Location Decisions of Competing Networks*

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Abstract

Early entrants in markets with network effects usually occupy a 'central location' and serve agents with 'intermediate tastes', while later entrants are niche players. Why would the first entrant choose to become a 'general' network, given that later entrants will not have enough room for differentiation, resulting in a more intense competition for market share? In a Hotelling model with two rival networks, we show that for intermediate values of the network externality parameter the location equilibrium is indeed asymmetric: the first entrant locates at the center while the second entrant chooses an extreme (niche) location.

Keywords: Product Selection; Endogenous Locations; Network externalities.
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1 Introduction

We examine 'product location' decisions of two competing networks. It is well-known that in the absence of network externalities firms choose maximum differentiation (d'Aspermont et al. 1979) in order to mitigate the ensuing competition. Most of the literature on network markets has ignored the issue of product selection and usually assumes maximum horizontal differentiation when the market has features of spatial competition.[1] In this paper, we endogenize location decisions of networks and we show that maximum horizontal differentiation does not always hold.

We formulate a model with two competing networks that make their location decisions sequentially.[2] Our most interesting and novel result is that for intermediate values of the (direct) network externality parameter the location equilibrium is *asymmetric*: the first-mover locates at the center of the market, while the follower locates at an extreme, with both networks having strictly positive market shares. This result may help to explain product location decisions that are observed in practice.

More specifically, early entrants in markets with network effects usually occupy a 'central location' and serve agents with 'intermediate tastes', while later entrants are niche players. In the TV industry, the first entrants were general content channels (e.g., ABC, CBS and NBC), while specialized channels (e.g., Weather channel, History channel) appeared later. In the market for online video websites, YouTube, which was launched before Hulu, carries a huge number of diverse videos and clips (suitable for more general audiences), while Hulu serves those who watch commercial movies and TV shows.[3] Other examples include search engines (e.g., Google, Yahoo vs. Google Scholar), online dating services (first entrants catered to the average man and woman and later entrants targeted specific groups, e.g., professionals and millionaires) and academic journals.

Our duopoly model can also be used to understand the geographic locations of physical markets.[4] When externalities are low (or equivalently transportation cost is high), markets

[1] See, for example, Griva and Vettas (2004), Armstrong (2006), Doganoglu and Wright (2006) and Zhu (2010). Exceptions are the papers by Gabszewicz et. al. (2002), Peitz and Valletti (2008) and Kind et al. (2007), in a two-sided market environment. Nevertheless, the models and predictions in those papers are very different from ours.

[2] A pure strategy equilibrium may fail to exist when networks locate simultaneously.

[3] Consistent with our model predictions, YouTube has a higher market share than Hulu. According to "Disney's Hulu Deal Raises Questions About YouTube Model," Wall Street Journal, April 30, 2009, YouTube had 100 million viewers in March of 2009, while in the same time period Hulu had 41 million viewers.

[4] Jin and Rysman (2009) investigate the pricing decisions of sportcard conventions. These conventions

locate at the periphery (maximum differentiation), i.e., far apart from each other. For stronger externalities (or low transportation costs) there are two active markets, one at the center and the other at the periphery. One implication of our model is that as the network externalities become stronger (or the transportation cost decreases) market shares will become more asymmetric and eventually one network will dominate.

The driving force behind the product selection decisions is demand creation. When network externalities are weak, price competition dominates demand creation. No network wants to be a 'general' network because if one network locates at the most attractive (central) location product differentiation is reduced and this creates stiff price competition. In contrast, when externalities are not weak, demand creation is important. A network in this case benefits by being 'general' and attracting many agents with intermediate preferences. The rival network serves a niche market.[5] Our results also apply to two-sided markets.

The rest of the paper is organized as follows. We present the model in the next Section. In Section 3, we solve the model and in Section 4 we solve the social planner's problem (first-best). We conclude in Section 5. All proofs can be found in the Appendix.

2 The description of the benchmark model

The market consists of two horizontally differentiated (and incompatible) networks, $k = A, B$. There is a continuum of agents that is uniformly distributed on the $[0,1]$ interval. Network A is located at point a and network B is located at point b, with $0 \leq a \leq b \leq 1$. We assume that transportation cost is quadratic in the distance d an agent has to 'travel' from his location to the location of the network, td^2, where the parameter $t > 0$ measures the per-unit cost of travel. We assume that each agent joins only one network (single-homing). Each agent who joins a given network cares about the number of agents that will join the same network. Denote by n_k the number of participants that network k attracts. The maximum willingness to pay of an agent who joins network k is given by $V + \alpha n_k$, where V is a stand-alone benefit each agent receives independent of the number of participants on

try to attract consumers and dealers. An important decision of these conventions is where to locate, given the competition they face from rival conventions.

[5] The tension between demand creation and price competition is at the heart of most models with endogenous location decisions, regardless of whether the market exhibits network externalities or not. For instance, one can mitigate the intensity of price competition, and hence create a less than maximum differentiation, by changing the transportation cost functions, as in Economides (1986), or by allowing for multiple purchases, as in Kim and Serfes (2006).

network k. The parameter $\alpha > 0$ measures the intensity of network externality. The indirect utility of an agent who is located at point $x \in [0,1]$ is given by,

$$U = \begin{cases} V + \alpha n_A^e - t\,(a-x)^2 - p_A, & \text{if he joins network } A \\ V + \alpha n_B^e - t\,(b-x)^2 - p_B, & \text{if he joins network } B \end{cases} \tag{1}$$

where p_k is network k's lump-sum charge (fee-based) and n_k^e denotes the expectations agents have about the number of agents that will join network k.[6] We assume that V is high enough which ensures that the market is covered. Marginal cost is zero. We assume that horizontal differentiation is more important than the network externality, $t > \alpha$.

Given that, in reality, location decisions happen sequentially, we impose a sequential location timing, with network A being the first-mover.[7] Then, networks set their prices simultaneously. Finally, agents, after observing network locations and prices, decide which network to join.

3 Analysis

We look for a subgame perfect Nash equilibrium. We solve the game backwards.

3.1 Stage 3: Agent decisions and market shares

The marginal agent can be found as follows.

$$V + \alpha n_A^e - t\,(a-x)^2 - p_A - \left(V + \alpha n_B^e - t\,(b-x)^2 - p_B\right) = 0$$

$$\Rightarrow \hat{x} = \frac{p_B - p_A + t\,(b^2 - a^2) - \alpha\,(n_B^e - n_A^e)}{2t\,(b-a)}. \tag{2}$$

The fraction of agents that joins network A is $n_A = \hat{x}$ and the fraction that joins network B is $n_B = 1 - \hat{x}$. In equilibrium, it must be that expectations are confirmed, that is, $n_A = n_A^e$and $n_B = n_B^e$. Using (2) this defines a system of two equations in two unknowns,

[6] More generally, networks can be fee-based and/or ad-based. We assume away the possibility of ad revenues. Casadesus-Masanell and Zhu (forthcoming) endogenize this decision.

[7] When networks locate simultaneously, for a range of parameter values, a pure strategy equilibrium, in location decisions, does not exist. More details can be found in Serfes and Zacharias (2009). In that paper, we assume that there are two distinct groups of agents (two-sided market) but the analysis and results are the same as in a one-sided market framework. The driving force behind the main results is the presence of a network externality.

n_A and n_B. By solving the system we obtain the market shares as a function of prices and parameters

$$n_A = \frac{p_B - p_A + t\left(b^2 - a^2\right) - \alpha}{2\left(t\left(b-a\right) - \alpha\right)} \tag{3}$$

and

$$n_B = \frac{p_A - p_B - b^2 t + a^2 t - 2at + 2bt - \alpha}{2\left(t\left(b-a\right) - \alpha\right)}. \tag{4}$$

3.2 Stage 2: Networks' pricing decisions

Network k chooses p_k to maximize its profits $\pi_k = p_k n_k$, where n_k, $k = A, B$, are given by (3) and (4). The profit functions are strictly concave in a network's own price if

$$\alpha < t\left(b-a\right). \tag{5}$$

Alternatively, the above condition can be written as

$$a < b - \frac{\alpha}{t} \text{ or } b > a + \frac{\alpha}{t}.$$

If (5) is satisfied, then the first order conditions are also sufficient for profit maximization. The equilibrium prices then are given by

$$p_A = \frac{t\left(b-a\right)}{3}\left(2+b+a\right) - \alpha \text{ and } p_B = \frac{t\left(b-a\right)}{3}\left(4-b-a\right) - \alpha. \tag{6}$$

The equilibrium market shares, by substituting (6) into (3) and (4), are given by

$$n_A = \frac{t\left(b-a\right)\left(2+b+a\right) - 3\alpha}{6\left(t\left(b-a\right) - \alpha\right)} \text{ and } n_B = \frac{t\left(b-a\right)\left(4-b-a\right) - 3\alpha}{6\left(t\left(b-a\right) - \alpha\right)}.$$

Note that if (5) is satisfied, then the denominators in the above expressions are positive. For an interior equilibrium we need the market shares to be in $(0,1)$. It turns out that $n_k \in (0,1)$ if and only if

$$\alpha < \min\left\{\frac{t\left(b-a\right)}{3}\left(4-b-a\right), \frac{t\left(b-a\right)}{3}\left(2+b+a\right)\right\} \tag{7}$$

or, equivalently, for n_A to be less than one (which implies that n_B is greater than zero) we must have

$$\alpha < \frac{t\left(b-a\right)}{3}\left(4-b-a\right) \Leftrightarrow a < \tilde{a} \equiv -\frac{1}{t}\left(-2t + \sqrt{3t\alpha + 4t^2 - 4bt^2 + b^2 t^2}\right) \tag{8}$$

and for n_A to be greater than zero (which implies that n_B is less than one) we must have

$$\alpha < \frac{t(b-a)}{3}(2+b+a) \Leftrightarrow a < \hat{a} \equiv -\frac{1}{t}\left(t - \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2}\right). \quad (9)$$

For any given location b of network B the market tips, either in favor of A or in favor of B, when network A locates close enough to network B, as the above thresholds indicate. The interior equilibrium profits as a function of the network s' locations, after we substitute (6) into the profit functions, are[8]

$$\pi_A(a,b) = \frac{(t(b-a)(2+b+a)-3\alpha)^2}{18(t(b-a)-\alpha)} \text{ and } \pi_B(a,b) = \frac{(t(b-a)(4-b-a)-3\alpha)^2}{18(t(b-a)-\alpha)}. \quad (10)$$

It can be easily verified that when $\alpha = 0$ (no network externality) the equilibrium profits reduce to those in d'Aspermont et al. (1979), where it is each network's dominant strategy to locate at the extreme points (maximum differentiation). As we show next, this is not always the case when $\alpha > 0$.

3.3 Stage 1: Networks' location decisions

Networks choose their locations a and b to maximize profits as they are given by (10). As we show in the proof of Lemma 1 (see also Figures 2 and 3), when the market tips it is the network that is closer to the middle point, 1/2, that attracts all the agents. If they are equidistantly located from the middle, $a = 1-b$, then we assume that all agents join network A.

Fix the location of network B at a specific point b. Let's examine the profits of network A when it moves from $a = 0$ to $a = b$. Two effects arise: i) price competition intensifies and ii) the network attracts more agents. Initially, the intensified competition effect is stronger, but eventually demand creation dominates. The latter is due to the network externality. The Lemma below summarizes the result.

Lemma 1 *The profit function of network A, for any fixed location b of network B, exhibits a U-shape.*

To gain a better intuition about the properties of the profit functions as a function of the locations, let's look at Figures 4 and 5 where the profit of network A is depicted as its

[8] We will deal with the tipping solution next when we will analyze the location game.

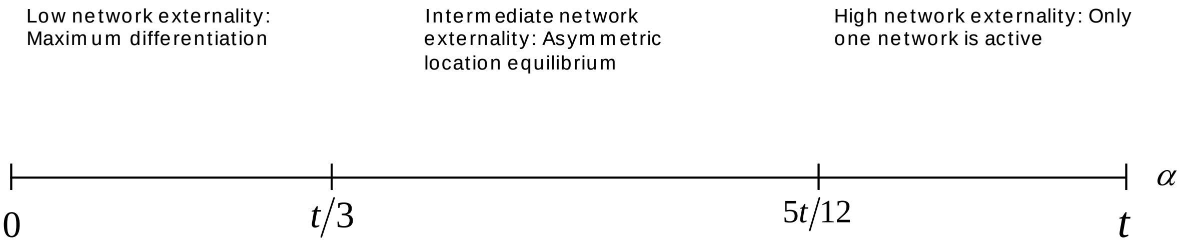

Figure 1: Equilibrium locations of the two networks

location a varies for a fixed location b of network B. The difference between the two figures is that in Figure 4 the location of network B is farther away from the most attractive point (the center) than in Figure 5. When B is farther away from the center, as in Figure 4, prices and profits initially fall, but after a threshold, namely $\tilde{a}$, the market tips in favor of A and its profits rise. In contrast, when B is closer to the center, as in Figure 5, the market tips in favor of B after a certain threshold, namely $\hat{a}$. When A moves closer to the center, that is when it exceeds $1 - b$, the market tips again but now in favor of A. These properties are quite intuitive. For a high degree of horizontal differentiation the market is shared and any movement closer to the rival intensifies competition. After a certain point, however, tipping happens. The question then is: which is the network that attracts all the agents? The answer is that it is the one closer to the most attractive location: the center.

Figure 1 summarizes the equilibrium locations as a function of the network externality α. These outcomes are presented formally in Proposition 2.

Proposition 2 *(**Location equilibria**) The subgame perfect Nash equilibrium is characterized as follows:*

- *(**Maximum differentiation**). If $\alpha \in [0, t/3]$, then networks differentiate maximally, that is $a = 0$ and $b = 1$. The equilibrium profits are*

$$\pi_A (a = 0, b = 1) = \pi_B (a = 0, b = 1) = \frac{t - \alpha}{2}.$$

- *(**Asymmetric location equilibrium**). If $\alpha \in (t/3, 5t/12)$, then the first mover (network A) locates at the center $a = 1/2$ and the follower (network B) locates at an extreme $b = 1$. Both networks have strictly positive market shares. The equilibrium profits are*

$$\pi_A (a = 1/2, b = 1) = \frac{(7t - 12\alpha)^2}{144 (t - 2\alpha)} \text{ and } \pi_B (a = 1/2, b = 1) = \frac{(5t - 12\alpha)^2}{144 (t - 2\alpha)}.$$

- ***(Tipping location equilibrium)****. If $\alpha \in [5t/12, t)$, then the first mover (network A) locates at the center $a = 1/2$ and the follower (network B) locates at an extreme $b = 1$. Network B's market share is zero. The equilibrium profits are*

$$\pi_A(a = 1/2, b = 1) = \alpha - \frac{t}{4} \text{ and } \pi_B(a = 1/2, b = 1) = 0.$$

When network externalities are weak, a network has no incentive to move close to the rival network in order to become the dominant network. The benefit from differentiation is minimized, when the two networks are close to each other, and the dominant network is only able to benefit from the larger market share. Since the externalities are weak, so is the benefit from the larger market share.

When network externalities are stronger, network A locates in the middle. Network B can attract some agents and make strictly positive profits when it locates at the extreme. Network A has no incentive to move away from the middle point, because in this case B has a profitable deviation to locate closer to the middle point than A and leave A with zero agents and profits.[9] For very strong network externalities, the market tips in favor of the first mover who locates at the center.

One important issue in the literature is the trade-off between 'standardization' and variety, e.g., Farrell and Saloner (1986). Having only one network in our model can be viewed as having one technical standard, at the expense of product variety. So, Proposition 2 predicts that for intermediate network externalities, there will be two 'standards' in equilibrium, with one being superior to the other (in terms of market share). Note also that in our model the 'type' of the standard is not given but it is being determined endogenously. When externalities are low, neither standard is superior to the other. Finally, when externalities are strong, there will be only one standard in equilibrium.

4 Welfare analysis

A social planner chooses the locations a and b of the two networks and the number of agents from each group that should join a network to maximize the difference between aggregate

[9] Tyagi (2000) examines location decisions of two firms that enter sequentially and have different costs. If the second mover has lower cost, then it locates close to the most attractive location, while the first mover locates far away from the most attractive location. Our asymmetric location equilibrium result has a similar flavor, but it is the first mover in our model that locates at the most attractive location. Moreover, the underlying mechanisms between the two models are different and in our model firms are ex-ante symmetric.

network externality and aggregate transportation cost. We denote by x the number of agents that join network A. Total welfare is given by

$$\begin{aligned} W &= \int_0^x \left(\alpha z - t\left(a-z\right)^2\right) dz + \int_x^1 \left(\alpha\left(1-z\right) - t\left(b-z\right)^2\right) dz = \\ &= \alpha x^2 + tax^2 - ta^2x + \frac{\alpha}{2} - \alpha x - \frac{t}{3} + tb - tbx^2 - tb^2 + tb^2. \end{aligned} \tag{11}$$

The next Proposition summarizes the solution to the social planner's problem.

Proposition 3 ***(First-best)*** *For $\alpha \leq t/4$, the optimal locations are $a = 1/4$ and $b = 3/4$ and the agents are split equally between the two networks. Total welfare is equal to $W = \alpha/4 - t/48$. For $\alpha \geq t/4$, all agents from both groups join one network which is located at the middle point $1/2$. Total welfare is equal to $W = \alpha/2 - t/12$.*

The intuition is simple. Aggregate network externality is maximized when all agents join one network. On the other hand, total transportation cost is minimized when the networks are located at the first and third quartiles. When the externality is weak, transportation cost is relatively more important and the social planner splits the agents equally between the two networks. For strong externalities one network is chosen to dominate the market, since externalities are now relatively more important.

Comparing the first-best with the non-cooperative outcome, we can see that the two coincide only when network externalities are strong, i.e., $\alpha \geq 5t/12$. For low externalities ($\alpha \in [0, t/4]$), horizontal differentiation is higher than in the first-best, i.e., 'too much' differentiation. The two networks differentiate maximally, while the social planner wants them at the first and third quartiles, as in a model with no network effects. When $\alpha \in [t/4, t/3]$, the social planner wants only one network to be active and locate at the center, while in the market equilibrium both networks are active and maximally differentiated. When $\alpha \in [t/3, 5t/12]$, market differentiation is still higher than the first-best.

5 Conclusion

We examine location decisions of two horizontally differentiated competing networks. Our model can yield both symmetric and asymmetric location equilibria, depending on the strength of the network externality. There are two effects when a network moves closer to the location of the rival: i) networks become less differentiated and prices tend to decline and ii) its market share increases. Due to network externality, the market share effect can be strong, and hence there is a tendency for less than maximum differentiation. In particular, we show that when the externality is weak, the principle of maximum differentiation holds. For intermediate externality, we obtain an asymmetric location equilibrium, where the first mover locates at the center and the follower at an extreme location. Both networks have strictly positive market shares. Our model offers an explanation for the coexistence (see the Introduction for examples) of 'general' networks (first mover) that cater to agents with intermediate tastes with niche networks (follower) that serve agents with extreme tastes.

Finally, we would like to highlight the role of 'product' selection. It is true that even with fixed locations at the extremes the market tips when the externalities are strong enough. This is typical in models with network externalities. Nevertheless, when locations are fixed, we do not obtain asymmetric market structures where *both* networks are active, which is the case when locations are endogenized (see Proposition 2). In that sense, a testable implication of our model is that market shares evolve 'more continuously' as the degree of network externalities (or the degree of differentiation) varies, starting from symmetric market structures when externalities are low, then moving to asymmetric market structures when externalities become stronger. Eventually, for very strong externalities, the market is dominated by one network .

A Appendix

A.1 Proof of Lemma 1

The thresholds (9) and (8) are very important at this stage. It turns out that $\hat{a} \geq \tilde{a}$ if and only if

$$b \geq \bar{b} \equiv \frac{1}{2} + \frac{\alpha}{2t}. \tag{12}$$

Symmetrically, we can define the thresholds for network B for a fixed location a of network A. When $\hat{a} \geq \tilde{a}$ the binding threshold is the $\tilde{a}$. In this case, as it will become evident below, the other threshold is irrelevant. The opposite is true when $\hat{a} \leq \tilde{a}$.

When the market tips it is the network that is closer to the middle point, 1/2, that attracts all the agents. If they are equidistantly located from the middle, $a = 1 - b$, then we assume that all agents join network A. In what follows, we assume that $a \leq b$. In the proof of Proposition 2, we allow $a > b$, but in this case the profit functions and the thresholds can

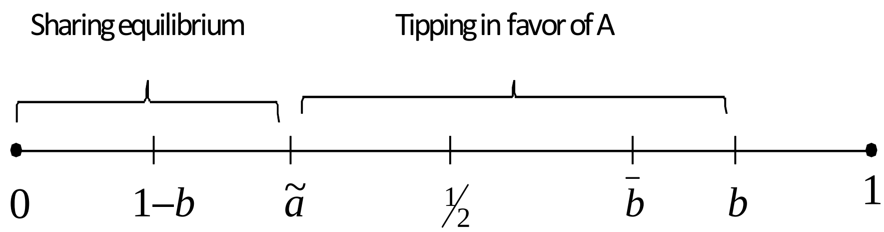

Figure 2: Type of equilibrium as the location a of network A varies from 0 to b, with the location of network B fixed at $b \geq \bar{b}$

be obtained via a simple relabeling.

When $b \geq \bar{b}$, as network A increases a it attracts all agents when $a = \tilde{a} \geq 1 - b$ (market tips).[10] After this point, the market remains tipped in favor of A until $a = b$. Figure 2 depicts this case.

When $b \leq \bar{b}$, network A's market share becomes zero when $a = \hat{a} \leq 1 - b$.[11] After this point, the market will tip in favor of A when $a = 1 - b$ (symmetric locations). The market will remain tipped in favor of A until $a = b$. Figure 3 depicts this case.

When $a > b$ the identities of the network s are reversed (A becomes B and B becomes A). The analysis in this case will follow from the above two cases and it will be equivalent to fixing a and allowing b to vary. This in turn is equivalent to the above two cases after we set $b = 1 - a$.

The second order condition (5) is always satisfied when the market has not tipped, i.e., when $a < \min\{\tilde{a}, \hat{a}\}$. The derivative of network A's profit function (10) with respect to a is

$$\frac{\partial \pi_A(a,b)}{\partial a} = \frac{(2at - 2bt + \alpha - 4abt + 4a\alpha + 3a^2t + b^2t)(2bt - 2at - 3\alpha - a^2t + b^2t)\,t}{18(bt - \alpha - at)^2}.$$

Note that

$$\frac{\partial \pi_A(a=0,b)}{\partial a} = \frac{(\alpha - 2bt + b^2t)(2bt - 3\alpha + b^2t)}{18(bt - \alpha)^2} < 0$$

[10] It can be shown that $\tilde{a} \geq 1 - b$ if and only if $b \geq \bar{b}$.

[11] It can be shown that $\hat{a} \leq 1 - b$ if and only if $b \leq \bar{b}$. In addition, $\hat{a} = 0$, when

$$b \leq -1 + \frac{\sqrt{t^2 + 3t\alpha}}{t} < \bar{b}.$$

This suggests that when platform B moves closer to the middle after the above threshold network A's market share will be zero even when $a = 0$.

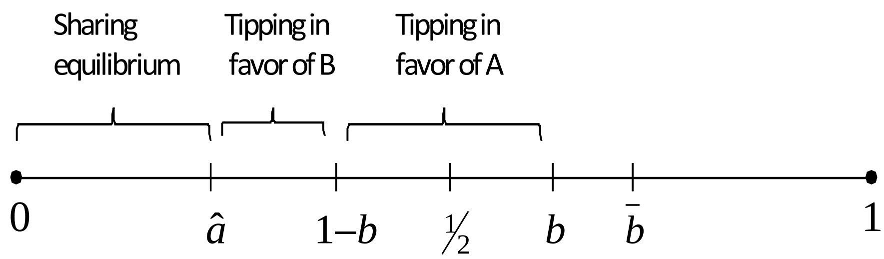

Figure 3: Type of equilibrium as the location a of network A varies from 0 to b, with the location of network B fixed at $b \leq \bar{b}$

if $b > \left(-t + \sqrt{3t\alpha + t^2}\right)/t$.[12] So, the derivative of the profit function at $a = 0$ is always strictly negative when the market is shared.

For any a now, the derivative $\partial \pi_A(a,b)/\partial a$ becomes zero at

$$a = a_1 \equiv -\frac{1}{t}\left(t - \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2}\right) \quad (13)$$

$$a = a_2 \equiv \frac{1}{3t}\left(-t - 2\alpha + 2bt + \sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2}\right) \quad (14)$$

$$a = a_3 \equiv -\frac{1}{t}\left(t + \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2}\right)$$

$$a = a_4 \equiv \frac{1}{3t}\left(-t - 2\alpha + 2bt - \sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2}\right).$$

Roots a_3 and a_4 are negative, so we rule them out.[13] Also note that $a_1 = \hat{a}$, where $\hat{a}$ is

[12] There are four roots when we solve

$$\frac{\partial \pi_A(a=0,b)}{\partial a} = \frac{\left(\alpha - 2bt + b^2t\right)\left(2bt - 3\alpha + b^2t\right)}{9\left(bt-\alpha\right)^2} = 0$$

with respect to b. One is negative and the other is greater than one, so we rule them out. The remaining two are

$$r_1 \equiv \frac{1}{t}\left(-t + \sqrt{3t\alpha + t^2}\right) \text{ and } r_2 \equiv \frac{1}{t}\left(t - \sqrt{-t\alpha + t^2}\right).$$

It can be shown that $r_1 \geq r_2$ if and only if $t \geq \alpha$, which we have assumed holds. It can be computed that $\hat{a}$ (from (9)) becomes zero when $b \leq r_1 \equiv \left(-t + \sqrt{3t\alpha + t^2}\right)/t$. Therefore, the other feasible root, root r_2, becomes irrelevant since when $b \leq r_1$ the market tips.

[13] Root a_3 is clearly negative. Root a_4 is negative because at $\alpha = 0$, a_4 becomes

$$\frac{-t\left(2-b\right)}{3t} < 0$$

and the derivative of a_4 with respect to α is

$$\frac{1}{t}\left(-\frac{1}{6}\frac{t + 8t\alpha - 8bt}{\sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2}} - \frac{2}{3}\right)$$

which is always negative for $b \in [0,1]$.

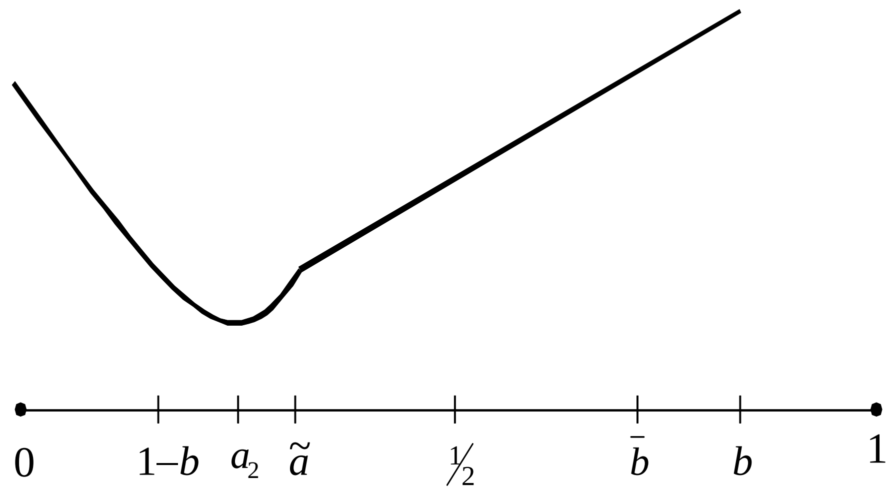

Figure 4: Profit function of network A as its location a varies from 0 to b for a fixed location b of network B at $b \geq \bar{b}$

given by (9). As we have mentioned before, we have $\hat{a} = a_1 \geq \tilde{a}$ if $b \geq \bar{b} \equiv 1/2 + \alpha/2t$ and $\tilde{a} \geq a_1 = \hat{a}$, if $b \leq \bar{b}$. This implies the following about the profit function of network A for any fixed location b of network B.

Case 1: $b \geq \bar{b} \equiv 1/2 + \alpha/(2t)$. Figure 4 depicts this case, where we have assumed that $a_2 \leq \tilde{a}$. Whether this holds or not depends, as we explain below, on the magnitude of α.

We have that $\hat{a} = a_1 \geq \tilde{a}$, so only root a_2 may be relevant. Given that it is the only relevant root coupled with the fact that the profit function of network A is strictly decreasing at $a = 0$ when sharing takes place, the profit function must attain a local minimum at $a = a_2$, if $a_2 \leq \tilde{a}$. The two network s share the agents (no tipping) when $a < \tilde{a}$. As network A moves closer to network B its prices fall (see (6)) but the market shares after a certain point may increase. This may happen after $a = 1 - b$ where network A is closer to the middle point 1/2 than B. That is why a minimum may be attained at $a = a_2$ and after this point the profit function increases. This is not always true as a_2 may be greater than $\tilde{a}$, in which case the profit function of network A is decreasing until $a = \tilde{a}$. network B is losing market share and at $a = \tilde{a}$ tipping occurs in favor of A. Profits increase for A as it moves closer to b because its distance to the marginal agents, who are located at 1, decreases. When $a > b$ the network s reverse identities. Overall, the profit function of network A as a function of its location a is U-shaped up to $a = b$ when $b \geq \bar{b}$.

Case 2: $b \leq \bar{b} \equiv 1/2 + \alpha/(2t)$. Figure 5 depicts this case.

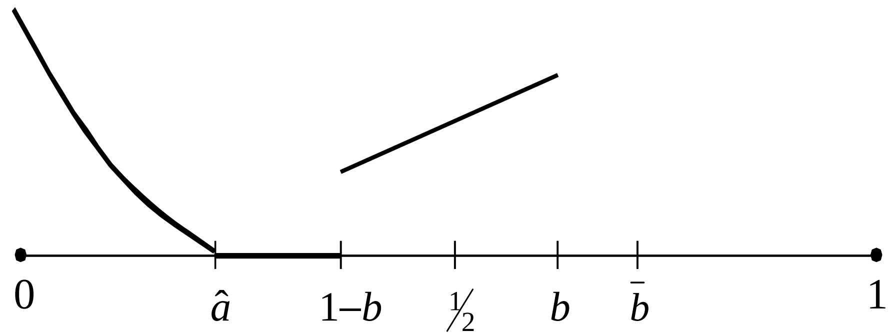

Figure 5: Profit function of network A as its location a varies from 0 to b for a fixed location b of network B at $b \leq \bar{b}$

Given that network A's profit function is strictly decreasing at $a = 0$ when the market is shared and that $\hat{a} = a_1$ is now a relevant root, root a_2 becomes irrelevant. This is because at $a = \hat{a} = a_1$ the market tips in favor of B and the slope of A's profit function becomes zero. This implies that a_2 cannot be less than $\hat{a} = a_1$, since if that was the case there should be one more root in that range. Hence, in this case it must be that $a_2 > \hat{a} = a_1$ and therefore a_2 is irrelevant. The two network s share the market when $a < \hat{a}$, unless $b \leq \left(-t + \sqrt{3t\alpha + t^2}\right)/t$, in which case network A's market share is zero at $a = 0$. As network A moves closer to b both prices and market shares fall (because now B is closer to the middle than in case 1 above) and at $a = \hat{a}$ the market tips in favor of B. Then, at $a = 1 - b$ the market tips in favor of A. As in case 1 above, the profit function is U-shaped up to $a = b$.

The above two cases will be used in the proof of Proposition 2. These cases illustrate that the profit function will also be U-shaped when $a > b$. This follows because the case of $a > b$ is a simple relabeling of the above two cases.

A.2 Proof of Proposition 2

Without loss of generality, we fix $a \leq 1/2$. We examine the optimal location of network B. We showed in Lemma 1 that network A's profit function is U-shaped with respect to a for any $a \leq b$. Via a simple relabeling the same holds for network B's profit function with respect to b for any fixed $b \geq a$, see Figures 4 and 5. This implies that, for any a, network B (the follower) will either locate right next to network A and tip the market in B's favor, or will locate at $b = 1$. Therefore, given network B's reaction, it is network A's dominated strategy to locate at $a \in (0, 1/2)$. This can be understood as follows. If network B finds it profitable to locate at $b = 1$, network A's profits are maximized at $a = 0$. This is because network B can tip the market in its favor (unless $a = 1/2$), but to choose to locate at $b = 1$, it must mean that the externality is not so strong. In this case A is better off locating at $a = 0$ first. If, on the other hand, network B finds it profitable to locate right next to A (and closer to the center $1/2$), then A is better off locating first at $1/2$. So, in any subgame perfect equilibrium we have either $a = 0$ or $a = 1/2$. Also, note that network A can always secure strictly positive profits for itself because it can always locate at the center and tip the market in its favor (recall that even if B locates at the center we have assumed that all agents join network A).

The above discussion suggests that network A will locate at $a = 0$ only if network B will locate at $b = 1$. If, instead, B locates arbitrarily close to 0, then it attracts all agents and its price (and profit) is α. If it locates at $b = 1$, B's profit is $(t - \alpha)/2$. Hence, if $\alpha \leq t/3$, network B has no incentive to deviate from $b = 1$ to $b \approx 0$. The equilibrium in this case is $a = 0$ and $b = 1$ (maximum horizontal differentiation).

If $\alpha > t/3$, then network A will locate at the center, $a = 1/2$ and $b = 1$. Given $a = 1/2$, network B has no profitable deviation. The profits of network A are

$$\pi_A(a = 1/2, b = 1) = \frac{(7t - 12\alpha)^2}{144(t - 2\alpha)}$$

and of network B are

$$\pi_B(a = 1/2, b = 1) = \frac{(5t - 12\alpha)^2}{144(t - 2\alpha)}.$$

The market share of network B (the follower) is positive as long as $\alpha < 5t/12$. If $\alpha \geq 5t/12$, network B's market share is zero. We assume that network B stays at $b = 1$ (its profit is zero regardless of where it locates; so, in that sense the equilibrium is not unique) and hence network A's price (and profit) is $\alpha - t/4$, which is derived as follows. The marginal consumer is at $b = 1$, suggesting that the difference in transportation cost between the two networks–in favor of B–is $t/4$ and the difference in the network benefit–in favor of A–is α. So, the difference in prices should be equal to $\alpha - t/4$. In equilibrium, B's price should be zero and thus A's price is $\alpha - t/4$.

A.3 Proof of Proposition 3

We differentiate (11) with respect to a, b and x. There is only one interior solution to the system of first order conditions

$$x = \frac{1}{2} \text{ and } a = \frac{1}{4}, b = \frac{3}{4}.$$

The interior solution yields welfare equal to $W = \alpha/4 - t/48$. The corner solution is the one where the social planner has only one network serving all agents (tipping). If we set $x = 1$, for example, then it is easy to verify that welfare is maximized at $a = 1/2$ and is equal to $W = \alpha/2 - t/12$. Finally, it can be easily verified that the interior solution dominates the corner solution if and only if $\alpha \leq t/4$.

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