Bundling, A La Carte Pricing and Vertical Bargaining in a Two-Sided Model*

Minghua Chen† Adam D. Rennhoff‡ Konstantinos Serfes§

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Abstract

We develop a two-sided market model with an upstream-downstream structure. More specifically, the platform consists of two rival upstream firms and a downstream monopolist. Each upstream firm negotiates the input price (license fee) with the downstream monopolist and also chooses the amount of advertising that is embedded in the good it sells to the downstream monopolist. The downstream monopolist, in turn, decides how to offer the two goods: a la carte, as a pure bundle, or in a mixed bundle. We use this model to understand the incentives to bundle and the properties of bundling in a two-sided market framework. We find that pure bundling may *strictly* dominate mixed bundling. We also contribute to the ongoing debate on a la carte pricing in the TV industry, where the two upstream firms can be viewed as two rival TV networks and the downstream monopolist as a cable operator.

JEL Classification Codes: L13, L50, L82

Keywords: Bundling, Two-sided market, Vertical relations, A la carte regulation

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†Research Institute of Economics and Management, Southwest University of Finance and Economics, Chengdu, Sichuan, China, 610074. Email: chenminghua@swufe.edu.cn.

‡Economics and Finance Department, P.O. Box 27, Middle Tennessee State University, Murfreesboro, TN 37132. Email: rennhoff@mtsu.edu.

§Department of Economics and International Business, Bennett S. LeBow College of Business, Drexel University, Matheson Hall, 32nd and Market Street, Philadelphia PA 19104. Email: ks346@drexel.edu.

1 Introduction

We examine the incentives of a monopolist to bundle its products and the welfare properties of bundling in a two-sided market (e.g., Caillaud and Jullien (2003), Armstrong (2006) and Rochet and Tirole (2006)). There is a substantial body of work on bundling in one-sided markets[1], but very few in two-sided market environments.[2] We model a downstream monopolist who purchases two goods from two rival upstream suppliers. Each upstream firm chooses the amount of advertising that is embedded in the good and negotiates with the downstream monopolist over the input price. The downstream monopolist chooses how to offer the two goods: a la carte (no bundle), as a pure bundle, or as a mixed bundle. We assume that consumer valuations for the two goods follow a bivariate normal distribution with a general correlation structure. Consumers dislike advertising, but advertisers' profit is increasing in the number of final consumers. Hence, the platform in our model consists of the two upstream firms and the downstream monopolist who try to attract advertisers and consumers (viewers).

Our model is the first one that combines advertising and input prices in a two-sided model. This allows us to shed light not only on the incentive to bundle, but also on how bundling interacts with the amount of advertising and the level of input prices, which are important determinants of the final prices and, hence, consumer welfare. We use the model to contribute to the ongoing debate about whether TV cable operators should be forced to offer channels a la carte. Due to the complexity of the model, our solutions are numerical.[3]

Cable prices, whether on an unadjusted or quality-adjusted basis, have been rising faster than inflation. Crawford et al. (2008), report that unadjusted cable prices increased by 84.1% over 1997-2005, while the increase was 50.5% when adjusted for quality improvements. In the same period, the increase in the CPI was only 18.8%. Consumer groups and politicians assert that the bundling practices by cable TV providers have contributed to the steep price increases. A la carte pricing is, therefore, promoted as a remedy for higher cable prices. By contrast, cable interest groups oppose the idea of a la carte pricing. They argue that bundling lowers transaction costs, realizes economies of scale, simplifies consumers' decision-making process, and a la carte pricing will hurt consumers by driving up prices. The attitude of the FCC about a la carte pricing is ambiguous. The FCC has published two reports on a la carte pricing. In the November 2004

[1] See, for example, Adams and Yellen (1976), Schmalensee (1984), McAfee, McMillan and Whinston (1989) and Bakos and Brynjolfsson (1999).

[2] Exception is the paper by Chao and Derdenger (2010) who examine the incentives of a monopolist to bundle in the context of the portable video game console market. Their model and results are very different from ours. Amelio and Jullien (2007), Choi (forthcoming) and Rochet and Tirole (2008) investigate tying in two-sided markets.

[3] Crawford and Cullen (2007) also examine the effects of a la carte pricing numerically, but in a one-sided environment.

report, they concluded that a la carte pricing would result in higher prices and thus provide little benefit to consumers. However, in the February 2006 report, their conclusion is the opposite from the previous one. The lack of consensus illustrated by the FCC's own reports is echoed in the academic literature. Crawford and Yurokoglu (forthcoming) predict that television license fees will increase following an a la carte regulation, while Rennhoff and Serfes (2009) claim that the fees will decrease. In this paper, we are able to illustrate that *either* price change may occur, depending on the underlying model parameters.[4]

We show that the inclusion of advertising can dramatically alter the bundling incentives of a monopolist. In a one-sided model, mixed bundling weakly dominates pure (and no) bundling, since mixed bundling contains the other two as special cases. This may no longer hold in a two-sided model. Pure bundling can *strictly* dominate mixed bundling (and no bundling). The intuition for this finding is that pure bundling can be used as a commitment to attract fewer consumers (viewers) and hence results in less advertising. The reduced advertising level increases the viewers' willingness to pay for the product and allows the monopolist to profitably increase the price of the bundle. Hence, we offer a novel explanation about why, in practice, pure bundling is the predominant way of offering TV channels.

When the cross-group externalities (that is, the nuisance cost to viewers from advertising and the profitability of more viewers to advertisers) are strong, then pure bundling is the most profitable method for the monopolist of offering the TV channels to end consumers. An a la carte regulation will force the monopolist to break the bundle and sell the goods individually. We identify three effects that affect consumer welfare as we move from pure bundling to a la carte: i) sorting effect, ii) license fee effect and iii) advertising effect. The bundle reduces consumer heterogeneity and allows the monopolist to extract more surplus, hence the sorting effect affects consumer welfare positively. The license fee (and thus the final price) may go either up or down, but when the cross-group externalities are strong it goes up, affecting consumer welfare negatively. Finally, advertising goes down following an a la carte regulation (because the number of viewers decreases and so does the demand for advertising on part of the advertisers), which increases consumer welfare. The net effect is in general ambiguous, but when the correlation of consumer valuations is positive (and strong) then the sorting effect is weak and an a la carte regulation hurts consumers. When the correlation is negative (and weak), on the other hand, consumers become better off.

Related Work. Rennhoff and Serfes (2009) and Crawford and Yurukoglu (forthcoming) are the

[4] While the discussion of a la carte pricing has been predominantly focused on the cable television industry, primarily as a response to rising cable prices, there has also been interest in a la carte pricing in a variety of other media areas, such as the market for satellite radio. Many industry analysts attribute Sirius XM Radio CEO Mel Karmazin's 2007 decision to begin offering a la carte pricing on a variety of satellite radio packages as a way of appeasing former Federal Communication Commission (FCC) Chairman Kevin Martin and helping secure FCC support for the proposed Sirius-XM merger.

closest papers to our work. Rennhoff and Serfes (2009) develop a theoretical upstream-downstream model to study the incentives of downstream firms to bundle their goods. Within this framework, the authors investigate the impact of an a la carte regulation. Crawford and Yurukoglu (forthcoming) also model the vertical channel and using a rich dataset from the TV industry they estimate the key model parameters and perform a counterfactual a la carte simulation. They find that an a la carte regulation will raise license fees and (most likely) reduce consumer welfare. Some of our results are consistent with this finding.[5] However, neither of the above two papers models advertising and its interaction with the viewer side in a two-sided market environment. Hence, our paper should be viewed as complementary to these two papers.

The remainder of the paper is organized as follows. The main model is presented in Section 2. The numerical simulation results, presented for a variety of different cases, are presented in Section 3. Section 4 contains welfare results and the welfare effects of an a la carte regulation. We conclude in Section 5.

2 The Model

A two-sided platform is comprised of two upstream firms, $i = a, b$, and one downstream firm. Each upstream firm sells one product to the downstream firm, who, in turn, sells them to final consumers. Each product also contains an amount of advertising, which is chosen by the upstream firms. Advertisers' profits depend positively on the number of final consumers, while consumers derive negative utility from advertising.[6] In this paper, we assume that the two goods are two TV channels/networks (e.g., CNN and ESPN) and the downstream firm is a cable operator (e.g., Comcast). We use the word 'channel' to refer to a 'good'. Figure 1 describes the structure of the model. The upstream TV channels have two sources of revenue: license fee and advertising fee, while the downstream monopolist earns revenues from the viewers.

The downstream monopolist packages and sells the two goods (a and b) to consumers. The monopolist can choose to offer these two goods either a la carte (interchangeable with "no bundling"), together as a bundle (defined as "pure bundling"), or a combination of both (defined as "mixed bundling"). Under a la carte, viewers are allowed to build the bundle on their own by paying the sum of the stand-alone prices. Under pure bundling, consumers choose to either buy the bundle or nothing. Under mixed bundling, consumers have four choices: good a only, good b only, a bundle of both a and b, and no purchase.

[5] In particular, we show in Section 4 that an a la carte regulation can result in higher license fees and hurt consumer welfare.

[6] However, our model is not restricted to the TV industry and can be easily adapted to deal with a positive externality, as it may be the case, for instance, with two rival newspapers, where readers benefit from ads (coupons, job listings, etc.).

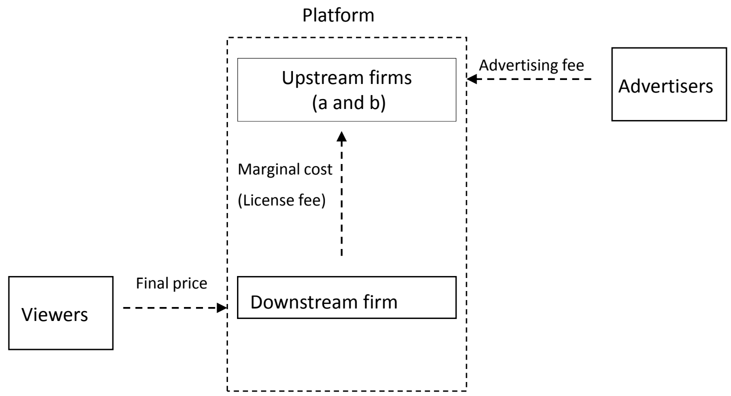

Figure 1: Structure of the upstream-downstream two-sided market model

We assume that consumer maximum willingness to pay for the two goods, (v_a, v_b), follows a bivariate normal distribution with mean (μ_a, μ_b), variance $\left(\sigma_a^2, \sigma_b^2\right)$ and covariance σ_{ab}. We denote the density by $f(v_a, v_b)$ and the marginal density by $f_i(v_i)$. The amount of advertising on channel i is α_i. Advertising imposes disutility on consumers (viewers) with t capturing the per-unit nuisance cost from advertising. Hence, consumer net utility from each channel is equal to $v_i - t\alpha_i - p_i$, where p_i is the price for channel i. Consumers, taking the advertising levels on each channel and the prices as given, buy the option which gives them the highest utility.

2.1 Downstream firm profit

The downstream monopolist has three options to offer the two channels: i) no bundling (a la carte), ii) pure bundling and iii) mixed bundling. We discuss each separately.

2.1.1 No bundling (a la carte)

Since the channels are offered separately, viewers will purchase channel i if and only if the channel valuation exceeds the disutility from advertising and the price, $v_i \geq t\alpha_i + p_i$, where p_i, $i = a, b$, are the stand-alone prices. Therefore, the corresponding demand for each channel is (see also Figure 2)

$$d_i = \int_{t\alpha_i + p_i}^{\infty} f_i(v_i)\, dv_i = 1 - F_i(p_i + t\alpha_i),\ i = a, b.$$

The profit function for the downstream monopolist is

$$\Pi = \pi_a + \pi_b = \sum_{i=a,b} (1 - F_i(p_i + t\alpha_i))(p_i - c_i) \tag{1}$$

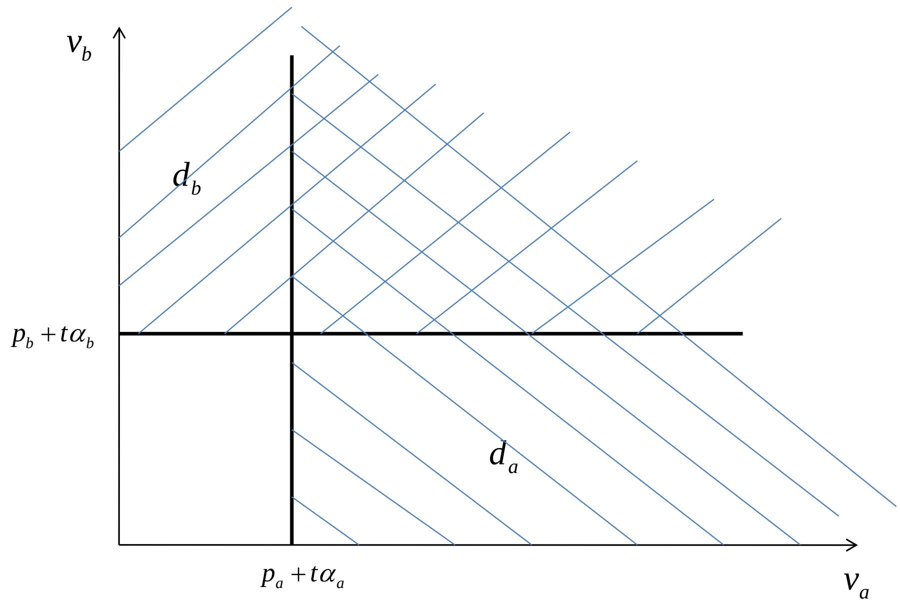

Figure 2: No bundling, a la carte, demand

where c_i is the marginal cost (license fee) charged by upstream firm i. The downstream monopolist chooses the stand-alone prices p_i to maximize its profit.

2.1.2 Pure bundling

We make a distinction between a subscriber and a viewer. Given that, under pure bundling, consumers cannot buy individual channels, we allow for the possibility that a consumer purchases the bundle but does not watch one of the two channels in the bundle. This can happen if the consumer has a sufficiently high valuation for one channel. The bundle price is p_B. The number of viewers for each channel can be calculated as follows (see also Figure 3).

Only channel a. A consumer purchases the bundle and watches only channel a if and only if

$$v_a \geq t\alpha_a + p_B \text{ and } v_b < t\alpha_b,$$

with mass of consumers given by

$$d_a = \int_{-\infty}^{t\alpha_b} \int_{p_B + t\alpha_a}^{\infty} f(v_a, v_b) dv_a dv_b.$$

Only channel b. A consumer purchases the bundle and watches only channel b if and only if

$$v_b \geq t\alpha_b + p_B \text{ and } v_a < t\alpha_a,$$

with mass of consumers given by

$$d_b = \int_{-\infty}^{t\alpha_a} \int_{p_B + t\alpha_b}^{\infty} f(v_a, v_b) dv_b dv_a.$$

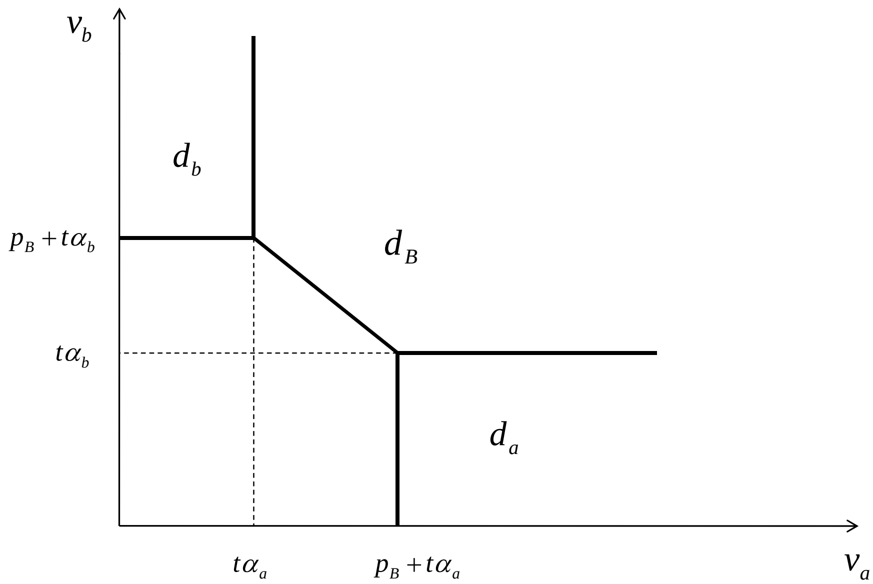

Figure 3: Pure bundling demand

Both channels. Finally, a consumer purchases the bundle and watches both channels if and only if

$$v_a + v_b \geq t\alpha_a + t\alpha_b + p_B,\ v_a \geq t\alpha_a \text{ and } v_b \geq t\alpha_b,$$

with mass of consumers given by

$$d_B = \int_{t\alpha_a + t\alpha_b + p_B - v_a}^{\infty} \int_{t\alpha_a}^{p_B + t\alpha_a} f(v_a, v_b) dv_a dv_b + \int_{t\alpha_b}^{\infty} \int_{p_B + t\alpha_a}^{\infty} f(v_a, v_b) dv_a dv_b.$$

The monopolist's demand function is $D_{PB} = d_a + d_b + d_B$, which represents the mass of subscribers, while the number of viewers for channel a is $d_a + d_B$ and for channel b is $d_b + d_B$. This distinction is necessary as the number of viewers will later be used to determine the advertising demand, since advertisers care about the number of viewers (and not about the number of subscribers).

The profit function of the monopolist is

$$\Pi_{PB} = D_{PB}(p_B - c_a - c_b) \tag{2}$$

which is maximized with respect to the bundle price p_B.

2.1.3 Mixed bundling

If the downstream monopolist offers the channels separately and together as a bundle, the viewers can choose the product that generates the highest net utility (see also Figure 4).

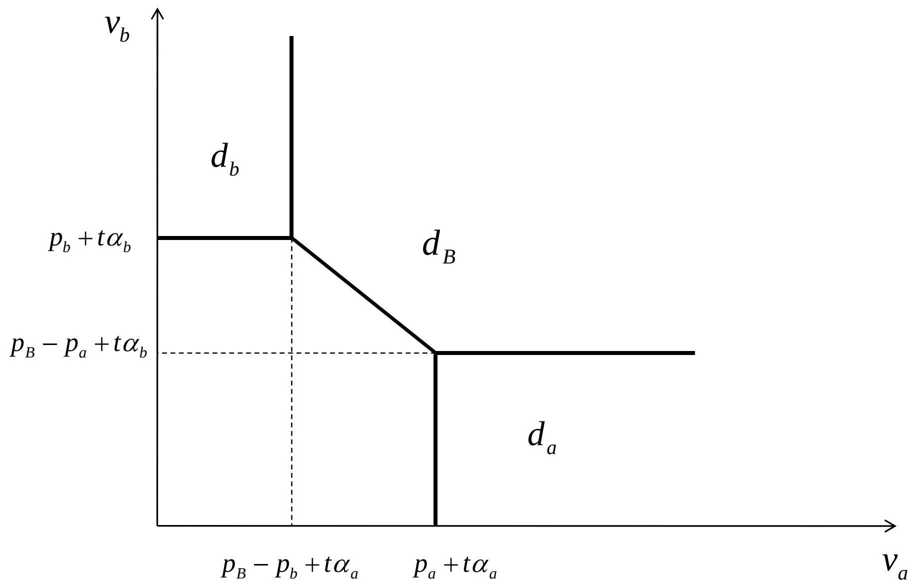

Figure 4: Mixed bundling demand

A consumer purchases channel a if and only if[7]

$$v_a - p_a - t\alpha_a \geq v_a + v_b - p_B - t(\alpha_a + \alpha_b) \text{ and } v_a - p_a - t\alpha_a \geq 0$$

with mass of consumers given by

$$d_a = \int_{-\infty}^{p_B - p_a + t\alpha_b} \int_{p_a + t\alpha_a}^{\infty} f(v_a, v_b) dv_a dv_b.$$

Similarly, a consumer purchases channel b if and only if

$$v_b - p_b - t\alpha_b \geq v_a + v_b - p_B - t(\alpha_a + \alpha_b) \text{ and } v_b - p_b - t\alpha_b \geq 0$$

with mass of consumers given by

$$d_b = \int_{-\infty}^{p_B - p_b + t\alpha_a} \int_{p_b + t\alpha_b}^{\infty} f(v_a, v_b) dv_b dv_a.$$

Finally, a consumer purchases the bundle if and only if

$$\begin{aligned} v_a + v_b - p_B - t(\alpha_a + \alpha_b) &\geq v_a - p_a - t\alpha_a,\ v_a + v_b - p_B - t(\alpha_a + \alpha_b) \geq v_b - p_b - t\alpha_b \\ \text{and } v_a + v_b - p_B - t(\alpha_a + \alpha_b) &\geq 0 \end{aligned}$$

which reduces to the following conditions

$$v_a \geq p_B - p_b + t\alpha_a;\ v_b \geq p_B - p_a + t\alpha_b; \text{ and } v_a + v_b \geq p_B + t(\alpha_a + \alpha_b).$$

[7]If a consumer prefers channel a over the bunlde, then the net utility from consuming only channel b is negative. Similarly, if a consumer prefers b over the bundle the net utility from consuming a exclusively is negative.

The mass of consumers is given by

$$d_B = \int_{p_B+t(\alpha_a+\alpha_b)-v_a}^{\infty} \int_{p_B-p_b+t\alpha_a}^{p_a+t\alpha_a} f(v_a, v_b)\, dv_a dv_b + \int_{p_B-p_a+t\alpha_b}^{\infty} \int_{p_a+t\alpha_a}^{\infty} f(v_a, v_b)\, dv_a dv_b.$$

Note that under mixed bundling there is no difference between a subscriber and a viewer, all subscribers are viewers. The profit function of the downstream firm can be expressed as follows

$$\Pi_{MB} = d_B(p_B - c_a - c_b) + d_a(p_a - c_a) + d_b(p_b - c_b). \tag{3}$$

The monopolist chooses the stand-alone prices p_a, p_b and the price of the bundle p_B to maximize (3).

2.2 Upstream firms' profits

The upstream firms' revenues are composed of two parts: the license fee paid by the downstream operator and the advertising fee paid by the advertisers. Let c_i be the license fee per subscriber. Cable operator needs to pay the license fee for each subscriber no matter whether he watches this channel or not. The profit from the license fee is

$$\Phi_i^L = c_i D_i$$

where D_i is the number of subscribers that channel i has, i.e. $D_a = d_a + d_b + d_B$ in pure bundling.

We assume that the advertiser willingness-to-pay will increase by n (where $n > 0$) if the number of viewers increases by 1. If we imagine the advertiser as the seller of a consumer product, this assumption is consistent with the notion that, holding all else constant, exposure to more viewers increases the number of purchases the firm expects to yield from a given advertisement.[8] The higher the value of n, the more valuable viewers are to the advertisers. We further assume that advertising spots are more valuable when a channel airs fewer advertisements. Therefore, we assume that the inverse demand for advertising on channel i is

$$g_i = nq_i - \alpha_i$$

where g_i is the advertising fee for advertising on channel i and q_i is the total number of channel i viewers. In pure bundling, for example, $q_i = d_i + d_B$.

The profit from advertising is given by

$$\Phi_i^A = (nq_i - \alpha_i)\alpha_i.$$

[8] Our modeling assumptions here are quite simplistic. We ignore, for example, the potential that different *types* of consumers might be more/less valuable to advertisers.

Thus, TV channel i's profit function is given by

$$\Phi_i = \Phi_i^L + \Phi_i^A = c_i D_i + (nq_i - \alpha_i)\alpha_i. \tag{4}$$

TV channel i chooses the amount of advertising α_i to maximize Φ_i. Moreover, it negotiates with the downstream cable operator over the license fee c_i. We turn to this negotiation process next.

2.3 Bargaining between an upstream firm (TV channel) and the downstream monopolist over the license fee

We assume that the license fee is determined via bilateral Nash Bargaining between each upstream firm and the downstream monopolist (Horn and Wolinsky (1988), Hart and Tirole (1990), McAfee and Schwartz (1994), and Segal and Whinston (2003)). The downstream firm will bargain with each upstream firm (i.e. Comcast-CNN and Comcast-ESPN) separately and simultaneously. We assume that upstream firms have 'passive beliefs', so if an upstream firm receives an out-of equilibrium offer from the downstream monopolist it still believes that the offer to the other upstream firm is at its equilibrium level. Each pair of upstream and downstream firms will choose the license fee c_i to maximize the Nash product. We assume that license fees are linear.[9] The Nash Product for the bargaining over the license fees between the downstream monopolist (cable operator) and TV channel i is defined as follows

$$NP_i = (\Pi^Y - \Pi^N)^\lambda (\Phi_i^Y - \Phi_i^N)^{(1-\lambda)},$$

where λ is the bargaining power of the downstream monopolist. We assume equal division of the bargaining power ($\lambda = 0.5$).[10]

2.3.1 No bundling

In the no bundling case, if the cable operator and network i reach an agreement, the cable operator's profit will be the maximum of (1) with respect to the stand-alone prices p_a and p_b, denoted by Π^Y. Network i's profit is $\Phi_i^Y = \Phi_i$, see (4). When the cable operator and network i do not reach an agreement, the cable operator's profit will become the profit from offering only channel j, $\Pi^N = \pi_j = (1 - F_i(p_j + t\alpha_j))(p_j - c_j)$, which is maximized with respect to p_j. Network i's profit is zero (there is also no advertising revenue because the number or viewers is zero), $\Phi_i^N = 0$. Therefore, we can explicitly write down the Nash bargaining product function as follows

$$NP_i^{NB} = (\Pi^Y - \pi_j)^\lambda (\Phi_i^Y)^{(1-\lambda)} = (\pi_i)^\lambda (\Phi_i)^{(1-\lambda)},$$

[9] While this is not general, linear per-subscriber fees are the norm in the TV industry.

[10] We also examine, in Section A.3, the robustness of our main results using different values for λ ($\lambda = 0.4$ and $\lambda = 0.6$). Our main results and insights are qualitatively unaffected.

which is maximized with respect to the license fee c_i.

2.3.2 Pure and Mixed Bundling

The Nash bargaining products for the remaining two cases can be derived similarly using (2), (3) and (4).

2.4 The three-stage game

Our model can be represented by the following three-stage game

- **Stage 1**: The downstream monopolist cable operator announces whether to offer the two goods (TV channels) as a mixed bundle, pure bundle or a la carte.
- **Stage 2**: The upstream television channels set advertising levels, α_i, which determines the advertising fee g_i. Upstream and downstream firms bilaterally negotiate over license fees, c_i.
- **Stage 3**: Downstream monopolist chooses final prices, p_i.

Due to the complexity of the model analytical solutions are impossible to obtain. Therefore, we solve for the equilibria numerically.[11]

3 Simulation Results

First, as a benchmark, we solve the model by eliminating both the vertical channel and the advertising side. This represents the textbook case of a monopolist who has to decide how to package its two goods to maximize profits. Second, we introduce the advertising side, but license fees are set to zero (no vertical channel). Third, we eliminate advertising, but we model the bargaining between upstream and downstream firms over the license fee. Finally, we solve the full-blown two-sided model with advertising and vertical bargaining over the license fee.

In our simulations, we hold the standard deviation of consumer valuations, σ_i, constant and equal to 10, while we vary the other parameters. The key parameters of the model that affect the equilibrium outcomes are: (1) Mean valuations of the two TV channels, μ_i. We define the market as a "big (mass) market" when μ_i is equal to 30, where almost every consumer has a positive valuation for the goods, and as a "small (niche) market" when μ_i is equal to 10 or 15, where a fraction of consumers have negative valuations; (2) The cross-group externalities, t and n that measure the

[11] Details of our numerical algorithm appear in the Appendix.

strength of the feedback loop in our two-sided model; and (3) The correlation coefficient ρ of the consumer valuations between the two goods.

In the tables that follow we only present the price of one good p_i, since in equilibrium, given that firms are assumed to be symmetric, we have $p_a = p_b$. Likewise, we also present one licence fee c_i. The total number of viewers is denoted by q, the profit of each upstream TV channel by π^U and the downstream monopolist's profits by π^D. Finally, CS denotes consumer surplus and AS the surplus to advertisers.

Consumer surplus is the area under the inverse demand curve above the final price and advertising surplus (AS) is the area under the inverse demand for advertising above the equilibrium advertising fee $g_i^*(\alpha_i^*, \alpha_{-i}^*)$, which is expressed as follows

$$AS_i = \int_0^{\alpha_i^*} \left[g_i(\alpha_i, \alpha_{-i}^*) - g_i^*(\alpha_i^*, \alpha_{-i}^*) \right] d\alpha_i.$$

Total surplus is defined as the sum of consumer surplus, downstream firm profit, upstream firms' profits and advertiser surplus

$$TS = CS + \pi^D + 2\pi^U + 2AS.$$

The results at the end of each subsection summarize the main novel findings from each case. Although the tables in the next four subsections also present welfare results, we do not offer any major welfare discussion until Section 4, where we focus on the full-blown model of Section 3.4 and, in particular, on the impact of an a la carte regulation relative to pure bundling.

3.1 Benchmark Case: No Advertising and Zero License Fees

Table 1: No Advertising and Zero License Fee, (Mass Market, $\mu_i = 30$)

	Corr.	p_i	p_B	q	π^D	CS	TS
NB	-	23.35	-	0.75	34.88	16.33	51.21
PB	-0.5	-	47.68	0.89	42.48	12.84	55.32
	0	-	46.48	0.83	38.60	14.80	53.40
	0.5	-	46.38	0.78	36.37	15.75	52.12
	0.9	-	46.62	0.75	35.14	16.23	51.37
	1	-	46.70	0.75	34.88	16.33	51.21
MB	-0.5	40.33	47.67	0.89	42.51	12.84	55.35
	0	34.37	46.65	0.83	38.64	14.77	53.41
	0.5	30.04	46.50	0.78	36.44	15.70	52.14

Tables 1 and 2 present the equilibria under no bundling, pure bundling and mixed bundling, respectively. In Table 1 the market is mass (mean valuation of each good is 30) while in Table 2

Table 2: No Advertising and Zero License Fee, (Niche Market, $\mu_i = 10$)

	Corr.	p_i	p_B	q	π^D	CS	TS
NB	-	11.32	-	0.45	10.13	6.73	16.86
PB	-0.5	-	16.68	0.63	10.59	5.87	16.37
	0	-	18.89	0.53	10.03	6.21	16.25
	0.5	-	20.87	0.48	10.02	6.48	16.50
	0.9	-	22.30	0.45	10.10	6.68	16.79
	1	-	22.63	0.45	10.13	6.73	16.86

(where we do not present the MB case, since it does not add anything new) the market is niche (mean valuation of each good is 10). Recall that the standard deviation is always equal to 10. The results are consistent with well-known results in the bundling literature, where mixed bundling weakly dominates pure bundling for a monopolist, e.g., Adams and Yellen (1976), Schmalensee (1984), and McAfee et al. (1989). By offering the mixed bundle the monopolist is able to sort the viewers and practice second-degree price discrimination. Under mixed bundling, viewers pay a higher price for a single channel than under no bundling, while the price for the bundle is slightly lower than under pure bundling. Not surprisingly, this flexibility boosts the monopolist's profit. The number of viewers, q, under mixed and pure bundling are significantly higher than under no bundling (the number of viewers under MB is slightly higher than under PB but not significantly). Total surplus (TS) is the highest under mixed bundling, while consumer surplus (CS) is the highest under no bundling (a la carte).

Consumer surplus under pure (and mixed) bundling decreases as the correlation of consumer valuations decreases. This is expected, because as the correlation decreases the consumer demand for the bundle becomes flatter, allowing the monopolist to extract more consumer surplus.

The change in monopoly profit as the correlation coefficient ρ varies is consistent with the findings in Johnson and Myatt (2006). To illustrate this, let's focus on pure bundling. When the market is big (mass market), as in Table 1, the valuation of the marginal consumers is consistently below the mean valuation for the bundle (which is 60). Higher correlation reduces the heterogeneity of consumer valuations and results in a clockwise rotation of the demand curve for the bundle (steeper). The willingness to pay of the marginal consumer decreases and so do the profits. On the other hand, when the market is small (niche market), as in Table 2, the valuation of the marginal consumer may be above the mean valuation for the bundle (which is 20). This is indeed true for high ρ's. Higher correlation in this case increases the willingness to pay of the marginal consumer and profits.

When $\rho = 1$, bundling makes no difference and the profits among the three different options (NB, PB, and MB) are equal.

3.2 Advertising and Zero License Fees

The upstream TV networks choose the level of advertising α_i (denoted by $Ad.$ in the tables), but the license fees are zero.

Table 3: Advertising and Zero License Fees, (Mass Market, $\mu_i = 30$, Low cross-group externalities $t = 1$, and $n = 1$)

	Corr.	p_i	p_B	$Ad.$	q	π^D	π^U	CS	AS	TS
NB	-	23.09	-	0.37	0.74	34.33	0.13	16.17	0.07	50.9
	-0.5	-	46.94	0.44	0.89	41.66	0.20	12.71	0.098	54.97
PB	0	-	45.83	0.41	0.83	37.91	0.17	14.66	0.085	53.08
	0.5	-	45.79	0.39	0.78	35.76	0.15	15.60	0.076	51.81
	-0.5	39.68	47.03	0.44	0.88	41.69	0.19	12.67	0.003	54.75
MB	0	33.90	46.00	0.41	0.82	38.00	0.17	14.58	0.003	52.93
	0.5	29.67	45.93	0.39	0.78	35.84	0.15	15.56	0.003	51.71

Table 4: Advertising and Zero License Fees, (Mass Market, $\mu_i = 30$, High cross-group externalities $t = 10$, and $n = 10$)

	Corr.	p_i	p_B	$Ad.$	q	π^D	π^U	CS	AS	TS
NB	-	14.04	-	1.46	0.55	15.57	5.96	9.43	4.46	45.84
	-0.5	-	20.33	1.74	0.68	14.84	8.71	7.35	3.47	46.55
PB	0	-	21.27	1.75	0.60	13.24	7.47	7.75	3.27	42.47
	0.5	-	23.17	1.76	0.54	12.53	6.39	7.80	3.15	39.41
	-0.5	17.00	19.69	1.87	0.89	13.57	9.48	7.89	5.01	50.44
MB	0	17.92	24.45	1.58	0.61	15.58	6.99	8.37	3.34	44.61
	0.5	17.22	24.75	1.55	0.68	15.60	6.96	9.13	2.90	44.45

Table 3 shows the equilibria when both cross-group externalities, t and n, are low and equal to 1. Table 4 shows the equilibria when the cross-group externalities are high, $t = 10$ and $n = 10$. A la carte (NB) attracts the least number of viewers. This is expected, since under a la carte consumers are not forced to purchase goods for which they have low valuations. Without advertising, as in Section 3.1, this yields the lowest profits to the downstream monopolist. However, this ranking of profits may fail to hold when advertising is introduced. Fewer viewers implies fewer ads, and given the negative externality of ads to viewers, it also implies higher willingness to pay on part of consumers and as a consequence higher profits to the downstream monopolist. This scenario is demonstrated in Table 4, where the nuisance cost of advertising t is high. Interestingly, and following a similar logic, the standard weak dominance of mixed bundling over pure bundling may not hold. *Pure bundling can strictly dominate mixed bundling*, precisely because pure bundling contains less advertising (because it inherently attracts fewer viewers). To see this, compare PB and MB when $\rho = -0.5$ in Table 4, $14.84 > 13.57$.

If the advertising levels were the same between MB and PB, then MB would be weakly dominant. But the upstream firms and the advertisers anticipate the higher number of viewers under MB and advertise more. To counteract this, the monopolist commits to PB as a way to reduce the number of viewers and hence advertisements. The assumption that the monopolist can commit to PB in stage 1 of the game is crucial, because for *any* given advertising level MB is weakly dominant. However, in reality, interaction is repeated and such commitments, if they are profitable, are easier to make.

Consumer surplus is still the highest under a la carte (NB). Mixed bundling is not always the most efficient method of offering the two goods. A la carte can dominate MB in terms of total surplus (see Table 4 when $\rho = 0$ and 0.5).

Result 1: *In a two sided market, it is possible that pure bundling strictly dominates mixed and no bundling for the downstream monopolist. This is because the advertising levels are lower under pure bundling, which increases demand for the goods (TV channels) and allows the monopolist to charge higher prices.*

3.3 No Advertising and Positive License Fees

Each upstream firm negotiates with the downstream monopolist the license fee c_i. There is no revenue from advertising.

Table 5: No Advertising and Positive License Fees, (Mass Market,$\mu_i = 30$)

	Corr.	p_i	p_B	c_i	q	π^D	π^U	CS	TS
NB	-	27.20	-	11.29	0.61	19.42	6.89	11.08	44.28
	-0.5	-	52.36	13.13	0.78	20.30	10.21	8.92	49.64
PB	0	-	52.94	12.59	0.69	19.19	8.70	9.86	46.45
	0.5	-	54.09	12.47	0.63	18.47	7.90	10.26	44.53
	-0.5	36.07	52.81	12.79	0.74	21.32	9.50	9.00	49.32
MB	0	33.44	53.25	12.15	0.67	20.24	8.18	10.01	46.61
	0.5	31.43	53.91	11.87	0.63	19.57	7.51	10.58	45.17

Table 6: No Advertising and Positive License Fees, (Niche Market,$\mu_i = 15$)

	Corr.	p_i	p_B	c_i	q	π^D	π^U	CS	TS
NB	-	17.27	-	6.72	0.41	8.65	2.76	5.91	20.08
	-0.5	-	27.70	6.24	0.59	9.00	3.69	5.25	21.63
PB	0	-	30.49	6.63	0.49	8.38	3.22	5.40	20.22
	0.5	-	33.21	7.19	0.43	8.03	3.07	5.42	19.59

What is the effect of a la carte pricing on the license fees? The answer to this question is important, because license fees affect downstream prices and consumer welfare. Crawford and Yurukoglu (forthcoming) show that an a la carte regulation will increase license fees while, Rennhoff and Serfes (2009) show the opposite. Our numerical results try to shed light on this issue. While our full discussion of a la carte welfare appears in the next section, it is instructive to consider the impact of market size on license fees in the current context, which eliminates the confounding impact of advertising. We find that in a mass market, as in Table 5, an a la carte regulation is more likely to lower license fees, while in a niche market, as in Table 6, license fees are possible to increase.

The reason why license fees decrease when the market is big ($\mu = 30$) and may increase when the market is small ($\mu = 15$) can be explained using elasticities, which can be computed from the first order condition of the Nash product with respect to the license fee, which after some derivations yields

$$\frac{dNP}{dc_i} = A\left(2 - |\varepsilon_p^q|(1 - \varepsilon_c^p)\right)$$

where ε_p^q is viewers' demand price elasticity, ε_c^p is the elasticity of final price with respect to the license fee (marginal cost), and A is $\frac{q^2 p}{-\varepsilon_p^q}$. We evaluate the above derivative at the license fees that prevail in the PB equilibrium, but we allow the downstream monopolist to respond by offering the two goods a la carte. If $\frac{dNP}{dc_i}$ is higher (lower) than zero, license fees increase (decrease) when a la carte pricing is imposed, relative to PB. It turns out that demand is more elastic when μ is high. Let us use $\rho = -0.5$ as an example. Table 7 depicts the elasticities for a small (niche) and a big (mass) market respectively.

Table 7: Table: Comparison of Elasticities

	c_i	p	q	$\frac{dq}{dp}$	ε_p^q	ε_c^p	$\frac{dNP}{dc}$
$\mu = 15$	6.24	16.99	0.42	0.039	-1.58	0.206	0.04
$\mu = 30$	13.13	27.97	0.58	0.039	-1.88	0.204	-0.16

As we can discern from Table 7, own-price elasticity $|\varepsilon_p^q|$ is much higher when the market is big, while ε_c^p is about the same. Hence, the elastic demand contributes to the license fee reduction in a big market. Furthermore, demand is elastic because when we compare a big with a small market the price increase is much more pronounced than the quantity increase (from 16.99 to 27.97 versus from 0.42 to 0.58), while the slope of the demand dq/dp, in equilibrium, between the two markets is about the same.

Result 2: *License fees are more likely to decrease (increase) in a mass (niche) market when there is an a la carte regulation, relative to pure bundling.*

3.4 Full-Blown Two-Sided Model with Advertising and License Fees

Each upstream TV network has two sources of revenue: from the advertising fee and from the license fee. Hence, in maximizing its profit each TV network balances optimally these two revenue sources. Too much advertising yields high advertising revenue, but, due to the negative externality, fewer viewers and as a consequence low revenue from the license fee. On the other hand, low license fee implies low final price, more viewers and higher willingness to pay on part of the advertisers.

Table 8: Two-Sided Market with Advertising and Positive License Fees, $\mu_i = 30$, $t = 1$ and $n = 1$

	Corr.	p_i	p_B	Ad	c_i	q	π^D	π^U	CS	AS	TS
NB	-	27.00	-	0.22	11.11	0.61	19.38	6.86	11.08	0.024	44.23
	-0.5	-	51.62	0.34	12.74	0.78	20.49	10.07	8.97	0.058	49.72
PB	0	-	52.32	0.29	12.25	0.69	19.25	8.59	9.88	0.043	46.40
	0.5	-	53.51	0.26	12.14	0.63	18.55	7.80	10.29	0.035	44.51
	-0.5	35.78	52.25	0.30	12.55	0.74	21.19	9.48	8.97	0.002	49.12
MB	0	33.19	52.78	0.25	11.95	0.67	20.18	8.14	9.99	0.002	46.45
	0.5	31.09	53.50	0.23	11.61	0.63	19.53	7.43	10.55	0.001	44.94

Table 9: Two-Sided Market with Advertising and Positive License Fees, $\mu_i = 30$, $t = 1$ and $n = 5$

	Corr.	p_i	p_B	Ad	c_i	q	π^D	π^U	CS	AS	TS
NB	-	24.81	-	1.50	7.54	0.64	22.24	7.43	12.20	1.335	51.97
	-0.5	-	47.11	2.14	8.93	0.80	23.33	11.21	9.68	2.310	60.05
PB	0	-	47.80	1.77	8.30	0.73	22.75	9.37	10.99	1.509	55.50
	0.5	-	48.92	1.63	8.26	0.67	21.84	8.41	11.51	1.327	52.82
	-0.5	34.69	47.98	1.85	8.81	0.78	24.55	10.65	9.75	0.271	56.14
MB	0	31.73	48.45	1.67	8.31	0.71	23.23	9.03	10.95	0.207	52.65
	0.5	29.37	49.19	1.57	8.11	0.67	22.36	8.18	11.58	0.173	50.65

Table 10: Two-Sided Market with Advertising and Positive License Fees, $\mu_i = 30$, $t = 3$ and $n = 1$

	Corr.	p_i	p_B	Ad	c_i	q	π^D	π^U	CS	AS	TS
NB	-	27.10	-	0.04	11.22	0.61	19.36	6.86	11.06	0.0009	44.14
	-0.5	-	50.95	0.24	12.50	0.78	20.20	9.83	8.89	0.028	48.81
PB	0	-	51.81	0.19	12.05	0.69	19.14	8.42	9.84	0.019	45.86
	0.5	-	53.04	0.18	11.98	0.63	18.40	7.66	10.24	0.065	44.09
	-0.5	36.59	52.20	0.12	12.56	0.74	20.96	9.36	8.91	0.001	48.59
MB	0	33.22	52.87	0.07	12.02	0.67	20.11	8.11	9.97	0.0004	46.30
	0.5	31.17	53.56	0.06	11.69	0.63	19.56	7.42	10.57	0.0003	44.97

To examine the effects of the cross-group externalities, t and n, on the equilibrium, we begin, in Table 8, with low externalities, $t = 1$ and $n = 1$, and then we increase them one at a time. In

Table 11: Two-Sided Market with Advertising and Positive License Fees, $\mu_i = 30$, $t = 5$ and $n = 5$

	Corr.	p_i	p_B	Ad	c_i	q	π^D	π^U	CS	AS	TS
NB	-	22.49	-	1.01	7.03	0.60	18.49	6.20	10.70	0.965	43.52
	-0.5	-	34.41	1.70	3.81	0.79	21.69	6.83	9.37	1.599	47.92
PB	0	-	36.55	1.56	3.61	0.71	20.87	5.68	10.43	1.322	45.30
	0.5	-	36.94	1.45	2.44	0.68	22.24	4.53	11.84	1.148	45.44
	-0.5	29.55	40.50	1.28	7.34	0.73	20.00	8.32	8.67	0.632	46.57
MB	0	27.75	42.43	1.15	7.31	0.66	19.01	7.25	9.62	0.485	44.10
	0.5	26.13	43.88	1.07	7.30	0.62	18.52	6.68	10.13	0.471	42.95

Table 9, we only increase the benefit of viewers to advertisers n, then in Table 10 we only increase the negative externality of advertising t ($t = 3$, since for $t = 5$, for example, the advertising level is negative) and in Table 11 we increase both.

An increase in n, holding t fixed, indicates higher advertisers' willingness to pay (from Table 8 to 9). Under this scenario, TV networks are able to attract more advertisers, therefore, the TV network as a platform of two different groups of agents is willing to lower the license fee so that it can increase the volume of trade (Rochet and Tirole (2006)). Indeed, in Table 9 TV networks will set, in equilibrium, lower license fees, relative to those in Table 8, to attract more viewers and as a result will increase their revenue from advertising. The amount of advertising also increases.

An increase in t, holding n fixed, implies higher nuisance cost from advertising (from Table 8 to Table 10). The amount of advertising decreases, relative to Table 8. The license fee may decrease or increase, depending on the correlation coefficient and on how the monopolist packages the two goods.

Finally, when both n and t increase (from Table 8 to Table 11) license fees decrease, while the amount of advertising increases.

Another interesting feature of the model is the direction of change of the license fees, when we move from PB to a la carte, as a function of the cross-group externalities. As we have already alluded to in Section 3.3, license fees are an important determinant of the final price and therefore it is imperative to understand what affects the direction of change when a la carte pricing is imposed. In Section 3.3, where there was no advertising, we argued that it is more likely for the license fees to go down in a big (mass) market. Here, the comparison is with respect to the cross-group externalities holding the size of the market fixed. When these externalities are low (Table 8) an a la carte regulation will decrease the license fees, while when the externalities are strong (Table 11) license fees increase. Final demand is less elastic when the externalities are strong and it pays to increase the license fees (more details on this in Section 4). Consumers are less willing to subscribe when the amount of advertising is high and their disutility from advertising is also high.

Result 3: *When both cross-group externalities t and n are low (as in Table 8), the advertising level and license fees are the highest under pure bundling. When the cross-group externality from viewers to advertisers, n, is strong while the disutility from advertisement to viewers, t, remains low (as in Table 9), the TV networks are willing to lower the revenue from license fees so that they can increase the revenue from advertisement. However, when the disutility from advertisement to viewers is high but the cross-group externality from viewer to advertiser is low (as in Table 10), the TV networks will focus on the revenue from license fees and give up the revenue from advertisement.*

Result 4: *When both cross-group externalities t and n are high (as in Table 11), the license fees will be the lowest under pure bundling. Pure bundling strictly dominates mixed and no bundling for the downstream monopolist.*

4 Welfare Analysis and the Effects of an A La Carte Regulation

Our welfare analysis discussion in this Section will focus on the full-blown two-sided market model of Section 3.4. In the discussion of the welfare properties, we focus on the equilibria where $\mu = [30, 30]$.

There are three effects associated with an a la carte regulation, relative to pure bundling: (1) sorting effect of PB, (2) license fee effect, and (3) advertising effect. First, pure bundling is used to reduce the heterogeneity of consumer valuations and extract more consumer surplus. The lower the correlation of consumer valuations the better the sorting effect works for the monopolist. An a la carte regulation is likely to benefit consumers and this benefit increases as the correlation decreases. Second, an a la carte regulation may increase or decrease the license fee, which directly affects the final price. Finally, a la carte attracts fewer viewers relative to pure bundling and therefore less advertising. Indeed, Tables 8-11 demonstrate that advertising under NB is always below the levels under PB. This tends to increase consumer surplus.

Our simulations reveal that when the cross-group externalities are strong (a situation which is arguably quite realistic), as it is the case in Table 11, pure bundling strictly dominates all other options for the downstream monopolist. This is consistent with what is observed in the TV industry where only big bundles of channels are available to consumers. It also explains why cable operators oppose a la carte pricing (*even* mixed bundling).

If we focus on Table 11, we can see the importance of advertising. *First, it is the inclusion of advertising that makes pure bundling strictly dominant for the downstream monopolist.* In Table 5, where advertising is absent, mixed bundling strictly dominates pure bundling in terms of π^D (downstream monopoly profits). Second, with advertising, it is possible to make the a la carte option consumer welfare decreasing, although in most cases it is consumer welfare increasing. A la carte pricing hurts consumers when $\rho = 0.5$ in Table 11.[12] In the absence of advertising, a la carte pricing increases consumer surplus, relative to pure bundling. This is illustrated in Section 3.3, where license fees are positive but there are no ads. To better understand this let's compare Table 11 with Table 5 (where there is no advertising). The bundle price p_B when $\rho = 0.5$ in Table 5 is 54.09, whereas in Table 11 for the same correlation coefficient $p_B = 36.94$. This is because advertising imposes a disutility to viewers and forces the firms to charge lower prices. Moreover, the number of viewers q increases, from 0.63 in Table 5 to 0.68 in Table 11. This suggests that the price elasticity of consumer demand decreases when advertising is included.[13] In turn, this explains the significant increase in the license fee (from 2.44 to 7.03) and the final price when a la carte pricing is imposed. This is consistent with the findings of Crawford and Yurukoglu (forthcoming) where they predict that a la carte will raise the license fees.

We can conclude that, moving from pure bundling (which is the most profitable for the monopolist) to a la carte, the sorting effect affects consumer welfare positively (and more strongly so as the correlation decreases), the license fee effect affects consumer welfare negatively and the advertising effect affects consumer welfare positively. For low values of the correlation coefficient the sorting effect is strong and consumers benefit, while for high levels of the correlation the sorting effect is weak and an a la carte regulation will hurt consumers.

The profits of TV networks in most cases we have examined increase when we move from PB to NB, while the profits of the downstream cable operator usually decrease. This difference in the direction of changes in profits is predominantly due to the license fee effect. In most cases, a la carte pricing decreases the license fee, which helps the downstream monopolist and hurts the upstream TV channels. We summarize our main findings below.

Result 5: *In the two-sided market model with advertising and license fees, when cross-group externalities are strong (as in Table 11), pure bundling strictly dominates mixed bundling and a la carte for the downstream monopolist. An a la carte regulation (relative to pure bundling) increases the license fees and results in less advertising. Consumer surplus may decrease or increase, depending on the correlation of the consumer valuations. If the correlation is positive then an a la carte regulation is more likely to hurt consumers, while the reverse is true under a negative correlation.*

[12]A number of our important conclusions are due to the predicted prices and advertising levels presented in Table 11. In order to verify, as best one can with numerical solutions, the accuracy of these results, we have solved these models numerous times from different starting values. While there is slight variation in the precise numerical values obtained from different starting values, these differences are not large enough to impact the conclusions that we draw from the reported tables. We report the estimated prices and profits for a variety of starting values in the Appendix.

[13]Performing a similar calculation as in Table 7 we find that $dNP/dc = 0.42$.

5 Conclusion

We conduct a numerical simulation to compute the equilibria of a two-sided market model with an upstream-downstream structure. More specifically, the platform consists of two rival upstream firms and a downstream monopolist. Each upstream firm negotiates with the downstream monopolist the input price (license fee) and also chooses the amount of advertising that is embedded in the good it sells to the downstream monopolist. The downstream monopolist, in turn, decides how to offer the two goods: a la carte, pure bundle or mixed bundle.

We use this model to understand the incentives to bundle and the properties of bundling in a two-sided market framework. We also contribute to the ongoing debate about a la carte pricing in the TV industry. The upstream firms can be viewed as two TV networks (e.g., ESPN and CNN) and the downstream firms as a monopolist cable operator (e.g., Comcast).

We derive a number of results, but the most interesting and novel ones can be summarized as follows. First, and contrary to standard results on bundling in one-sided markets, we show that in a two-sided market, pure bundling can *strictly* dominate mixed bundling (and a la carte). This is because PB attracts fewer viewers and hence less advertising, which increases the consumers' willingness to pay and the monopolist's profits. This finding provides a novel explanation about why pure bundling is the predominant method cable operators choose to offer TV channels. Second, we shed more light on the conditions that determine whether a la carte pricing lowers or increases the license fees. In our model both possibilities arise, depending on the size of the market. Finally, we examine the effects of an a la carte regulation. There are three effects that affect welfare: i) sorting effect, ii) license fee effect and iii) advertising effect. Our paper is the first one that combines all these three effects, allowing also for correlation between the two goods.

When the cross-group externalities are strong (that is, the nuisance cost of advertisements to viewers and the profitability of an extra viewer to advertisers), then pure bundling strictly dominates all other options for the downstream monopolist. This is a quite realistic scenario for the TV industry, where, arguably, both the disutility from ads to viewers and the profitability of additional viewers to advertisers are high. A la carte pricing will increase the licence fees (and final prices), but it will result in fewer ads (and the freedom to viewers to pick and choose). The net impact on consumer welfare is ambiguous. If the correlation of consumer valuations is positive then consumer welfare is likely to decrease, but the opposite may be true when valuations are negatively correlated.

In this paper, we have relied on numerical simulation methods to solve our two-sided model. There are a number of limitations inherent in our choice. Without closed-form solutions it is difficult to truly conduct comparative statics, which would allow us to isolate and identify the impact that each model parameter has on equilibrium prices and advertising levels. We have made a modest attempt at conducting comparative statics using different parameter values, but we acknowledge that the insights we derive may be limited.

Our reliance on time-consuming numerical methods also requires us to make a number of simplifying assumptions. For example, we assume the upstream firms are symmetric. There are a variety of ways in which asymmetry might meaningfully be incorporated into the model (asymmetric value to the advertisers, costs, etc.). Additionally, we have ignored the possibility of downstream competition, which could certainly have additional implications for the profitability of bundling decisions.

A Appendix

A.1 Simulation Algorithm

We use Matlab to conduct the numerical simulation needed to find the model equilibria. Since it is not possible to derive analytical expressions for the final prices a cable operator will charge for given license fees and advertising levels, we are unable to use first order conditions to find the equilibrium prices. Our inability to rely on first order conditions substantially increases the computation time of our simulation. Instead of simultaneously solving a system of equations for all unknown prices, we must rely on an iterative approach. In our model, the final prices p_i are determined in a different stage than the stage of setting advertising levels and license fees. Therefore, in each iterative step, we need to find the optimal prices for the downstream cable operator in response to the changes in either advertising levels or license fees. All minimizations (maximizations) are done using the Nelder-Mead method (Matlab's *fminsearch* command). The time needed to complete one simulation ranges from eight hours to one day. More specifically, we use the iterative approach we describe below to derive the equilibrium values in Tables 3-6, 8-11 and 13-14. Tables 3 and 4 include only advertising, so we skip steps 4 and 5 in the estimation, while Tables 5 and 6 include only license fees, so steps 2 and 3 are skipped.

The detailed simulation algorithm is summarized as follows:

Step 1: We start by choosing initial values for advertising levels α_1^{old} and α_2^{old}, the license fees $c_1^{old}$and c_2^{old}, and the final prices set by the cable operator.

Step 2: We fix α_2^{old}, c_1^{old}, and c_2^{old} and allow α_1 to change. The cable operator responds to any change in α_1 by setting (new) prices p_i to maximize its profit. Channel 1 will choose its advertising level until its profit is maximized. We rename this new (conditionally optimal) advertising level as α_1^{new}. Given the values of α_2^{old}, c_1^{old}, and c_2^{old}, this is assumed to be an optimal best response.

Step 3: The steps are similar for channel 2 to choose its advertising level. Taking α_1^{new}, c_1^{old}, and c_2^{old} and the response from the cable operator p_i as given, channel 2 sets their advertising level to maximize its profit. The cable operator respond to any change in α_2 by setting (new) prices p_i to maximize its profit.This new level is denoted α_2^{new}.

We then switch to the determination of license fees:

Step 4: Taking the current advertising values α_1^{new} and α_2^{new} and the initial value of c_2 as given, we find the new optimal license fee for channel 1 (c_1^{new}) by maximizing the Nash product. Once again, we allow the cable operator to respond to any change in c_1 by setting new prices to maximize its profit.

Step 5: We then switch to the estimation of the license fee for channel 2. Taking α_1^{new}, α_2^{new}, and c_1^{new} as given, we find the new optimal license fee for channel 2 by maximizing the Nash product. As before, in each iteration, the cable operator responds to the updated c_2 with new optimal prices.

Steps 2 through 5 are repeated until the changes in α_1, α_2 , c_1, and c_2 are smaller than a specified threshold, which is 10^{-12} in our estimations in Tables 3-6, 8-11 and 13. In Table 14, the threshold is 10^{-6} and in Table 4, when $\rho = -0.5$, the threshold is 0.001, since we were not able to have convergence when setting 10^{-12} as exit point.

For Tables 1-2, we just need to find the prices that maximize the monopolist's profit. To do so, we use the command "fminsearch".

A.2 Starting Value Sensitivity

A number of our primary conclusions are based - at least in part - on the simulation results in Table 11. Because our methods are numerical, determining the accuracy of our predicted prices is not quite as simple as verifying that a particular first-order condition holds. To anecdotally test whether it is reasonable to believe that our simulation algorithm has identified the appropriate prices, we present the results (of Pure Bundling in Table 11) for a variety of different starting values below.

Table 12:

Initial values			Estimated values			Function values		
p_B	Ad	c	p_B	ad	c	π^D	π^U	Log(NP)
40	0.415	10	38.83	1.42	3.98	20.24	5.25	1.73
30	1.415	2.5	37.99	1.48	3.65	20.29	5.10	1.70
30	5	15	38.83	1.42	3.98	20.25	5.25	1.73
40	0	1	38.38	1.42	3.93	20.28	5.27	1.72
30	2	5	38.83	1.42	3.98	20.24	5.25	1.73

A.3 Different Bargaining Power Parameters

The results in Tables 8-11 were derived based on an assumption of equal bargaining power. In order to assess the impact that bargaining power differences might have on our key set of results, we have replicated the results from Table 11 under the assumptions of $\lambda = 0.4$ and 0.6 (recall that λ is defined as the monopolist's bargaining power). These results, which are presented in Tables 13 and 14, are fundamentally similar to those in Table 11. The license fees are lower under pure bundling than they are under no bundling, although the level of these fees varies depending on the extent of the monopolist's power. Additionally, when $\rho = 0.5$ pure bundling yields higher consumer surplus than no bundling.

Table 13: Two-Sided Market with Advertising and Positive License Fees, $\mu_i = 30$, $t = 5$, $n = 5$, and $\lambda = 0.6$

	Corr.	p_i	p_B	Ad	c_i	q	π^D	π^U	CS	AS	TS
NB	-	20.99	-	1.17	4.55	0.62	20.53	5.12	11.54	1.275	44.86
	-0.5	-	33.14	1.81	1.59	0.81	24.38	5.29	9.86	1.824	48.41
PB	0	-	34.51	1.66	1.49	0.73	23.18	4.44	11.15	1.512	46.23
	0.5	-	36.94	1.45	2.34	0.68	22.24	4.46	11.84	1.126	45.26
	-0.5	28.78	37.47	1.49	4.60	0.76	22.45	6.89	9.27	0.760	47.02
MB	0	26.75	39.36	1.33	4.68	0.69	21.34	6.02	10.37	0.604	44.96
	0.5	24.91	40.79	1.25	4.71	0.65	20.67	5.53	10.98	0.511	43.73

Table 14: Two-Sided Market with Advertising and Positive License Fees, $\mu_i = 30$, $t = 5$, $n = 5$, and $\lambda = 0.4$

	Corr.	p_i	p_B	Ad	c_i	q	π^D	π^U	CS	AS	TS
NB	-	24.13	-	0.83	9.65	0.57	16.48	7.16	9.84	0.66	41.96
	-0.5	-	36.98	1.58	6.10	0.76	18.97	8.16	8.58	1.39	46.65
PB	0	-	39.29	1.41	6.23	0.68	18.26	7.04	9.56	1.07	44.07
	0.5	-	41.89	1.27	6.76	0.62	17.66	6.55	9.95	0.88	42.47
	-0.5	30.38	43.72	1.06	10.00	0.69	17.71	9.46	8.07	0.48	45.66
MB	0	28.92	45.80	0.94	10.03	0.62	16.92	8.30	8.87	0.35	43.09
	0.5	27.79	47.11	0.84	10.33	0.59	16.58	7.84	9.45	0.25	42.21

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