m(i-1)$'s incremental profit, and hence its willingness to pay for entrepreneur i .

Due to PAM in the VC market, it is clear that the higher expertise VC $m(i)$ can always outbid the lower expertise VC $m(i-1)$. Nevertheless, the improved alternative for the higher idea quality entrepreneur i might require the higher expertise VC $m(i)$ to adjust its contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ in order to transfer more surplus to the entrepreneur, and thus to ensure entrepreneur i accepts the contract. Technically, the higher expertise VC $m(i)$ needs to adjust the contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ whenever $u_{i,m(i-1)}$ (the maximum utility that entrepreneur i can

²⁰Note that $u_{i,m(i-1)}$ is the quasi-inverse of the bargaining frontier, $\bar{\Pi}(u_{i,m(i-1)}, \Omega_{i,m(i-1)})$, between entrepreneur i and VC $m(i-1)$. See Legros and Newman (2007) for the formal definition of the quasi-inverse of the bargaining frontier.

obtain from VC $m(i-1)$ with endogenous matching) exceeds $U_{i,m(i)}^V$ (the expected utility of entrepreneur i paired with VC $m(i)$ when the entire bargaining power rests with the VC and there is no endogenous matching).

When is the higher expertise VC $m(i)$ compelled to offer the higher idea quality entrepreneur i a more attractive contract? Put differently, when does the condition $U_{i,m(i)}^V < u_{i,m(i-1)}$ hold? Our first proposition establishes a condition under which matching is relevant for a given entrepreneur-VC pair $(i, m(i))$, and hence affects the associated VC contract, giving rise to the *endogenous matching effect*.

Proposition 1 (The Endogenous Matching Effect) *Consider the entrepreneur-VC match $(i, m(i))$ with $i \geq 2$. Then, there exists a threshold match quality $\bar{\Omega}_{i,m(i-1)}$, with $\bar{\Omega}_{i,m(i-1)} < \Omega_{i,m(i)}$, such that for*

- (i) $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$, *endogenous matching alters the contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ between entrepreneur i and VC $m(i)$, i.e., $U_{i,m(i)}^V < u_{i,m(i-1)}$;*
- (ii) $\Omega_{i,m(i-1)} \leq \bar{\Omega}_{i,m(i-1)}$, *endogenous matching does not alter the contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$, i.e., $U_{i,m(i)}^V \geq u_{i,m(i-1)}$.*

Proposition 1 provides an important implication about the role of endogenous matching in the VC market. Suppose a match between entrepreneur i and the lower ranked VC $m(i-1)$ results in a sufficiently high match quality $\Omega_{i,m(i-1)}$ (i.e., $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$). This is the case whenever the predominant force behind a match quality improvement is the idea quality μ_i of entrepreneur i . Intuitively, this can be observed whenever entrepreneurial ideas are sufficiently heterogenous, while the expertise of VCs is very similar. We infer the following:

Corollary 1 (Identifying the Endogenous Matching Effect) *The endogenous matching effect arises in VC markets if entrepreneurial idea qualities are relatively dispersed and/or the expertise of VCs is sufficiently concentrated.*

If $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$, entrepreneur i 's maximum expected utility $u_{i,m(i-1)}$ —which he could get by contracting with the lower ranked VC $m(i-1)$ —exceeds his expected utility $U_{i,m(i)}^V$ when accepting the previously derived contract $\Gamma_{ij}^* = \{\lambda_{ij}^*, K_{ij}^*\}$ from VC $m(i)$; see Lemma 1. However, we can infer from Lemma 4 that the higher expertise VC $m(i)$ can profitably match $m(i-1)$'s contract offer $\{\lambda_{i,m(i-1)}, K_{i,m(i-1)}\}$, while also making the higher idea quality entrepreneur $m(i)$ strictly better off. This in turn implies that VC $m(i)$ offers entrepreneur i a contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ that guarantees him an expected utility of at least $u_{i,m(i-1)}$ (his maximum expected utility when contracting with VC $m(i-1)$), which exceeds $U_{i,m(i)}^V$ (his expected utility in the absence of endogenous matching). Therefore, while the entrepreneur-VC match $(i, m(i))$ is mutually beneficial, and—according to Lemma 6—constitutes the equilibrium outcome, matching considerations nonetheless alter entrepreneur i 's equilibrium contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ in his favor as endogenous matching forces VC $m(i)$ to cede some of the surplus in order to attract entrepreneur i . This in turn suggests that entrepreneurs generally benefit from the matching process in the VC market. In other words, if $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$ holds, each matched entrepreneur $i \in \{1, \dots, n_E - n_{VC} + 2\}$ (except for the one with the lowest idea quality $\mu_{n_E - n_{VC} + 1}$) earns a *matching rent*.

By applying the Envelope Theorem to (13), one can show that entrepreneur i 's maximum expected utility when being matched with the lower ranked VC $m(i-1)$, $u_{i,m(i-1)}$, is increasing in the corresponding match quality $\Omega_{i,m(i-1)}$. Thus, entrepreneur i extracts a higher *matching rent* whenever the match quality difference $\Omega_{i,m(i)} - \Omega_{i,m(i-1)}$ becomes smaller. Intuitively, the competition among VC $m(i)$ and VC $m(i-1)$ for entrepreneur i 's business idea then becomes more severe, allowing entrepreneur i to capture more of the surplus from the venture in equilibrium. We therefore state the following:

Corollary 2 (The Magnitude of the Endogenous Matching Effect) *The endogenous matching effect is stronger in VC markets the more dispersed are entrepreneurial idea qualities and/or the more concentrated is the expertise of VCs.*

So far, we have examined whether VCs are forced to adjust their contracts due to endogenous matching. We now investigate in detail how matching alters the profit share $\lambda_{i,m(i)}$ for the entrepreneur as well as the capital investment $K_{i,m(i)}$. According to Lemma 1, we need to distinguish between two cases. First, if $\Omega_{i,m(i-1)} \leq \bar{\Omega}_{i,m(i-1)}$, entrepreneur i 's participation constraint (12) does not bind for the contract $\{\lambda_{i,m(i)}^*, K_{i,m(i)}^*\}$ as derived in Section 3 in the absence of endogenous matching. Thus, there is clearly no need for the corresponding VC $m(i)$ to adjust its contract offer as entrepreneur i is better off accepting this contract rather than pursuing his best alternative (i.e., contracting with VC $m(i-1)$). As a result, if $\Omega_{i,m(i-1)} \leq \bar{\Omega}_{i,m(i-1)}$, the matching process does not affect the optimal VC contracts as derived in Lemma 2.

On the other hand, if $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$, then endogenous matching results in an adjusted contract offer $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ from VC $m(i)$. This can be observed because without such an adjustment, VC $m(i-1)$ would simply outbid VC $m(i)$ in order to attract entrepreneur i . Technically, entrepreneur i 's participation constraint (12) is violated for the contract $\{\lambda_{i,m(i)}^*, K_{i,m(i)}^*\}$ (see Lemma 2). Matching in the VC market thereby has an impact on the contractual arrangements between VCs and entrepreneurs.

For the remainder of this paper, we assume that $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$ holds: the competition for entrepreneur i 's business idea with quality μ_i forces VC $m(i)$ to cede more of the surplus through adjusting its contract offer $\{\lambda_{i,m(i)}^M, K_{i,m(i)}^M\}$.²¹ The next lemma characterizes the optimal adjusted VC contract $\{\lambda_{i,m(i)}, K_{i,m(i)}\}$ in case matching matters (i.e., $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$).

Lemma 7 (VC Contracting with Endogenous Matching) *Suppose $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$. Then, the optimal contract between entrepreneur i and VC $m(i)$ comprises the profit share*

$$\lambda_{i,m(i)}^M = \frac{\sqrt{2u^*(i)}}{\pi(K_{i,m(i)}^M, \Omega_{i,m(i)})} \quad (14)$$

²¹This case is also empirically more relevant; see Sørensen (2007).

for the entrepreneur, and the capital investment $K_{i,m(i)}^M$ by the VC implicitly defined by

$$\frac{\partial \pi(K_{i,m(i)}^M, \Omega_{i,m(i)})}{\partial K_{i,m(i)}} = \frac{r}{\sqrt{2u^*(i)}}. \quad (15)$$

The optimal contract in the presence of endogenous matching, $\{\lambda_{i,m(i)}^M, K_{i,m(i)}^M\}$, as emphasized by Lemma 7 will play an important role in the next section when we investigate how economic cycles affect the efficiency of VC contracts. From a closer inspection of (15) it becomes clear that VC $m(i)$'s investment $K_{i,m(i)}^M$ is increasing in the value of entrepreneur i 's outside option $u^*(i)$. Since a higher investment also motivates the entrepreneur to implement more effort (see (6)), it follows that his outside option $u^*(i)$ also has an indirect and positive effect on his equilibrium effort e_{ij}^* , and thus on the probability that his venture will be successful. To summarize, endogenous matching in the VC market improves the incentives for VCs to invest in new ventures, and at the same time, enhances entrepreneurs' incentives to exert effort in order to turn their innovative ideas into marketable products. However, does endogenous matching eventually lead to the first-best investment K_{ij}^{fb} and effort level e_{ij}^{fb} ? The next proposition proves that—despite the adjustments engendered by the matching process—the investment and effort levels remain sub-efficient from a social perspective.

Proposition 2 (VC Contracting with versus without Endogenous Matching) *The optimal contract $\{\lambda_{i,m(i)}^M, K_{i,m(i)}^M\}$ between entrepreneur i and VC $m(i)$ with endogenous matching exhibits greater investment and effort levels compared to the contract $\{\lambda_{i,m(i)}^*, K_{i,m(i)}^*\}$ in the absence of endogenous matching, i.e., $K_{i,m(i)}^M > K_{i,m(i)}^*$ and $e_{i,m(i)}^M > e_{i,m(i)}^*$. However, the investment $K_{i,m(i)}^M$ and effort $e_{i,m(i)}^M$ with endogenous matching are still below their socially efficient levels, i.e., $K_{i,m(i)}^M < K_{i,m(i)}^{fb}$ and $e_{i,m(i)}^M < e_{i,m(i)}^{fb}$.*

Finally, the following lemma shows that, as one would expect, VCs and entrepreneurs with superior ideas are better off:

Lemma 8 (Entrepreneurial Ideas and Payoffs) *The expected profit of VCs and the expected utility of entrepreneurs are increasing in the quality of entrepreneurial ideas, i.e., $\Pi^*(m(i)) \geq \Pi^*(m(i-1))$ and $u^*(i) \geq u^*(i-1)$ for $i \in E = \{n_E - n_{VC} + 1, \dots, n_E\}$.*

5 Economic Cycles and Venture Capital Contracts

We now examine how economic expansions and contractions affect equilibrium contracts in the VC market taking into account the *endogenous matching effect*. We address the following questions. First, are entrepreneurs, VCs, or both, better off during expansions? Second, how does the economic environment impact investments in new ventures? Finally, how sensitive is the survival rate of new ventures to economic cycles? Clearly, substantially more capital is available during economic expansions, which results in new VCs entering the market (see, e.g. Hochberg, Ljungqvist, and Vissing-Jørgensen (2008)). We therefore focus on the effects on VC contracts resulting from entry and exit of VCs during economic expansions and contractions, while keeping the number of entrepreneurs constant.

5.1 Economic Expansions

We begin by elaborating on the effects of economic expansions on equilibrium contracts in the VC market. Suppose a new VC enters the market, which then competes with incumbent VCs for entrepreneurs with high quality ideas.²² Intuitively, due to its limited experience relative to incumbents, one can expect the new VC to exhibit a relatively lower expertise compared to some (if not most) incumbents.

²²One can easily derive a condition for market entry being optimal for a VC during an economic expansion. Intuitively, entry is more likely to be optimal whenever expected profits of new ventures increase (as e.g. in economic booms), or alternatively, costs of entry decrease. However, the subsequently discussed implications of entry in the VC market—which originate from the identified *endogenous matching effect*—still hold when allowing for endogenous entry.

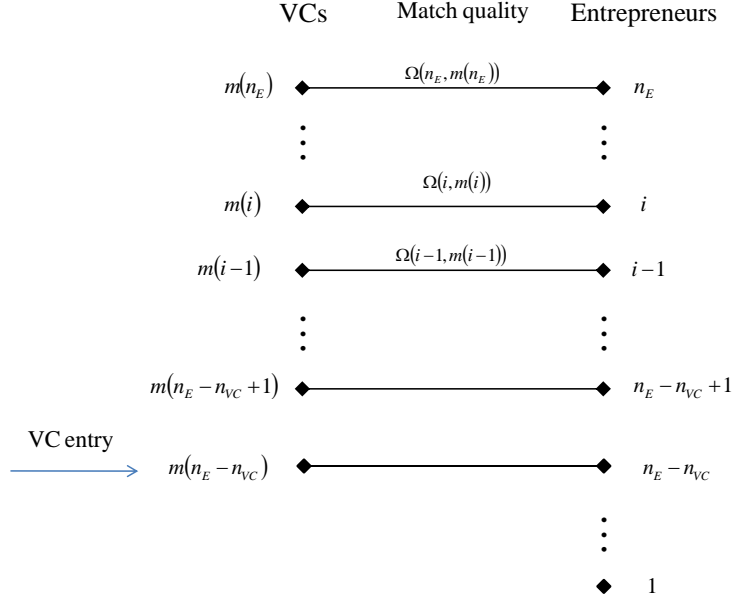


Figure 3: The VC Market in an Economic Expansion

Suppose the new VC has the lowest expertise among all VCs in the market, and thus enters at the bottom of the ladder. This situation is illustrated by Figure 3. In the new matching equilibrium, the entrant VC $m(n_E - n_{VC})$ (that is endowed with the lowest expertise) is matched with entrepreneur $n_E - n_{VC}$, who, without entry, would have remained unmatched. Thus, entrepreneur $n_E - n_{VC}$ clearly benefits from the entry of VC $m(n_E - n_{VC})$ as he is now equipped with sufficient capital to pursue his own business idea. More interesting, however, is the implication of entry at the bottom of the ladder for the entrepreneur-VC pairs above the point of entry. To identify the different effects, consider first entrepreneur $n_E - n_{VC} + 1$ who is matched with VC $m(n_E - n_{VC} + 1)$ in equilibrium. Without entry, the reservation utility of entrepreneur $n_E - n_{VC} + 1$ would have been zero as no other VC would have offered him a contract. After entry occurred, however, the new VC $m(n_E - n_{VC})$ is now willing to also offer a contract to entrepreneur $n_E - n_{VC} + 1$, which in turn improves his outside option. We can infer from Lemma 6 that in equilibrium, entrepreneur $n_E - n_{VC} + 1$ is still matched with VC $m(n_E - n_{VC} + 1)$. According to Lemma 2, however, VC $m(n_E - n_{VC} + 1)$ must now transfer more of the surplus to its entrepreneur $n_E - n_{VC} + 1$ to account for his improved outside option.

Specifically, VC $m(n_E - n_{VC} + 1)$ is now forced to provide its entrepreneur $n_E - n_{VC} + 1$ with more capital, which in turn motivates a higher effort level, and thus improves the venture's likelihood of survival. On the other hand, the overall profitability for VC $m(n_E - n_{VC} + 1)$ is reduced, which in turn makes it optimal to offer the next best entrepreneur $n_E - n_{VC} + 2$ a more attractive contract (which follows from (13)). The higher ranked VC $m(n_E - n_{VC} + 2)$ is then forced to adjust the contract offer for its entrepreneur $n_E - n_{VC} + 2$ in order to match his improved outside option, and so on. By inductive reasoning, we can infer that VC entry has a *ripple effect* across the entire ladder of entrepreneur-VC matches. According to this logic, in an economic expansion that results in entry by a VC at the bottom of the ladder, all VC-backed ventures will experience greater capital investments, and ultimately, higher success rates.

To summarize, our analysis reveals that entry of VCs following an economic boom benefits all entrepreneurs, and not just the ones that are financed by entrants. The incumbent VCs, on the other hand, are then compelled to transfer more of the surplus to their entrepreneurs (through more attractive VC contracts), which is rooted in the *endogenous matching effect*. Nonetheless, entry improves the overall efficiency of the VC market: new ventures are now equipped with more capital, which in turn enhances their prospects of success. The key mechanism behind this implication is the interdependence of all ventures through the endogenous outside options of entrepreneurs. In this sense, endogenous matching has a *ripple effect* in the VC market as entrepreneurs benefit from intensified competition originating from VC entry. This prediction is consistent with evidence in Gompers and Lerner (2000) that the more intense the competition, the higher the amount of funding each portfolio company receives.

5.2 Economic Contractions

The *ripple effect* can turn into an *unraveling effect* during an economic contraction. Suppose that—due to a reduction in the supply of capital—some VCs are forced to exit the market, which clearly mitigates the extent of competition among the remaining VCs. Exit potentially triggers

a wave of failures as it travels up the ladder. To see this, note that exit of a VC near the bottom of the ladder imposes a negative externality on all the above ventures, which follows by simply reversing the logic behind VC entry as described above.²³ All the above ventures become more likely to fail because of their lower capital endowments in the new matching equilibrium. Thus, failures near the bottom of the ladder can trigger a crisis in the VC market, implying that low quality ventures (financed by low expertise VCs) may be "too small to fail".²⁴ Interestingly, while entrepreneurs are adversely affected by economic downturns, all VCs that remain in the market benefit from the limited competition for entrepreneurial ideas.

An implication of our analysis is that policymakers should assist ventures at the lower points of the ladder rather than ventures near the top because such interventions have the maximum impact on the efficiency of the *entire* VC market. Indeed, failures near the top of the ladder do not impose a negative externality on all entrepreneur-VC pairs below. On the other hand, very high quality matches may be immune from turbulence at the bottom. Suppose there is a significant "break" in the ladder between two groups of entrepreneur-VC matches; for example, there is a collection of ventures that are of very high quality (e.g., the "outliers" that receive much attention in the popular press), while there is another collection consisting of lower quality ventures. According to Proposition 1, if the differences in quality between the two groups are sufficiently large, such that entrepreneurs in one group are "out of reach" for VCs in the other group, then failure in the bottom group may not affect the prospect of success in the top group.

6 Venture Capitalists Financing Multiple Entrepreneurs

To keep our analysis as simple and intuitive as possible without compromising the quality of our insights, we have focused on one-to-one matching in the VC market. In this section, we

²³We note that failure can occur anywhere in the ladder, but it is more likely to happen at the lower points.

²⁴Note that a venture with a low match quality is characterized by a low equilibrium capital investment, and is therefore by nature "small".

briefly discuss how allowing each VC to contract with multiple entrepreneurs would affect our main results.²⁵

For simplicity, suppose the VC with the highest expertise, VC n_{VC} , now wants to contract with entrepreneurs n_E and $n_E - 1$, who have the most promising business ideas. We know from Lemma 4 that each VC would strictly benefit from being matched with a higher quality entrepreneur. The next lower ranked VC $n_{VC} - 1$ therefore competes for both high quality entrepreneurs n_E and $n_E - 1$, but can always be overbid by the higher ranked VC n_{VC} ; see also our discussion in Section 4.2. Thus, the matching equilibrium is still positive assortative: VC n_{VC} (possessing the highest expertise x_{VC}) matches with entrepreneurs n_E and $n_E - 1$ (providing the highest idea qualities μ_{n_E} and μ_{n_E-1}), whereas VC $n_{VC} - 1$ matches with entrepreneur $n_E - 2$.

What are the specific implications for the VC contracts? Consider first the contract between VC n_{VC} and entrepreneur $n_E - 1$. Clearly, if matching matters, VC n_{VC} needs to transfer more surplus in order to retain entrepreneur $n_E - 1$. However, compared to our previous one-to-one matching equilibrium, the higher expertise VC $n_{VC} - 1$ instead of the lower ranked VC $n_{VC} - 2$ is now competing for entrepreneur $n_E - 1$, which results in a higher bid. This in turn forces VC n_{VC} to provide its entrepreneur $n_E - 1$ with even more capital, making the venture even more likely to succeed.

Next, consider the contract between VC n_{VC} and entrepreneur n_E (i.e., the entrepreneur with the highest quality idea). Clearly, the best outside option for entrepreneur n_E is to contract with the next best VC $n_{VC} - 1$. However, compared to our previous one-to-one matching equilibrium, VC $n_{VC} - 1$ is now matched with the lower quality entrepreneur $n_E - 2$ (instead of entrepreneur $n_E - 1$). Thus, VC $n_{VC} - 1$ would now benefit even more from being matched with entrepreneur n_E , which results in a higher bid. To contract with entrepreneur n_E , VC

²⁵Inderst, Müller, and Münnich (2007) and Fulghieri and Sevilir (2009) explicitly model the problem of a VC that finances multiple ventures, though in the absence of endogenous matching.

n_{VC} is now compelled to provide even more capital as compared to the one-to-one matching equilibrium, thereby boosting the venture's prospect of survival.

To summarize, allowing VCs to contract with multiple entrepreneurs would reinforce our main implication: Endogenous matching leads to more efficient capital endowments of new ventures, which in turn enhances their prospects of success.

7 Empirical Implications

The model yields a series of empirically testable predictions with regard to the roles of entry and exit in the VC market. We make predictions about the impact of VC entry on entrepreneur-VC pairs above and below the match quality of the entrants; these predictions pertain to the magnitude of the investment and the likelihood of success (or survival) of individual ventures. For such tests to be feasible, it is necessary to rank ventures according to their match quality. Sørensen (2007) resolved the econometric issues associated with two-sided matching in the VC market, while Hochberg, Ljungqvist, and Lu (2010) examined the properties of entry and exit in the VC market. Thus, the required ingredients are available to test our empirical predictions.

There are numerous empirical proxies one may use to rank the expertise of VCs, which is a requisite for understanding the matching process. VCs are significantly heterogeneous in terms of their broad characteristics, experience, skills, and reputation (see Sørensen (2007), Kaplan and Strömberg (2003, 2004), Hsu (2004), Kaplan and Schoar (2005), Kaplan, Sensoy, and Strömberg (2009), and Hamza and Kooli (2010)). In addition to providing entrepreneurs with the required financial resources to start their own businesses, VCs bring in specific expertise, which is particularly invaluable for entrepreneurs with little or no business-related skills. Specifically, VCs monitor and are commonly represented on the boards of directors (Lerner (1995), and Gompers and Lerner (1999)); provide technical and commercial advice (Bygrave and Tymmons (1992)); provide access to valuable networks and meet with suppliers and cus-

tomers (Gorman and Sahlman (1989), and Hochberg, Ljungqvist, and Lu (2007, 2010)); help in designing business strategy and human resource policies, and have contacts with potential managers (Hellman and Puri (2002)); and assist with the creation of strategic alliances (Lindsey (2008)). The expertise of VCs can therefore be crucial for the exploration of innovative ideas with market potential, and hence for the success of new ventures.²⁶ Given their specific expertise, the key challenge for VCs is not only to identify entrepreneurs with suitable ideas, but also to attract them by offering mutually beneficial contracts.

Our entry and exit analysis also has implications in terms of the business cycle. The technology bubble and subsequent crash in 2001 highlighted the volatile nature of VC financing. The model provides a framework within which to examine the peaks and troughs of such cycles. We showed that endogenous matching leads to a *ripple effect* in an expansion, and an *unraveling effect* in a contraction. That is, endogenous matching between entrepreneurs and VCs *amplifies* business cycle effects. Hence, observed volatility in the VC market may in part be due to such considerations, rather than the underlying economic forces driving the business cycle.

The empirical literature has highlighted two effects pertaining to the role of VC financing in the determination of entrepreneurial success. First, there is the *selection effect*: higher ability VCs select superior projects to invest in. Second, there is the *treatment effect*: higher ability VCs are endowed with superior expertise, thereby providing greater value-added to the venture. By virtue of modeling the endogenous matching process between heterogeneous entrepreneurs and VCs, which engenders an endogenous reservation utility for each entrepreneur, we introduced the notion of the *endogenous matching effect*. Sørensen (2007) quantified the selection and treatment effects. Future empirical work should follow along these lines by evaluating the importance of the *endogenous matching effect* relative to the selection and treatment effects. Corollary 1 provides conditions under which the *endogenous matching effect* arises,

²⁶Indeed, Kaplan and Schoar (2005) provide empirical evidence suggesting that the experience of an individual general partner (which can be considered as a proxy for the expertise of the financier) has a positive effect on the overall performance of equity partnerships.

while Corollary 2 provides conditions under which the effect becomes stronger. These conditions may be used as guidelines by empiricists to identify and quantify the effect. Furthermore, the *endogenous matching effect* amplifies the impact of the business cycle on the VC market; thus, empiricists should take into account that the relative importance of the effect may vary with the business cycle.

8 Conclusion

We consider a VC market that is comprised of VCs that are heterogeneous with respect to their expertise, and entrepreneurs that are heterogeneous with respect to the quality of their ideas. Within this market, we study how the two sides are matched endogenously. Our main interest is how endogenous matching affects equilibrium contracts and the survival rate of new ventures. First, we show that, under reasonable conditions, matching is positive assortative (PAM): VCs with high expertise match with entrepreneurs that have high quality business ideas. The key property of our matching framework is that each entrepreneur's outside option is endogenous as he could contract with an alternative VC. We show that endogenous matching—which originates in the competition among VCs for high quality ideas—increases the outside option of entrepreneurs because it allows them to contract alternatively with a different VC. This forces VCs to make more attractive contract offers to entrepreneurs, which is achieved via greater capital provisions (*endogenous matching effect*). Better capital endowments of new ventures, on the other hand, boost the effort levels of entrepreneurs, and therefore improve the survival rate of their start-ups. While investment and effort levels always remain sub-optimal (i.e., below first-best), we find that endogenous matching improves their respective efficiency from a social perspective.

Our framework provides another important insight: intensified competition for entrepreneurial ideas—triggered by market entry of new VCs—improves the overall efficiency of the VC

market (the *ripple effect*), which is consistent with evidence in Gompers and Lerner (2000). Specifically, new ventures are then equipped with more venture capital, which in turn enhances their prospects of success. From a policy perspective, the positive externality stemming from the *endogenous matching effect* should be exploited in order to spur innovative activities. In this sense, lowering barriers to entry for VCs, e.g. by reducing capital requirements or administrative costs, would not only benefit entrepreneurs whose ventures will then get funded. Also entrepreneurs contracting with incumbent VCs would benefit as more intensive competition improves the capital endowments of their ventures (the *endogenous matching effect*). Spurring entry into the VC market has therefore a multiplier effect on innovative activities, which are crucial for economic growth. To take advantage of this multiplier effect, and thus to motivate innovation, it is indispensable to eliminate regulatory hurdles for investors when financing entrepreneurial ventures.

Our model suggests the following theoretical extensions to gain a more complete understanding of the VC market. First, the contractual framework could be more sophisticated by allowing for convertible securities, as in Schmidt (2003). Second, the roles of control rights and non-pecuniary benefits could be explored, following along the lines of Hellmann (1998) and Kirilenko (2001), for example.

Appendix

Proof of Lemma 1

The socially efficient effort level e_i^{fb} as well as the efficient capital investment K_{ij}^{fb} are characterized by the following two first-order conditions:

$$\pi(K_{ij}, \Omega(\mu_i, x_j)) = e_i \quad (16)$$

$$\frac{\partial \pi(\cdot)}{\partial K_{ij}} e_i = r. \quad (17)$$

Substituting (16) into (17) yields the lemma. \square

Proof of Lemma 2

By accounting for the entrepreneur's effort choice e_{ij}^* as given by (6), the optimal contract components λ_{ij}^* and K_{ij}^* are implicitly characterized by the following two first-order conditions:

$$2\lambda_{ij}^*(1 - \lambda_{ij}^*)\pi(K_{ij}^*, \Omega_{ij}) \frac{\partial \pi(K_{ij}^*, \Omega_{ij})}{\partial K_{ij}} = r \quad (18)$$

$$(1 - 2\lambda_{ij}^*)\pi^2(K_{ij}^*, \Omega_{ij}) = 0. \quad (19)$$

Solving (19) for λ_{ij}^* , and substituting the resulting expression into (18), yields the lemma. \square

Proof of Lemma 3

Recall from Lemma 2 that $K_{ij}^*(\Omega_{ij})$ is implicitly characterized by

$$\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}} = 2r. \quad (20)$$

Implicit differentiating (20) yields

$$\frac{dK_{ij}^*(\Omega_{ij})}{d\Omega_{ij}} = -\frac{\frac{\partial\pi(\cdot)}{\partial\Omega_{ij}}\frac{\partial\pi(\cdot)}{\partial K_{ij}} + \pi(K_{ij}, \Omega_{ij})\frac{\partial^2\pi(\cdot)}{\partial K_{ij}\partial\Omega_{ij}}}{\frac{\partial}{\partial K_{ij}}\left[\pi(K_{ij}, \Omega_{ij})\frac{\partial\pi(\cdot)}{\partial K_{ij}} - 2r\right]} \quad (21)$$

Note that the denominator is strictly negative due to the second-order condition. Moreover, recall that $\partial^2\pi(\cdot)/(\partial K_{ij}\partial\Omega_{ij}) \geq 0$. Thus, $dK_{ij}^*(\Omega_{ij})/d\Omega_{ij} > 0$. Finally, observe that $d\lambda_{ij}^*/d\Omega_{ij} = 0$. \square

Proof of Lemma 4

Applying the Envelope Theorem to (8) yields $d\Pi_{ij}^{VC}/d\Omega_{ij} > 0$. Next, the entrepreneur's expected utility under the unconstrained contract $\Gamma_{ij}^*(\Omega_{ij}) = \{\lambda_{ij}^* = 1/2, K_{ij}^*(\Omega_{ij})\}$ can be written as

$$U_i(e_{ij}^*, K_{ij}^*, \Omega_{ij}) = \frac{1}{2}\pi(K_{ij}^*, \Omega_{ij})e_{ij}^* - \frac{(e_{ij}^*)^2}{2}. \quad (22)$$

Applying the Envelope Theorem yields

$$\frac{dU_i}{d\Omega_{ij}} = \frac{1}{2}\left[\frac{\partial\pi}{\partial K_{ij}^*}\frac{dK_{ij}^*}{d\Omega_{ij}} + \frac{\partial\pi}{\partial\Omega_{ij}}\right]. \quad (23)$$

Recall from Lemma 3 that $dK_{ij}^*/d\Omega_{ij} > 0$. Thus, $dU_i/d\Omega_{ij} > 0$. \square

Proof of Lemma 5

At the bargaining frontier, the constraint (12) must be binding. Using e_{ij}^* as defined by (6), the binding constraint can be written as

$$\frac{1}{2}\lambda_{ij}^2(\pi(K_{ij}, \Omega_{ij}))^2 = u_{ij}.$$

Consequently, λ_{ij}^* must satisfy

$$\lambda_{ij}^* = \frac{\sqrt{2u_{ij}}}{\pi(K_{ij}, \Omega_{ij})}. \quad (24)$$

Substituting (24) and e_{ij}^* , as defined by (6), into (11) yields the unconstrained maximization problem for venture capitalist j :

$$\max_{\{K_{ij}\}} \sqrt{2u_{ij}}\pi(K_{ij}, \Omega_{ij}) - 2u_{ij} - rK_{ij}.$$

The optimal capital provision $K^*(u_{ij}, \Omega_{ij})$ is implicitly characterized by the first-order condition:

$$\sqrt{2u_{ij}} \frac{\partial \pi(K_{ij}, \Omega_{ij})}{\partial K_{ij}} - r = 0. \quad (25)$$

Thus, the frontier contract $\{\lambda_{ij}^*, K_{ij}^*\}$ satisfies

$$\frac{\partial \pi(K_{ij}^*, \Omega_{ij})}{\partial K_{ij}} = \frac{r}{\sqrt{2u_{ij}}} \text{ and } \lambda_{ij}^* = \frac{\sqrt{2u_{ij}}}{\pi(K_{ij}^*, \Omega_{ij})}. \quad (26)$$

Next, we can infer from (25) that

$$\frac{dK_{ij}^*}{du_{ij}} = -\frac{\frac{\partial \pi(\cdot)}{\partial K_{ij}}}{2u_{ij} \frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2}} > 0 \text{ and } \frac{dK_{ij}^*}{d\Omega_{ij}} = -\frac{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}}}{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2}} > 0. \quad (27)$$

Thus, a higher outside option u_{ij} and/or a higher match quality Ω_{ij} lead to a higher capital provision K_{ij}^* . Next, by using (24) and (27), one gets

$$\frac{d\lambda_{ij}^*}{du_{ij}} = \frac{\pi(\cdot) + \left(\frac{\partial \pi(\cdot)}{\partial K_{ij}}\right)^2 / \left(\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2}\right)}{\sqrt{2u_{ij}}\pi^2(\cdot)}. \quad (28)$$

Substituting λ_{ij}^* (as defined by (24)) and e_{ij}^* (as defined by (6)) into (11) eventually yields the bargaining frontier $\bar{\Pi}(\cdot)$ with

$$\bar{\Pi}(u_{ij}, \Omega_{ij}) = \sqrt{2u_{ij}}\pi(K_{ij}^*(u_{ij}, \Omega_{ij}), \Omega_{ij}) - 2u_{ij} - rK_{ij}^*(u_{ij}, \Omega_{ij}). \quad (29)$$

Differentiating the bargaining frontier $\bar{\Pi}(u_{ij}, \Omega_{ij})$ with respect to u_{ij} leads to

$$\frac{d\bar{\Pi}(\cdot)}{du_{ij}} = \frac{1}{\sqrt{2u_{ij}}}\pi(\cdot) + \sqrt{2u_{ij}}\frac{\partial\pi(\cdot)}{\partial K_{ij}^*}\frac{dK_{ij}^*}{du_{ij}} - 2 - r\frac{dK_{ij}^*}{du_{ij}}, \quad (30)$$

which, by using (25), can be simplified to

$$\frac{d\bar{\Pi}(\cdot)}{du_{ij}} = \frac{1}{\sqrt{2u_{ij}}}\pi(\cdot) - 2. \quad (31)$$

When the derivative $\frac{d\bar{\Pi}(\cdot)}{du_{ij}}$ is evaluated at the lowest point $u_{ij} = U_{ij}^V$ (see (10)), it is zero. Hence, for any u_{ij} with $u_{ij} > U_{ij}^V$, the slope of the bargaining frontier $\bar{\Pi}(u_{ij}, \Omega_{ij})$ is negative. \square

Proof of Lemma 6

We verify whether the sufficient conditions for positive assortative matching (PAM) are satisfied. First, recall from Proof of Lemma 5 VC j 's constrained profit function $\bar{\Pi}_{ij}(\cdot)$ when matched with entrepreneur i :

$$\bar{\Pi}_{ij}(u_{ij}, \Omega(\mu_i, x_j)) = \sqrt{2u_{ij}}\pi(K_{ij}^*(u_{ij}, \Omega(\mu_i, x_j)), \Omega(\mu_i, x_j)) - 2u_{ij} - rK_{ij}^*(u_{ij}, \Omega_{ij}). \quad (32)$$

Differentiating $\bar{\Pi}_{ij}(\cdot)$ with respect to VC j 's expertise x_j yields

$$\frac{d\bar{\Pi}_{ij}(\cdot)}{dx_j} = \sqrt{2u_{ij}}\frac{\partial\pi(\cdot)}{\partial K_{ij}^*}\frac{dK_{ij}^*}{d\Omega_{ij}}\frac{\partial\Omega_{ij}}{\partial x_j} + \sqrt{2u_{ij}}\frac{\partial\pi(\cdot)}{\partial\Omega_{ij}}\frac{\partial\Omega_{ij}}{dx_j} - r\frac{dK_{ij}^*}{d\Omega_{ij}}\frac{\partial\Omega_{ij}}{\partial x_j}, \quad (33)$$

which, by using (25), can be simplified to

$$\frac{d\bar{\Pi}_{ij}(\cdot)}{dx_j} = \sqrt{2u_{ij}} \frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} \frac{\partial \Omega_{ij}}{\partial x_j}. \quad (34)$$

Next, differentiating $d\bar{\Pi}_{ij}(\cdot)/dx_j$ with respect to entrepreneur i 's idea quality μ_i leads to

$$\frac{d^2\bar{\Pi}_{ij}(\cdot)}{dx_j d\mu_i} = \sqrt{2u_{ij}} \left[\frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij}^2} \frac{\partial \Omega_{ij}}{\partial \mu_i} \frac{\partial \Omega_{ij}}{\partial x_j} + \frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij} \partial K_{ij}} \frac{dK_{ij}^*}{d\Omega_{ij}} \frac{\partial \Omega_{ij}}{\partial \mu_i} \frac{\partial \Omega_{ij}}{\partial x_j} + \frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} \frac{\partial^2 \Omega_{ij}}{\partial x_j \partial \mu_i} \right]. \quad (35)$$

Using (27), we can rewrite (35) as

$$\frac{d^2\bar{\Pi}_{ij}(\cdot)}{dx_j d\mu_i} = \sqrt{2u_{ij}} \left[\frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij}^2} \frac{\partial \Omega_{ij}}{\partial x_j} \frac{\partial \Omega_{ij}}{\partial \mu_i} - \frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij} \partial K_{ij}} \frac{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}}}{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2}} \frac{\partial \Omega_{ij}}{\partial \mu_i} \frac{\partial \Omega_{ij}}{\partial x_j} + \frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} \frac{\partial^2 \Omega_{ij}}{\partial x_j \partial \mu_i} \right]. \quad (36)$$

Since $\partial^2 \Omega_{ij} / (\partial x_j \partial \mu_i) \geq 0$, (36) is positive if

$$\sqrt{2u_{ij}} \left[\frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij}^2} - \frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij} \partial K_{ij}} \frac{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}}}{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2}} \right] \frac{\partial \Omega_{ij}}{\partial \mu_i} \frac{\partial \Omega_{ij}}{\partial x_j} \quad (37)$$

is positive. Recall that $\pi(K_{ij}, \Omega_{ij})$ is concave in both of its arguments. This implies that the Hessian determinant must be positive, i.e.,

$$\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2} \frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij}^2} - \frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij} \partial K_{ij}} \frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}} \geq 0. \quad (38)$$

Thus, (37) is positive, which in turn implies that $d^2\bar{\Pi}_{ij}(\cdot)/(dx_j d\mu_i) \geq 0$.

Next, differentiating $d\bar{\Pi}_{ij}(\cdot)/dx_j$ (see (34)) with respect to entrepreneur i 's reservation utility u_{ij} yields

$$\frac{d^2\bar{\Pi}_{ij}(\cdot)}{dx_j du_{ij}} = \frac{1}{\sqrt{2u_{ij}}} \frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} \frac{\partial \Omega_{ij}}{\partial x_j} + \sqrt{2u_{ij}} \frac{\partial^2 \pi(\cdot)}{\partial \Omega_{ij} \partial K_{ij}} \frac{dK_{ij}^*}{du_{ij}} \frac{\partial \Omega_{ij}}{\partial x_j}. \quad (39)$$

It is straightforward to show that $dK^*/du_{ij} > 0$. Hence, based on our assumptions about $\pi(\cdot)$ and Ω_{ij} , (39) is positive, i.e., $d^2\bar{\Pi}_{ij}(\cdot)/dx_j du_{ij} \geq 0$. \square

Proof of Proposition 1

Without loss of generality, consider an arbitrary entrepreneur-VC pair $(i, j = m(i))$, where entrepreneur i 's reservation utility is denoted u_{ij} . Then, VC j 's constrained maximization problem is

$$\max_{\{\lambda_{ij}, K_{ij}\}} \Pi_{ij}(\cdot) = (1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij})e_{ij}^* - rK_{ij} \quad (40)$$

s.t.

$$\lambda_{ij}\pi(K_{ij}, \Omega_{ij})e_{ij}^* - (e_{ij}^*)^2/2 \geq u_{ij}, \quad (41)$$

where (41) constitutes entrepreneur i 's participation constraint. Consider first the contract $\{\lambda_{ij}^*, K_{ij}^*\}$ which is optimal in the absence of matching; see Lemma 2. Clearly, (41) is non-binding as long as u_{ij} is sufficiently low. In contrast, for sufficiently high values of u_{ij} , (41) becomes a binding constraint. For convenience, let \bar{u}_{ij} denote entrepreneur i 's reservation utility where his participation constraint (41) just binds for the optimal contract $\{\lambda_{ij}^*, K_{ij}^*\}$. Clearly, $u_{ij} \leq \bar{u}_{ij}$ if $u_{i,m(i-1)} \leq U_{i,m(i)}^V$, and $u_{ij} > \bar{u}_{ij}$ otherwise. Thus, VC j needs to adjust the contract $\{\lambda_{ij}, K_{ij}\}$ if $u_{i,m(i-1)} > U_{i,m(i)}^V$, implying that endogenous matching is then relevant. Recall that $u_{i,m(i-1)}$ is defined by (13), and thus a function of the match quality $\Omega_{i,m(i-1)}$ which would result when entrepreneur i matches with the lower ranked VC $m(i-1)$. Observe that $u_{i,m(i-1)} = 0$ if $\Omega_{i,m(i-1)} = 0$. Thus, (41) does not bind for the optimal contract $\{\lambda_{ij}^*, K_{ij}^*\}$ if $\Omega_{i,m(i-1)} \rightarrow 0$. One can show by using the Envelope Theorem that $du_{i,m(i-1)}/d\Omega_{i,m(i-1)} > 0$. Moreover, $u_{i,m(i-1)} > U_{i,m(i)}^V$ for $\Omega_{i,m(i-1)} \rightarrow \Omega_{i,m(i)}$. Thus, there exists a threshold $\bar{\Omega}_{i,m(i-1)}$, with $\Omega_{i,m(i-1)} < \Omega_{i,m(i)}$, such that $u_{i,m(i-1)} > U_{i,m(i)}^V$ for $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$, and $u_{i,m(i-1)} \leq U_{i,m(i)}^V$ for $\Omega_{i,m(i-1)} \leq \bar{\Omega}_{i,m(i-1)}$. \square

Proof of Lemma 7

The optimal equity share $\lambda_{i,m(i)}^M$ and capital investment $K_{i,m(i)}^M$ for $\Omega_{i,m(i-1)} > \bar{\Omega}_{i,m(i-1)}$ follow directly from the derivations in the proof of Lemma 5. \square

Proof of Proposition 2

First, we can infer from (27) that $K_{ij}^M > K_{ij}^*$. Moreover, from (6), we have

$$\frac{de_i^*}{du_{ik}} = \frac{d\lambda_{ij}^*}{du_{ik}} \pi(\cdot) + \lambda_{ij}^* \frac{\partial \pi(\cdot)}{\partial K_{ij}} \frac{dK_{ij}^*}{du_{ik}}. \quad (42)$$

Using (24), (27), and (28), it follows that

$$\frac{de_i^*}{du_{ik}} = \frac{1}{\sqrt{2u_{ij}}} > 0. \quad (43)$$

Next, recall that K_{ij}^M satisfies (26) if $u^*(i) = u_{ik}$. Then, the optimality condition for K_{ij}^M , (26), is only equivalent to the optimality condition for the socially efficient capital provision K_{ij}^{fb} , (3), when $u_{ij} = \pi^2(\cdot)(K_{ij}, \Omega_{ij})/2$. However, on the bargaining frontier, we have

$$u_{ij} = \frac{1}{2} \lambda_{ij}^2 \pi^2(K_{ij}, \Omega_{ij}) < \frac{1}{2} \pi^2(K_{ij}, \Omega_{ij}), \quad (44)$$

because $\lambda_{ij} < 1$. This in turn implies that $K_{ij}^M < K_{ij}^{fb}$, and hence, $e_{ij}^M < e_{ij}^{fb}$. Finally, (27) implies that a higher outside option u_{ij} (due to endogenous matching) leads to a higher capital provision K_{ij}^M . \square

Proof of Lemma 8

By way of contradiction, suppose that for some i , $\Pi^*(m(i)) < \Pi^*(m(i-1))$. Then, VC $m(i)$ can offer entrepreneur $i-1$ exactly the same contract as VC $m(i-1)$ offers entrepreneur $i-1$. Because $x_{m(i)} > x_{m(i-1)}$, and therefore $\Omega_{(i-1,m(i))} > \Omega_{(i-1,m(i-1))}$, the expected utility of en-

trepreneur $i - 1$ is greater than $u^*(i - 1)$, and the expected profit of VC $m(i)$ is greater than $\Pi^*(m(i - 1))$, implying that the matching cannot be stable. Thus, $\Pi^*(m(i)) < \Pi^*(m(i - 1))$ is a contradiction, so that $\Pi^*(m(i)) \geq \Pi^*(m(i - 1))$ must hold. A similar argument can be used to show that $u^*(i)$ is increasing in i . \square

Entrepreneur Has the Entire Bargaining Power

Suppose that the entire bargaining power rests with the entrepreneur. When completely ignoring the policy of the VC, it is intuitively clear that the entrepreneur would prefer (i) to be the residual claimant of the entire profit of the new venture (i.e., $\lambda_{ij} = 1$); and (ii) an unlimited investment of capital (i.e., $K_{ij} \rightarrow \infty$). However, without any adequate revenues for its capital investment, the VC would never invest in the new venture. Technically, the participation constraint of the VC would be violated. This in turn implies that we need to account for the policies of the VC when considering the contractual preferences of the entrepreneur.

Suppose entrepreneur i is matched with VC j , and chooses his preferred share λ_{ij}^E on the venture's profit, while taking the corresponding investment $K_{ij}^*(\lambda_{ij}^E)$ by VC j into account. Let U_{ij}^E denote entrepreneur i 's expected utility, which constitutes his highest expected utility level when forming a partnership with VC j . Then, λ_{ij}^E is defined by

$$\lambda_{ij}^E \in \operatorname{argmax}_{\tilde{\lambda}_{ij}} U_i^E(e_i^*, \tilde{\lambda}_{ij}, K_{ij}^*(\tilde{\lambda}_{ij}), \Omega_{ij}) = \tilde{\lambda}_{ij} \pi(K_{ij}^*(\tilde{\lambda}_{ij}), \Omega_{ij}) e_i^* - \frac{(e_i^*)^2}{2},$$

where $K_{ij}^*(\lambda_{ij})$ is implicitly characterized by (18) (ignoring condition (19)). Obviously, it is in the entrepreneur's best interest to leave some shares for the VC (i.e., $\lambda_{ij}^E < 1$). Otherwise, without any share on the generated revenue of the new venture (i.e., $\lambda_{ij} = 1$), the VC would not invest at all (i.e., $K_{ij}^*(1) = 0$).²⁷

²⁷In fact, one can show that there exists an inverse U-shaped relationship between the entrepreneur's expected utility $U_i^E(\cdot)$ and his share λ_{ij} on the venture's profit, similar to the relationship between the venture capital firm's expected profit $\Pi_{ij}(\cdot)$ and λ_{ij} .

Drawing on the entrepreneur's contractual preference—as characterized above—allows us to identify whether the entrepreneur favors a higher share on the venture's profit—compared to the unconstrained contract $\Gamma^*(\Omega_{ij})$ —at the expense of a higher capital investment, or vice versa. The next lemma elaborates on the relationship between the entrepreneur's preferences and those of the VC, and how this relationship is affected by the specific match quality $\Omega(\mu_i, x_j)$.

Lemma 9 *The entrepreneur's preferred share $\lambda_{ij}^E(\Omega_{ij})$ on the venture's profit always lies above the share $\lambda_{ij}^* = 1/2$ preferred by the venture capitalist, even though this would concurrently imply a lower capital investment $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij})$ (i.e., $\lambda_{ij}^E(\Omega_{ij}) > \lambda_{ij}^* = 1/2$ and $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij}) < K^*(\lambda_{ij}^* = 1/2, \Omega_{ij})$). Moreover, the entrepreneur's preferred share $\lambda_{ij}^E(\Omega_{ij})$ is strictly increasing in the match quality Ω_{ij} (i.e., $d\lambda_{ij}^E(\Omega_{ij})/d\Omega_{ij} > 0$).*

Proof: By accounting for the entrepreneur's effort choice e_{ij}^* , we can rewrite his expected utility as a function of λ_{ij} :

$$U_{ij}(\lambda_{ij}, K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) = \frac{1}{2} \lambda_{ij}^2 \pi^2(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}).$$

Entrepreneur i 's preferred share $\lambda_{ij}^E(\Omega_{ij})$ is implicitly characterized by the first-order condition:

$$\pi(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) + \lambda_{ij} \frac{\partial \pi(\cdot)}{\partial K_{ij}^*} \frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}} = 0. \quad (45)$$

The first-order condition implies that the preferred share $\lambda_{ij}^E(\Omega_{ij})$ is where $dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$.

Implicit differentiating $K_{ij}^*(\cdot)$ as defined by (18) yields

$$\frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}} = - \frac{2(1 - 2\lambda_{ij})\pi(\cdot) \frac{\partial \pi(\cdot)}{\partial K_{ij}^*}}{\frac{\partial}{\partial K_{ij}^*} \left[2\lambda_{ij}(1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}^*} - r \right]}. \quad (46)$$

Note first that the denominator is strictly negative due to the second-order condition. Thus, $dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$ if $\lambda_{ij} > 1/2$. This in turn implies that $\lambda_{ij}^E(\Omega_{ij}) > \lambda_{ij}^* = 1/2$. Moreover, since

$dK_{ij}^*(\cdot)/d\lambda_{ij} < 0$ for $\lambda_{ij} > 1/2$, it follows that $K_{ij}^*(\lambda_{ij}^E(\Omega_{ij}), \Omega_{ij}) < K^*(\lambda_{ij}^* = 1/2, \Omega_{ij})$. Next, define

$$F \equiv \pi(K_{ij}^*(\lambda_{ij}, \Omega_{ij}), \Omega_{ij}) + \lambda_{ij} \frac{\partial \pi(\cdot)}{\partial K_{ij}^*} \frac{dK_{ij}^*(\cdot)}{d\lambda_{ij}}$$

$$G \equiv 2\lambda_{ij}(1 - \lambda_{ij})\pi(K_{ij}, \Omega_{ij}) \frac{\partial \pi(\cdot)}{\partial K_{ij}} - r.$$

Applying Cramer's rule yields

$$\frac{\partial \lambda_{ij}^E(\cdot)}{\partial \Omega_{ij}} = \frac{\det(A)}{\det(B)}, \quad (47)$$

where

$$A = \begin{pmatrix} -\frac{\partial F}{\partial \Omega_{ij}} & \frac{\partial F}{\partial K_{ij}} \\ -\frac{\partial G}{\partial \Omega_{ij}} & \frac{\partial G}{\partial K_{ij}} \end{pmatrix} \quad B = \begin{pmatrix} \frac{\partial F}{\partial \lambda_{ij}} & \frac{\partial F}{\partial K_{ij}} \\ \frac{\partial G}{\partial \lambda_{ij}} & \frac{\partial G}{\partial K_{ij}} \end{pmatrix}.$$

Note that

$$\det(A) = -\frac{\partial F}{\partial \Omega_{ij}} \frac{\partial G}{\partial K_{ij}} + \frac{\partial G}{\partial \Omega_{ij}} \frac{\partial F}{\partial K_{ij}}. \quad (48)$$

Clearly, $\partial G/\partial K_{ij} < 0$ due to the second-order condition. Moreover,

$$\frac{\partial F}{\partial \Omega_{ij}} = \frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} + \lambda_{ij} \frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}} \frac{dK_{ij}^*}{d\lambda_{ij}} > 0$$

$$\frac{\partial G}{\partial \Omega_{ij}} = 2\lambda_{ij}(1 - \lambda_{ij}) \left[\frac{\partial \pi(\cdot)}{\partial \Omega_{ij}} \frac{\partial \pi(\cdot)}{\partial K_{ij}} + \pi(\cdot) \frac{\partial^2 \pi(\cdot)}{\partial K_{ij} \partial \Omega_{ij}} \right] > 0$$

$$\frac{\partial F}{\partial K_{ij}} = \frac{\partial \pi(\cdot)}{\partial K_{ij}} + \lambda_{ij} \underbrace{\frac{\partial^2 \pi(\cdot)}{\partial K_{ij}^2}}_{\leq 0} \underbrace{\frac{dK_{ij}^*}{d\lambda_{ij}}}_{< 0} > 0.$$

Thus, $\det(A) > 0$. Next, we have

$$\det(B) = \frac{\partial F}{\partial \lambda_{ij}} \frac{\partial G}{\partial K_{ij}} - \frac{\partial G}{\partial \lambda_{ij}} \frac{\partial F}{\partial K_{ij}}. \quad (49)$$

Notice that $\partial F/\partial \lambda_{ij}, \partial G/\partial K_{ij} < 0$ due to the respective second-order conditions. Moreover, recall that $\partial F/\partial K_{ij} > 0$. Furthermore,

$$\frac{\partial G}{\partial \lambda_{ij}} = 2(1 - 2\lambda_{ij})\pi(K_{ij}, \Omega_{ij})\frac{\partial \pi(\cdot)}{\partial K_{ij}} < 0. \quad (50)$$

Therefore, $\det(B) > 0$. Combining previous observations, we have $\partial \lambda_{ij}^E(\cdot)/\partial \Omega_{ij} > 0$. \square

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