Emergency Department Overcrowding as a Nash Equilibrium: Hypothesis and Test by Survey Methodology

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Introduction

Data show that between 1994 and 2004, emergency department (ED) visits have increased by 26%, while the number of EDs has decreased by 9% and hospitals have closed 198,000 beds. (Kellerman, 2006) Ostensibly, this would explain the observed overcrowding in EDs. As a symptom of this overcrowding, wait times are increasing, diminishing quality and speed of care (Wilper et. al, 2008; Derlet and Richards, 2000; Burt and McCaig, 2006)

However, the problem may not be as simple as diminished capacity and increasing demand. While ED visits have increased and total number of EDs have diminished, the bed and treatment capacity of many EDs has actually increased. Between 2003 and 2004, 16.1 percent of all hospitals recently expanded their ED physical space, and approximately one-third of others plan to do so within the next 2 years. Over all, about 43.2 percent of all EDs had either recently expanded or had planned to do so. While it may be counterintuitive, the EDs most likely to be overcrowded were the ones

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which had recently expanded. Their characteristics were higher volume; located in a hospital classified as proprietary, voluntary, or nonprofit; affiliated with medical schools; experienced ambulance diversion; and finally demonstrated longer average visit durations. (Burt and McCaig, 2006) One California study demonstrated that ED bed capacity increased between 1990 and 2001 in roughly 40% of EDs and that over time, the number of ED beds per 100,000 people actually increased by 5 to 10%. (Melnick et. al, 2004)

Although ED overcrowding is a multifactorial problem, insured patients with less urgent conditions may be driving the volume increase. (Cunningham and May, 2004) This may account for why some hospitals have found it cost effective to expand ED capacity and others have not, perhaps depending upon the insurance status of the population they serve. (Melnick et. al, 2004) Additionally, across the health care system, patients report increasing difficulty in accessing office based care for both routine and urgent health care needs, driving patients to seek care in EDs. (Burt et. al, 2004)

While patients seek healthcare in the ED for a variety of reasons, but it suffices to say that a portion have an acute emergency and a portion seeks care because of lack of an acceptable alternative. In other words, a large portion of patients turn to EDs because they are diverted from other forms of medical care that is either at capacity or unavailable to them. Examples of this include but are not limited to: primary care physician not available within the desired time, specialist not available within the desired time, patients with no health care insurance or money to pay a specialist or primary care physician, or alternative health care source is inconvenient. In addition, it is a common practice for physicians themselves to use the ED as alternative access to the hospital for admission, lab work, evaluation, or procedures. Even the hospital itself “diverts” patients to the ED when the admissions are held in the ED until a bed becomes available. One final source of patients is those that are diverted from other closed EDs in the area. The capacity for handling patients in any given ED is limited by facility, number of physicians, and number of nurses, number of support staff and services.

One generally accepted solution to ED overcrowding and congestion is to increase capacity. But does this follow from an accurate diagnosis of the problem? Drawing on noncooperative game theory, we will argue that ED overcrowding is the equilibrium state of the current health care system. If this hypothesis is correct, then increasing capacity will merely reproduce the crowding problem on a larger scale.

Small-Scale Game-Theoretic Models

As a preliminary model consider Table 1. This is the simplest nontrivial game model of the problem, a 2x2 game in which the two players are two potential patients. Each patient has a medical problem that needs medical attention. Each can choose to go to their primary care provider’s (PCP) office, but if both patients do so, then the office will be overloaded and they may be turned away until an appointment is available. However, they could go to the ED and get care sooner – unless they both decide to do so and the ED becomes crowded. We assume the best response is to maximize your personal satisfaction based on your perceived need for immediate attention (rather than obey rules imposed by your health insurance for example). We will express the payoffs in terms of a patient satisfaction scale, where 1 is the least satisfied and 10 is the most satisfied. Furthermore, a crowded ED or office is less satisfying in its delivery of health care than one that is not crowded (i.e. "quiet"). In this example, we are using the following assumptions: satisfaction with quiet ED = satisfaction with quiet PCP office > satisfaction with crowded ED = satisfaction with crowded PCP office. Either service is crowded if both patients choose it. The satisfaction ratings shown in Table 3 are consistent with these assumptions.

Table1. The Patients' Coordination Problem

Payoffs represented as Patient 1,2		Patient 2	
		Go to ED	Go to PCP office
Patient 1	Go to ED	1,1	8,8
	Go to PCP office	8,8	1,1

In this case, no common the strategy "Go to the ED" or "Go to PCP office" is an equilibrium strategy combination. Whenever Patient 1 chooses to go to his PCP, Patient 2's best response is to take advantage of the ER, and conversely. In short, this is an anticoordination game.

While the two-person game captures an important aspect of the problem, it will be worthwhile to make our model a little more realistic by allowing for more than two patients, for different health states among patients, and for errors and uncertainty. As a first step the model is extended to N agents, as a numerical example. (Compare McCain, 2004, pp. 158-162). Suppose we have ten potential patients and that all are alike (this is a counterfactual simplifying assumption) so that their benefits from medical care in the ED differ only to extent that their waiting times differ. Waiting time is roughly proportional to the position one has in line, but the position in line cannot be predicted – there is some probability that I will be first, some that I will be last, depending on when I and others arrive. Suppose that the person first in line derives a satisfaction of 10 from the medical services of the ED, that being one place further back in the line reduces this satisfaction by two, and that the alternative to the ED provides a satisfaction level of five. We then have Table 2.

Table 2. The Patients' Coordination Game with a Population of 10

Number of ED Patients and Position in line	net benefit	average position in line	average net benefit	benefit of alternative
1	10	1	10	5
2	8	1.5	9	5
3	6	2	8	5
4	4	2.5	7	5
5	2	3	6	5
6	0	3.5	5	5
7	-2	4	4	5
8	-4	4.5	3	5
9	-6	5	2	5
10	-8	0	1	5

Let the number of these people who choose the ED be the state variable[4] for this larger game. The second column gives the benefit as it depends on the position in line. The third and fourth show average line position and average net benefit as they decline with the number choosing the ED. What we can see is that, if less than six patients choose the ED, then the ED is an individual's best response to the decisions of others; while if more than six choose the ED, the best response is to choose the alternative. Thus, for a Nash equilibrium to occur, just six choose the ED while the other four choose their alternative. This example is still not very realistic, as the population would be much

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[4] This term is adapted from the theory of differential games to refer to a variable that determines the payoffs of alternative strategies without any further information.

bigger in the real world, patients would be heterogeneous, there might be more than one hospital ED to choose from, and the penalty of standing in line would probably be less relative to the benefits of treatment. Nevertheless the larger example illustrates three conclusions which will extend (with a more formal treatment) to a realistic general case:

1) Just as in the two-person anticoordination game, equilibrium requires some agents to choose different strategies even if they themselves do not differ.
2) When the strategies are modes of service, the number choosing the different services in equilibrium will be such that the different services yield the same benefits, in expected value terms.
3) The equilibrium is not efficient, in general. In this example, the fourth person using the ED adds only 4 units of net benefits, while the alternative service yields 5. Thus, the efficient patient population for the ED in the example is 3. Queuing for service leads to inefficient congestion. We should add that queuing is difficult to avoid for a service, like emergency medical care, the mission of which is inconsistent with other forms of rationing that (while more efficient) would require that some people in need of medical care be turned away. However, it does mean that the ED is likely to be a relatively inefficient means of providing routine medical care, over and above its mission.

Nevertheless, we cannot expect that expanding the capacity of the ED will eliminate the overcrowding and inefficiency. Just the contrary: an increase in the capacity of the ED (so that there are two queues, for example, or that the queue moves twice as fast) will be offset by an increase in the number using the ED, to the point that the expected benefit of ED care is again the same as the expected benefit of the alternative service, i.e. 5 in Table 2. This is likely to mean increased waste, in that a larger proportion of the population chooses the less efficient mode of service. (This assumes that alternative medical care is not rationed by queuing, and so is less inefficient).

Agent-Based Computer Simulation

To further extend the model and allow for 1) much larger numbers of potential patients, 2) heterogeneity of health states, experience, and expectation, 3) boundedly rational learning, and 4) initialization effects, dynamic adjustment and transients, we undertook agent-based computer simulation. (Holland and Miller 1991, Tesfatsion 2000) The formal structure of this larger model is expressed as computer program code[5]. For these simulations, the agents are potential patients, while the ED is not a "player in the game" but a mechanism that mediates the interaction of the agents. It is assumed that (at each iteration of the simulation) agents are randomly sorted into four health states. The largest group, in state zero, have no health concerns that would lead them to consider seeking medical care either from the ED or the alternative mode. Agents in states 1 and 3 have health concerns such that treatment in the ED offers a higher benefit than the alternative in the absence of congestion. Recall that the difference in benefits may reflect the inconvenience of the alternative as well as a condition that requires emergency treatment. For agents in state 2, there is a health concern such that treatment through the alternative mode offers higher expected benefit than treatment by the ED, even in the absence of congestion. For all three types, experience of treatment in the ED depends on congestion and a random term. Each agent chooses the mode that offers the greater

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[5] The compiled software is available from the authors on request. The computer code is included as Appendix 1. The current version runs only on a Macintosh computer under System X.

expectation of benefits[6], with the expectations formed by trial-and-error learning in which the expectation approaches the average experience via a Koyck (1954) updating procedure.

The results can best be illustrated by reviewing some representative simulations. Figure 1 shows the experience of agents of type 1 who choose the ED and those who choose the alternative over 500 successive iterations of the simulation. In this simulation agents of type 1 receive a benefit that averages 3 times as great from ED medical care than from the alternative, in the absence of crowding. However, as we see, crowding reduces the benefit for agents of type 1 to approximate equality with that from the alternative mode, as in simpler Nash equilibrium models. We note moreover that the expected benefit from ED care reported by agents who have chosen the ED remains above the true value. This occurs because of the heterogeneity of expectations. All of these pseudodata are reported only for agents who chose the ED. On the whole, the agents who choose the ED are the agents who have relatively favorable expectations for it, while those in the original population with less favorable expectations do not choose the ED and thus are not polled. Accordingly, we would expect to see their reported expectations biased upward, as here. The ability to allow for heterogeneity is a major advantage of agent-based computer simulation, and as we will see, can be important in interpreting empirical data.

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[6] A minimum of 5% of potential patients choose the ED regardless of expected benefits. This exception is necessary to avoid trap phenomena in a trial-and-error learning model and should be thought of as a minimum of exploratory behavior.

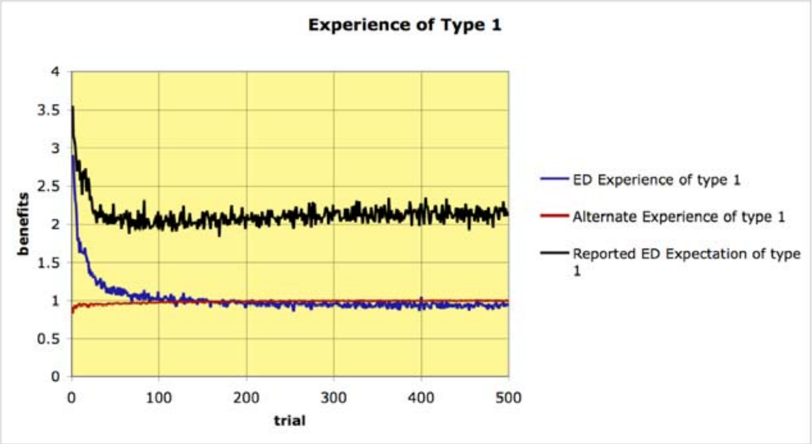

Figure 1. Some Results from a Simulation

These tendencies are observed also for agents of type 3, and similar tendencies have been observed in a number of other, exploratory simulations.[7] However, the tendencies observed for agents of type 2 were different. Figure 2 shows (for agents of type 2) the same results as Figure 1. Here, we see no convergence of the ED experience to the expected value of the alternative. Recall, for agents of type 2, the ED is the wrong choice even if there is no congestion. Conversely, those few agents of type 2 who nevertheless choose the ED on a given round do so primarily on the basis of highly biased estimates of the benefits of doing so.

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[7] Appendix 2 gives systematic discussion of a series of simulations with differing random number series.

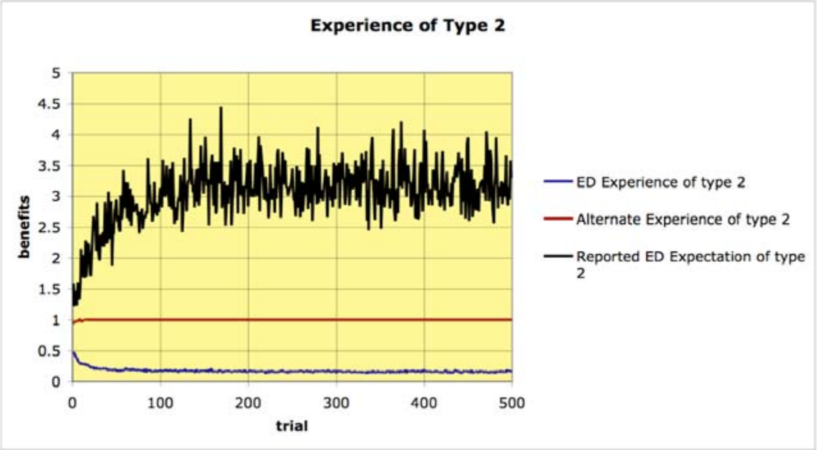

Figure 2. Further Results from the Same Simulation

A key suggestion from the simpler noncooperative game models was that an expansion of the capacity of the ED would not eliminate overcrowding but would only reproduce it on a larger scale. In the simulation just examined the capacity of the ED was 500 for a potential patient population of between 6,500 and 7,500 on any given iteration. We repeated the simulation with only the capacity changed, and the capacity doubled to 1000. Consider Figure 3, which shows the experience of agents of type 1, in direct parallel to Figure 1. The expansion of the ED capacity has a visible, but very slight, tendency to improve the experience of agents of type 1, due to the recurrence of approximately equilibrial crowding.

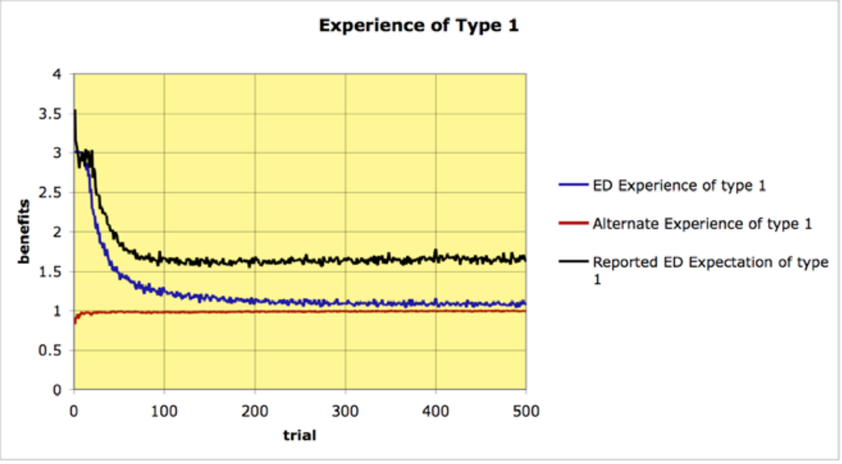

Figure 3. Type A in a Simulation with Doubled Capacity

Survey Method and Results

To test this model, a phone survey was conducted of patients who visited the ED of an urban university teaching hospital. The complete script used for the survey is shown in Appendix 3. These phone surveys were conducted by an independent research group who were given a list of all patients who had visited the ED during summer, 2007. Names and telephone numbers were randomly chosen from this list to complete 301 interviews. Interviewers made up to six calls, on different days and at different times, to reach and interview 301 patients.

Seven survey items were used to assess patient satisfaction. These items asked the patient to evaluate his/her satisfaction with the perceived quantity and promptness of care, the administrative and medical staff's effectiveness and consideration, staff time spent, and overall satisfaction with her or his care (sample item: "How satisfied were you with the promptness of care you received at the ED?"). A five point, Likert-type response scale was used for each item, ranging from 1 = Very satisfied to 5 = Very dissatisfied (the scores were reverse scored for the analyses, so higher numbers reflected greater satisfaction).

The same facets of satisfaction were also used to assess the patient's expected experience in the ED and the patient's expected experience with an alternative mode of care (sample items: "How satisfied did you expect to be with the promptness of care you were going to receive in the ED?" and "Keeping in mind the facility you most likely would have gone to if you had not visited the ED: How satisfied would you have expected to be with the promptness of care?"). In addition to these individual items, we also created three multi-item scales measuring overall: satisfaction with the ED, expected satisfaction with the ED and expected satisfaction with the alternative mode of care. The means of the seven items were used for each scale and each showed strong internal consistency (satisfaction with ED, $\alpha = .96$; expected satisfaction with ED, $\alpha = .95$; expected satisfaction with alternative mode of care, $\alpha = .95$). Note that since only patients who actually used the ED were surveyed, we have no data on the experienced satisfaction of the users of the alternative, nor on their expectations for the two modes[9].

Paired t-tests were used to assess differences in satisfaction levels between what the patients experienced at the ED and expected satisfaction with the alternative (Table 2), as well as the difference in the expected and experienced satisfaction with the ED (Table 3). In addition to the paired means for each item and the overall satisfaction scale, the tables show t-statistics and p-values The sign of the t-statistics is positive if the expectation for the alternative was greater (by however little margin) than the reported experience in the ED. The equilibrium hypothesis is that the there would be no

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[9] It would be possible, in the simulations, to determine these things; but the simulations were designed to produce pseudodata that correspond to the data of the survey.

statistically significant differences between the scores. The tests for difference of means are consistent with this hypothesis. Conventionally, the difference of means would be considered "statistically significant" if the p-value is less than 5%. This is true of every comparison except the promptness of care, in which "significantly" more satisfaction was expected of the alternative than of the ED. However, it is overall satisfaction that directly tests the equilibrium hypothesis, since overall satisfaction is the motive for choosing one alternative over the other. Moreover, the underlying mechanism – that equilibrium is brought about by increasing congestion, which offsets other advantages of the ED – is consistent with the statistically significant difference for promptness of care.

Table 2. Comparison of Experienced Satisfaction from ED with Expected Satisfaction from Alternative

Criterion	Expected from alternative	Experienced at the ED	t	p-value
Overall Quality of Care	3.83	3.77	.534	0.593
Quantity of Care	3.77	3.86	-.822	0.412
Promptness of Care	3.71	3.46	2.04	0.042
Administrative Staff Quality	3.79	3.87	-.728	0.467
Staff Ability to Resolve Your Medical Condition	3.89	3.91	-.164	0.870
Personal Care and Consideration	3.96	3.93	.265	0.791
Amount of Staff Time Spent	3.69	3.60	.892	0.373
Overall satisfaction	3.80	3.77	.379	0.134

Table 3 shows a similar series of comparisons for the expected and experienced satisfaction with ED care. The t statistic is positive if the expected satisfaction with ED care is greater than the experienced satisfaction. As we have noted, positive differences are to be expected if the expectations of the population polled were heterogeneous. These results are consistent with the assumption of heterogeneity.

Table 3. Comparisons of Expected and Experienced Satisfaction with ED Service

Criterion	Expected from the ED	Experienced at the ED	t	p-value
Overall Quality of Care	4.27	3.76	6.02	0.000
Quantity of Care	4.26	3.84	5.59	0.000
Promptness of Care	3.99	3.45	6.08	0.000
Administrative Staff Quality	4.15	3.86	4.05	0.000
Staff Ability to Resolve Your Medical Condition	4.41	3.93	6.15	0.000
Personal Care and Consideration	4.32	3.93	5.23	0.000
Amount of Staff Time Spent	4.06	3.60	5.34	0.000
Overall satisfaction	4.21	3.77	6.40	0.000

All in all, the results of the survey are consistent with the predictions arising from the agent-based simulation of the Nash equilibrium model for ED overcrowding.

Concluding Summary

The project reported in this paper was highly interdisciplinary, drawing ideas and techniques from several sources. There are novel contributions for each.

- For health care policy, we have specified, tested, and verified a Nash equilibrium hypothesis of the cause and nature of ED overcrowding. This hypothesis implies that increasing ED capacity may have little or no impact on overcrowding, in the absence of important changes in access to the alternative modes of medical care.
- For game theory, we have provided an example of testing a game-theoretic equilibrium model by questionnaire methods, using a realistically scaled agent-based computer simulation with boundedly rational learning to extend the insights of two- and small N-person game models to generate hypotheses for the survey.
- For questionnaire methods, we have provided an example of application to hypotheses from game theory and some evidence of the importance and

consequences of heterogeneity, and the possibility of modeling heterogeneity explicitly by means of agent-based computer simulation.

For practical purposes, the policy implications would seem to be the most important, and a major limitation is that they reflect data from only one hospital. In future research, we hope to extend the questionnaire studies to a larger sample including hospitals in different regions and in different social contexts, e.g. rural and suburban as well as urban.

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